Appendix to the Lecture Notes for ENGI 4421 Probability and Statistics

Some Statistical Tables

On the next five pages are the three most frequently used statistical tables:

- critical values of the t distribution;
- values of the standard normal cumulative distribution function $\Phi(z)$; and
- critical values of the χ^2 distribution.

A copy of these tables will be provided in the final examination (and, if any question requires them, in Test 2 also).

These and other tables can be found, as Excel spreadsheet files, at the web site

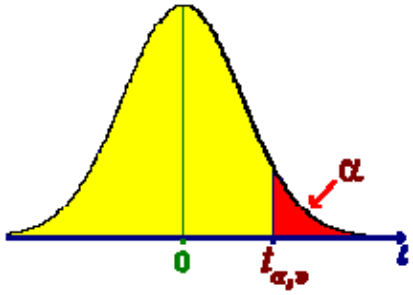
`www.engr.mun.ca/~ggeorge/4421/demos/`

Note the distinction between the tables displayed in this appendix:

In the t-tables, for chosen values of ν and α, values of t are tabulated, such that

$$\mathrm{P}[T_\nu > t] = \alpha,$$

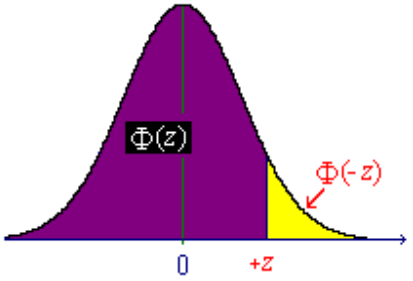
where T_ν is a random quantity following the t distribution with ν degrees of freedom.
The values of the c.d.f. are therefore the complements of the values in the top margin (the column headers). The χ^2 tables are similar.



In the z-tables, for chosen values of z, values of α are tabulated, such that

$$\mathrm{P}[Z < z] = \alpha \quad \text{or, equivalently,}$$

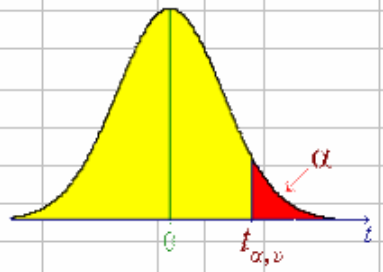
values of the function $\Phi(z)$ are tabulated.
The values of the c.d.f. are therefore in the body of the table.



Also note that three very small Excel spreadsheet files on the web site are of much more use in practice (but will not be available in the final examination):
`tCalculator.xls`, `chiSqCalculator.xls`, and `zCalculator.xls`.
In these spreadsheets, the c.d.f. $\alpha = F(t)$ [or $\Phi(z)$] can be found for any chosen t [or z] and the inverse c.d.f. $t = F^{-1}(\alpha)$ [or $z = \Phi^{-1}(\alpha)$] can be found for any chosen probability α.

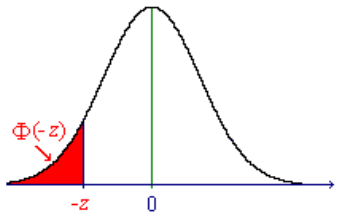
α = P[$T > t_{\alpha,\nu}$] (= right-tail probability)

