ENGI 4421

Term Test 2

2020 July 09

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1. The joint probability mass function $p(x, y)$ of two Bernoulli random quantities X, Y is displayed in this table.

$p(x,y)$		y	
		0	1
x	0	0.3	0.1
	1	0.5	0.1

(a) Find the covariance of X, Y. [8]

(b) Are X, Y independent? [2]

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2. A truck contains 18 good sections of culvert lining and 2 defective sections.
A random sample of 4 sections is taken from the truck.
Let X = the number of defective sections in the random sample.

(a) Is the probability distribution of X exactly binomial, approximately binomial or not binomial at all? [4]

(b) Find the probability that there are at least two defective sections in the sample. [8]

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3. The breaking load W of a cable from a production line is known to be a normally distributed random quantity with mean $\mu = 500.0$ N and standard deviation $\sigma = 5.0$ N. The breaking load of each cable is independent of the breaking loads of all other cables manufactured on this production line. Find the probability that

(a) the next cable has a breaking load less than 492.0 N. [3]

(b) the average breaking load $\overline{W}$ of the next 25 cables is more than 501.0 N. [4]
[You may assume that $\overline{W}$ is normally distributed.]

(c) the difference between the breaking loads of the next two cables exceeds 10.0 N. [7]

Note: You do ***not*** need to use linear interpolation in this question.
The table of the standard normal c.d.f. was provided with the question paper.

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4. A continuous random quantity X has the probability density function (pdf)

$$f(x) = \frac{18}{(x+2)^3} \quad (x>1)$$

(a) Show that the cumulative distribution function (cdf) is [3]

$$F(x) = \begin{cases} 0 & (x \le 1) \\ 1 - \dfrac{9}{(x+2)^2} & (x>1) \end{cases}$$

(b) Show that $f(x)$ is a valid probability density function. [3]

(c) Find the median value $\tilde{\mu}$ of X. [3]

(d) Find the population mean value $\mu = \mathrm{E}[X]$. [5]

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5. *BONUS QUESTION* [+6]

The rate Q (cm^3/s) at which water flows out from a pipe of circular cross section, radius r (cm), when the bulk speed of the water flow inside the pipe is v (cm/s), is

$$Q = \pi r^2 v$$

Find the uncertainty of the estimate of the bulk speed of the water flow v, given the estimates Q = (824.7 ± 20.0) cm^3/s and r = (4.9 ± 0.1) cm.
You may assume independence between Q and r.

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