FLUIDS AT REST

HYDRAULIC GATES

The pressure/weight method for hydraulic gates starts by boxing the gate with vertical and horizontal surfaces. The fluid within these surfaces is considered frozen to the gate. Then the horizontal and vertical pressure forces on the box surfaces are calculated. Force balances, which subtract the weight frozen to the gate, then give the horizontal and vertical forces on the gate. Moment balances give the location of the forces. The panel method for hydraulic gates starts by subdividing the surface of the gate into a finite number of flat panels. The pressure depth law gives the pressure at the centroid of each panel. Pressure times panel area gives the force at the centroid. The unit normal pointing at the panel allows one to break the force into components. Summation gives the total force on the gate in each direction. Moment balances give the location of the forces. If the gate has a pivot, a summation of each force times its moment arm gives the total moment about the pivot. The important equations for the panel method are:

$$\Delta\mathbf{F} = P\Delta A\ n_x\mathbf{i} + P\Delta A\ n_y\mathbf{j} + P\Delta A\ n_z\mathbf{k}$$

$$F_x = \Sigma\ P\Delta A\ n_x \qquad F_y = \Sigma\ P\Delta A\ n_y \qquad F_z = \Sigma\ P\Delta A\ n_z$$

$$\Sigma\ \mathbf{r} \times \Delta\mathbf{F} \qquad \mathbf{r} = r_x\mathbf{i} + r_y\mathbf{j} + r_z\mathbf{k}$$

$$P = \rho g h \qquad W = \rho g V$$

The panel force is $\Delta\mathbf{F}$. Its pivot moment is $\mathbf{r} \times \Delta\mathbf{F}$. Summations give total force components and the pivot moment.

METACENTER

The metacenter M occurs at the intersection of two lines. One line passes through the center of gravity or G and the center of buoyancy or B of a floating body when it is not rotated: the other line is a vertical line through B when the body is rotated. Inspection of a sketch of these lines shows that, if M is above G, gravity and buoyancy generate a restoring moment, whereas if M is below G, gravity and buoyancy generate an overturning moment. One finds the location of M by finding the shift in the center of buoyancy generated during rotation and noting that this shift could result from a rotation about an imaginary point which turns out to be the metacenter. The important equations are:

$$M_V = V\ S \qquad V = \Sigma\Delta V \qquad (\Sigma d\Delta V)/V$$

$$M_W = \int r\ r\Theta\ w\ dr = K\ \Theta \qquad S = BM\ \Theta$$

$$M_W = \Sigma\ r\ r\Theta\ w\ \Delta r = K\ \Theta \qquad GM = BM - BG$$

When a body is rotated by an angle Θ, M_V is the volume moment generated by the shift S in the center of buoyancy B, and M_W is the volume moment generated by the wedge shaped volumes created by rotation. Setting M_V equal to M_W gives an equation of the form $S = K/V\ \Theta$. Rotation of B about the metacenter M gives $S = BM\ \Theta$. The S equations give $BM = K/V$. The distance between B and the center of gravity G is BG. Geometry gives GM. If GM is positive, M is above G and the body is stable.

FLUIDS IN MOTION

SCALING LAWS

Scaling laws allow prototype behavior to be predicted from model data. For a pump, it is customary to let N be the rotor RPM and D be the rotor diameter. All flow speeds U scale as ND and all areas A scale as D^2. Pressures are set by the dynamic pressure $\rho U^2/2$. Ignoring constants, one can define a reference pressure $[\rho N^2 D^2]$ and a reference flow $[ND^3]$. Since fluid power is just pressure times flow, one can also define a reference power $[\rho N^3 D^5]$. Dividing dimensional quantities by reference quantities gives the scaling laws:

$$C_P = P \ / \ [\rho N^2 D^2] \qquad C_Q = Q \ / \ [ND^3] \qquad C_{\mathbf{P}} = \mathbf{P} \ / \ [\rho N^3 D^5]$$