ν = # degrees of freedom



	α											
ν	**0.1**	**0.05**	**0.025**	**0.01**	**0.005**		ν	**0.1**	**0.05**	**0.025**	**0.01**	**0.005**
1	3.07768	6.31375	12.70615	31.82096	63.65590		**26**	1.31497	1.70562	2.05553	2.47863	2.77872
2	1.88562	2.91999	4.30266	6.96455	9.92499		**27**	1.31370	1.70329	2.05183	2.47266	2.77068
3	1.63775	2.35336	3.18245	4.54071	5.84085		**28**	1.31253	1.70113	2.04841	2.46714	2.76326
4	1.53321	2.13185	2.77645	3.74694	4.60408		**29**	1.31143	1.69913	2.04523	2.46202	2.75639
5	1.47588	2.01505	2.57058	3.36493	4.03212		**30**	1.31042	1.69726	2.04227	2.45726	2.74998
6	1.43976	1.94318	2.44691	3.14267	3.70743		**31**	1.30946	1.69552	2.03951	2.45283	2.74404
7	1.41492	1.89458	2.36462	2.99795	3.49948		**32**	1.30857	1.69389	2.03693	2.44868	2.73849
8	1.39682	1.85955	2.30601	2.89647	3.35538		**33**	1.30774	1.69236	2.03452	2.44479	2.73329
9	1.38303	1.83311	2.26216	2.82143	3.24984		**34**	1.30695	1.69092	2.03224	2.44115	2.72839
10	1.37218	1.81246	2.22814	2.76377	3.16926		**35**	1.30621	1.68957	2.03011	2.43772	2.72381
11	1.36343	1.79588	2.20099	2.71808	3.10582		**36**	1.30551	1.68830	2.02809	2.43450	2.71948
12	1.35622	1.78229	2.17881	2.68099	3.05454		**37**	1.30485	1.68709	2.02619	2.43144	2.71541
13	1.35017	1.77093	2.16037	2.65030	3.01228		**38**	1.30423	1.68595	2.02439	2.42857	2.71157
14	1.34503	1.76131	2.14479	2.62449	2.97685		**39**	1.30364	1.68488	2.02269	2.42584	2.70791
15	1.34061	1.75305	2.13145	2.60248	2.94673		**40**	1.30308	1.68385	2.02107	2.42326	2.70446
16	1.33676	1.74588	2.11990	2.58349	2.92079		**50**	1.29871	1.67591	2.00856	2.40327	2.67779
17	1.33338	1.73961	2.10982	2.56694	2.89823		**60**	1.29582	1.67065	2.00030	2.39012	2.66027
18	1.33039	1.73406	2.10092	2.55238	2.87844		**80**	1.29222	1.66413	1.99007	2.37387	2.63870
19	1.32773	1.72913	2.09302	2.53948	2.86094		**100**	1.29008	1.66023	1.98397	2.36421	2.62589
20	1.32534	1.72472	2.08596	2.52798	2.84534		**200**	1.28580	1.65251	1.97189	2.34513	2.60063
21	1.32319	1.72074	2.07961	2.51765	2.83137		**1000**	1.28240	1.64638	1.96234	2.33008	2.58075
22	1.32124	1.71714	2.07388	2.50832	2.81876							
23	1.31946	1.71387	2.06865	2.49987	2.80734		∞	**1.28155**	**1.64485**	**1.95996**	**2.32635**	**2.57583**
24	1.31784	1.71088	2.06390	2.49216	2.79695							
25	1.31635	1.70814	2.05954	2.48510	2.78744							

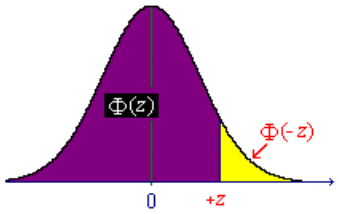
Tabulated is
$\Phi(z) = P[Z \le z]$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
–3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
–2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
–2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
–2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
–2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
–2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
–2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
–2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
–2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
–2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
–2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
–1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
–1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
–1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
–1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
–1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
–1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
–1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08692	0.08534	0.08379	0.08226
–1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
–1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
–1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
–0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
–0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
–0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
–0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
–0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
–0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
–0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
–0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
–0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
–0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414

–3.5	0.00023267337
–4.0	0.00003168603
–5.0	0.00000028710
–6.0	0.00000000099
–7.0	1.28808E-12

Tabulated is
$\Phi(z) = P[Z \le z]$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91308	0.91466	0.91621	0.91774
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900
3.5	0.99976732663									
4.0	0.99996831397									
5.0	0.99999971290									
6.0	0.99999999901									
7.0	1.00000000000									

