ENGI 4421
Term Test 2
2021 July 08

1. A function of a ***continuous*** variable x is defined by

$$f(x) = \begin{cases} \frac{1}{12}\left(x^2 - 6x + 8\right) & (0 \le x \le 6) \\ 0 & (\text{otherwise}) \end{cases}$$

Can $f(x)$ be a probability density function? Why or why not? [8]

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2. A storage yard contains 570 good steel beams and 30 defective beams.
A random sample of 5 beams is taken from the yard.
Let X = the number of defective beams in the random sample.

(a) Is the probability distribution of X exactly binomial, approximately binomial or not binomial at all? [4]

(b) Find the probability that there are at least two defective sections in the sample. [8]

(c) Find the mean $\mathrm{E}[X]$ and variance $\mathrm{V}[X]$ of X. [4]

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3. Two independent sets of measurements of the times to failure of stressed beams are taken, one with an old clock and the other with a somewhat improved device. [12]
The results are (702.52 ± 0.80) s and (703.38 ± 0.60) s.

Combine these measurements to find the estimate of mean failure time that has the minimum uncertainty and find that minimum uncertainty.

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4. The average time between consecutive maintenance call-outs to a pumping station is $\mathrm{E}[T] = 30$ days. Call-outs are independent of each other. The time T to the next call-out follows an exponential distribution.

(a) Find the probability, correct to three significant figures, that the next call-out occurs within 21 days of the most recent call-out. [5]

(b) Find the probability, correct to three significant figures, that more than two call-outs occur during the next 90 days. [9]

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5. *Bonus Question* [+4]
Items are sampled randomly from a very large population and tested until the third defective item is found. It is known that the proportion of defective items in the population is p. Let X be the number of items that have been tested when the third defective item is found. Derive the probability mass function $p(x)$ for X.

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