These equations show that, if D of a pump is doubled, **P** increases 32 fold, whereas if N is doubled, **P** increases 8 fold. For bodies in a flow, important numbers are:

$$C_D = \mathbf{D}/(A[\rho U^2/2]) \qquad C_L = \mathbf{L}/(A[\rho U^2/2])$$

$$Re = UD/\nu \qquad Fr = U/\sqrt{[gL]} \qquad St = [D/U]/T$$

In these equations, **D** is drag and **L** is lift. These can be represented in dimensionless form as drag and lift coefficients C_D and C_L. These coefficients are often a function of Reynolds Number Re or Froude Number Fr. When flow has an oscillatory character, an important number is the Strouhal Number St. Re is a ratio of inertia and viscous forces in a flow. Fr is a ratio of inertia and gravity forces in a flow. St is a ratio of transit time and flow period.

CONSERVATION LAWS

Conservation of Mass states that the time rate of change of mass of a specific group of fluid particles in a flow is zero. Conservation of Momentum states that the time rate of change of momentum of a group of particles must balance with the net load acting on it. Conservation of Energy states that the time rate of change of energy of a group of particles must balance with heat and work interactions with its surroundings. Mathematically one can write:

Conservation of Mass

$$\frac{D}{Dt}\int_V \rho \, dV = \int_V \partial\rho/\partial t \, dV + \int_S \rho \, \mathbf{v}.\mathbf{n} \, dS = 0$$

Conservation of Momentum

$$\frac{D}{Dt}\int_V \rho\mathbf{v} \, dV = \int_V \partial\rho\mathbf{v}/\partial t \, dV + \int_S \rho\mathbf{v} \, \mathbf{v}.\mathbf{n} \, dS$$

$$= \int_S \boldsymbol{\sigma} \, dS + \int_V \rho\mathbf{b} \, dV$$

Conservation of Energy

$$\frac{D}{Dt}\int_V \rho e \, dV = \int_V \partial\rho e/\partial t \, dV + \int_S \rho e \, \mathbf{v}.\mathbf{n} \, dS$$

$$= - \int_S \mathbf{q}.\mathbf{n} \, dS + \int_S \mathbf{v}.\boldsymbol{\sigma} \, dS$$

In these equations, V is fluid volume, S is fluid surface area, t is time, **n** is outward unit normal, **v** is velocity, ρ is density, **σ** denotes surface stresses such as pressure and viscous traction, **b** denotes body forces such as gravity, e is energy density and **q** denotes heat flux.

For streamtube or pipe flow, the conservation laws reduce to

Conservation of Mass

$$\sum(\rho CA)_{OUT} - \sum(\rho CA)_{IN} = 0 \qquad \dot{M} = \rho CA$$

Conservation of Momentum

$$\sum(\rho \mathbf{v} CA)_{OUT} - \sum(\rho \mathbf{v} CA)_{IN} = - \sum(PA\mathbf{n})_{OUT} - \sum(PA\mathbf{n})_{IN} - \mathbf{R}$$

$$\sum \dot{M}\,(U_{OUT}-U_{IN})=F_x \qquad \sum \dot{M}\,(V_{OUT}-V_{IN})=F_y \qquad \sum \dot{M}\,(W_{OUT}-W_{IN})=F_z$$

Conservation of Energy

$$\sum(\rho eCA)_{OUT} - \sum(\rho eCA)_{IN} = \sum(CPA)_{IN} - \sum(CPA)_{OUT} + \sum\dot{T} - \sum\dot{L}$$

$$h = C^2/2g + P/\rho g + z \qquad e = u + C^2/2 + gz$$

$$h_{OUT} - h_{IN} = h_T - h_L \qquad Re=CD/\upsilon=\rho CD/\mu \qquad \varepsilon = e/D$$

$$h_L = (fL/D + \Sigma K)\ C^2/2g$$

In these equations, ρ is density, C is flow speed, A is pipe area, **v** is velocity, P is pressure, **n** is outward normal, U V W are velocity components, h is head, u is internal energy, g is gravity, f is pipe friction factor, L is pipe length, D is pipe diameter and K indicates losses at constrictions.

