ENGI 4421
Term Test 2
2019 July 11

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1. The lengths X of a type of girder are known to follow a normal distribution with a population mean of 3.100 m and a population standard deviation of 0.005 m.
   (a) Find the probability that the next girder has a length less than 3.095 m. [3]
   (b) Find the probability that the difference D between the lengths of the next *two* girders is greater than 0.005 m. Do **not** use interpolation. State any assumption that you need to make. Quote your final answer to two significant figures. [7]

[Tables of the standard normal cdf were provided with the question paper.]

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2. A pair of random quantities X, Y has the joint probability mass function $p(x, y)$ given by the table below.

$p(x,y)$		y			
		–1	0	1	
	–1	.10	.04	.06	
x	0	.15	.01	.14	
	1	.25	.15	.10	

   (a) Complete the table to display the marginal probability functions $p_X(x)$ and $p_Y(y)$. [2]
   (b) Find $\mathrm{P}[Y < 0 \mid X > 0]$ (the probability that Y is negative given that X is positive). [3]
   (c) Calculate the covariance of X and Y, Cov[X, Y]. [3]
   (d) Are the random quantities X, Y independent? Why or why not? [2]

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3. The distance x from an observer to the top of a tree is related to the angle θ (between that line of sight and the horizontal), and the known distance $b = 40 \text{ m}$ (between the observer and the base of the tree), by [9]

$$x = b \sec \theta$$

The measured angle is reported to be $\theta = (28.41 \pm 0.12)°$.

Find the uncertainty in the distance x correct to two decimal places.

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4. The time T from installation to failure of a particular type of valve in a high-volume pipe junction follows an exponential distribution and is independent of the times to failure of all other valves. Upon failure a valve is replaced immediately with a new valve whose lifetime follows the same probability distribution. The mean time to failure μ is known to be 168 hours.

(a) Write down the p.d.f. (probability density function) for T. [1]

(b) Find $P[T < 100]$. [2]

(c) Find the time $\tilde{\mu}_T$ (correct to the nearest hour) before which half of all valves fail. [3]

(d) The number N of these valves that fail in 2,184 hours (one quarter of a year) follows a Poisson distribution. Show that $E[N] = 13$. [2]

(e) Find the probability that at least ten valves fail during the quarter. [3]

[A table of the Poisson pmf and cdf for $\mu = 13$ was provided with the question paper.]

BONUS QUESTION

5. For the joint probability density function [+5]

$$f(x, y) = k(ax + by), \qquad (0 \le x \le 1 \text{ and } 0 \le y \le 1)$$

where a, b are positive constants, find the value of k (in terms of a and b) **and** determine whether or not the random quantities X, Y are independent.

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