			α		
ν	0.995	0.99	0.975	0.95	0.90
1	0.00004	0.00016	0.00098	0.00393	0.01579
2	0.01003	0.02010	0.05064	0.10259	0.21072
3	0.07172	0.11483	0.21580	0.35185	0.58437
4	0.20699	0.29711	0.48442	0.71072	1.06362
5	0.41174	0.55430	0.83121	1.14548	1.61031
6	0.67573	0.87209	1.23734	1.63538	2.20413
7	0.98926	1.23904	1.68987	2.16735	2.83311
8	1.34441	1.64650	2.17973	2.73264	3.48954
9	1.73493	2.08790	2.70039	3.32511	4.16816
10	2.15586	2.55821	3.24697	3.94030	4.86518
11	2.60322	3.05348	3.81575	4.57481	5.57778
12	3.07382	3.57057	4.40379	5.22603	6.30380
13	3.56503	4.10692	5.00875	5.89186	7.04150
14	4.07467	4.66043	5.62873	6.57063	7.78953
15	4.60092	5.22935	6.26214	7.26094	8.54676
16	5.14221	5.81221	6.90766	7.96165	9.31224
17	5.69722	6.40776	7.56419	8.67176	10.08519
18	6.26480	7.01491	8.23075	9.39046	10.86494
19	6.84397	7.63273	8.90652	10.11701	11.65091
20	7.43384	8.26040	9.59078	10.85081	12.44261
21	8.03365	8.89720	10.28290	11.59131	13.23960
22	8.64272	9.54249	10.98232	12.33801	14.04149
23	9.26042	10.19572	11.68855	13.09051	14.84796
24	9.88623	10.85636	12.40115	13.84843	15.65868
25	10.51965	11.52398	13.11972	14.61141	16.47341
26	11.16024	12.19815	13.84391	15.37916	17.29189
27	11.80759	12.87850	14.57338	16.15140	18.11390
28	12.46134	13.56471	15.30786	16.92788	18.93924
29	13.12115	14.25645	16.04707	17.70837	19.76774
30	13.78672	14.95346	16.79077	18.49266	20.59923
31	14.45777	15.65546	17.53874	19.28057	21.43356
32	15.13403	16.36222	18.29076	20.07191	22.27059
33	15.81527	17.07351	19.04666	20.86653	23.11020
34	16.50127	17.78915	19.80625	21.66428	23.95225
35	17.19182	18.50893	20.56938	22.46502	24.79666
36	17.88673	19.23268	21.33588	23.26861	25.64330
37	18.58581	19.96023	22.10563	24.07494	26.49209
38	19.28891	20.69144	22.87848	24.88390	27.34295
39	19.99587	21.42616	23.65432	25.69539	28.19579
40	20.70654	22.16426	24.43304	26.50930	29.05052
> 40	$\chi^2_{\alpha,\nu} \approx \nu\left(1 - \frac{2}{9\nu} + z_\alpha\sqrt{\frac{2}{9\nu}}\right)^3$				

			α		
ν	0.10	0.05	0.025	0.01	0.005
1	2.70554	3.84146	5.02389	6.63490	7.87944
2	4.60517	5.99146	7.37776	9.21034	10.59663
3	6.25139	7.81473	9.34840	11.34487	12.83816
4	7.77944	9.48773	11.14329	13.27670	14.86026
5	9.23636	11.07050	12.83250	15.08627	16.74960
6	10.64464	12.59159	14.44938	16.81189	18.54758
7	12.01704	14.06714	16.01276	18.47531	20.27774
8	13.36157	15.50731	17.53455	20.09024	21.95495
9	14.68366	16.91898	19.02277	21.66599	23.58935
10	15.98718	18.30704	20.48318	23.20925	25.18818
11	17.27501	19.67514	21.92005	24.72497	26.75685
12	18.54935	21.02607	23.33666	26.21697	28.29952
13	19.81193	22.36203	24.73560	27.68825	29.81947
14	21.06414	23.68479	26.11895	29.14124	31.31935
15	22.30713	24.99579	27.48839	30.57791	32.80132
16	23.54183	26.29623	28.84535	31.99993	34.26719
17	24.76904	27.58711	30.19101	33.40866	35.71847
18	25.98942	28.86930	31.52638	34.80531	37.15645
19	27.20357	30.14353	32.85233	36.19087	38.58226
20	28.41198	31.41043	34.16961	37.56623	39.99685
21	29.61509	32.67057	35.47888	38.93217	41.40106
22	30.81328	33.92444	36.78071	40.28936	42.79565
23	32.00690	35.17246	38.07563	41.63840	44.18128
24	33.19624	36.41503	39.36408	42.97982	45.55851
25	34.38159	37.65248	40.64647	44.31410	46.92789
26	35.56317	38.88514	41.92317	45.64168	48.28988
27	36.74122	40.11327	43.19451	46.96294	49.64492
28	37.91592	41.33714	44.46079	48.27824	50.99338
29	39.08747	42.55697	45.72229	49.58788	52.33562
30	40.25602	43.77297	46.97924	50.89218	53.67196
31	41.42174	44.98534	48.23189	52.19139	55.00270
32	42.58475	46.19426	49.48044	53.48577	56.32811
33	43.74518	47.39988	50.72508	54.77554	57.64845
34	44.90316	48.60237	51.96600	56.06091	58.96393
35	46.05879	49.80185	53.20335	57.34207	60.27477
36	47.21217	50.99846	54.43729	58.61921	61.58118
37	48.36341	52.19232	55.66797	59.89250	62.88334
38	49.51258	53.38354	56.89552	61.16209	64.18141
39	50.65977	54.57223	58.12006	62.42812	65.47557
40	51.80506	55.75848	59.34171	63.69074	66.76596
> 40	$\chi^2_{\alpha,\nu} \approx \nu\left(1 - \frac{2}{9\nu} + z_\alpha\sqrt{\frac{2}{9\nu}}\right)^3$				