Mass Flow Rate: $\dot{M}$ Shaft Work: $\dot{T}$ Losses: $\dot{L}$

SYSTEM DEMAND

System Demand consists of two components: pressure/gravity head and head losses. Head H versus Flow Q is given by:

$$H = X + Y\,Q^2 \qquad Q = C\,A$$

$$X = \Delta\,(P/\rho g + z) \qquad Y = (fL/D + \Sigma K)/(2gA^2)$$

X accounts for piezometric or pressure/gravity head and Y accounts for losses due to friction and constrictions.

PUMP SELECTION

One first calculates the specific speed based on the system operating point. This is a nondimensional number which does not have pump size in it: $N\sqrt{Q}/H^{3/4}$. This allows one to pick the appropriate type of pump. Next one scans pump catalogs of the type indicated by specific speed and picks the size of pump that will meet the system demand, while it is operating at its best efficiency point (BEP) or best operating point (BOP). Finally, to prevent cavitation, the pump is located in the system at a point where it has the Net Positive Suction Head or NPSH recommended by the manufacturer:

$$NPSH = P_s/\rho g + U_s U_s/2g - P_v/\rho g$$

In this equation, P_v is the vapor pressure of the fluid being pumped, and P_s and U_s are pressure and speed at the pump inlet: they can be estimated from conservation of energy.

PIPE NETWORKS

In a pressure iteration method one would first assume pressure at each node in the network where it is not known. Then for each node one would assume pressures at the surrounding nodes to be fixed. Next for each pipe connected to the node one balances head loss with piezometric or pressure/gravity head: here pumps are treated as negative head losses while turbines are treated as positive head losses. This allows us to calculate the flow in each pipe and its direction. One then calculates the sum of the flows into the node treating flows in as positive and flows out as negative. If the $\Sigma Q>0$ then the node acts like a sink and the pressure there is too low and must be increased a bit. If the $\Sigma Q<0$ then the node acts like a source and the pressure there is too high and must be lowered a bit. Each node in the network is treated the same way. One sweeps through the network nodes again and again until the ΣQ for each node is approximately zero. In a flow iteration method one assumes a distribution of flow which satisfies $\Sigma Q=0$ at each node in the network. The flow iteration method modifies flows throughout the network in a way which maintains $\Sigma Q=0$ at each node. In the method one identifies pipe loops in the network. Then for each loop one calculates the sum of the head losses as one moves around it in a clockwise sense. If flow in a pipe is clockwise head loss is taken to be positive whereas if flow is counterclockwise head loss is taken to be negative. For a loop if the $\Sigma h_L>0$ then there is too much clockwise flow: so flows must be reduced a bit in a clockwise sense. This decreases clockwise flows and increases counterclockwise flows. If the $\Sigma h_L<0$ then there is not enough clockwise flow:

so flows must be increased a bit in a clockwise sense. This increases clockwise flows and decreases counterclockwise flows. Each loop in the network is treated the same way. One sweeps through the network loops again and again until the Σh_L for each loop is approximately zero. Special pseudo loops are used to connect reservoirs.

TURBOMACHINES

Swirl is the only component of fluid velocity that has a moment arm around the axis of rotation or shaft of a turbomachine. Because of this, it is the only one that can contribute to shaft power. The shaft power equation is:

$$\mathbf{P} = \Delta\ (T\ \omega) = \Delta\ (\rho Q\ V_t\ \ R\ \omega)$$

The swirl or tangential component of fluid velocity is V_t. The symbol Δ indicates we are looking at changes from inlet to outlet. The tangential momentum at an inlet or an outlet is $\rho Q\ V_t$. Multiplying momentum by moment arm R gives the torque T. Multiplying torque by the speed ω gives the power **P**. The power equation is good for pumps and turbines. Power is absorbed at an inlet and expelled at an outlet. If the outlet power is greater than the inlet power, then the machine is a pump. If the outlet power is less than the inlet power, then the machine is a turbine. Geometry can be used to connect V_t to the flow rate Q and the rotor speed ω.