TURBULENT HYDRODYNAMIC FLOWS

CONSERVATION OF X MOMENTUM

$$\rho \left(\partial U/\partial t + U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z \right) + A = - \partial P/\partial x$$

$$+ \left[\partial/\partial x \left(\mu\, \partial U/\partial x\right) + \partial/\partial y \left(\mu\, \partial U/\partial y\right) + \partial/\partial z \left(\mu\, \partial U/\partial z\right) \right]$$

CONSERVATION OF Y MOMENTUM

$$\rho \left(\partial V/\partial t + U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z \right) + B = - \partial P/\partial y$$

$$+ \left[\partial/\partial x \left(\mu\, \partial V/\partial x\right) + \partial/\partial y \left(\mu\, \partial V/\partial y\right) + \partial/\partial z \left(\mu\, \partial V/\partial z\right) \right]$$

CONSERVATION OF Z MOMENTUM

$$\rho \left(\partial W/\partial t + U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z \right) + C = - \partial P/\partial z - \rho g$$

$$+ \left[\partial/\partial x \left(\mu\, \partial W/\partial x\right) + \partial/\partial y \left(\mu\, \partial W/\partial y\right) + \partial/\partial z \left(\mu\, \partial W/\partial z\right) \right]$$

CONSERVATION OF MASS

$$\partial P/\partial t + \rho\, c^2 \left(\partial U/\partial x + \partial V/\partial y + \partial W/\partial z \right) = 0$$

WATER SURFACE TRACKER

$$\partial F/\partial t + U\partial F/\partial x + V\partial F/\partial y + W\partial F/\partial z = 0$$

KINETIC ENERGY OF TURBULENCE

$$\partial k/\partial t + U\partial k/\partial x + V\partial k/\partial y + W\partial k/\partial z = T_P - T_D$$

$$+ \left[\partial/\partial x\ (\mu/a\ \partial k/\partial x) + \partial/\partial y\ (\mu/a\ \partial k/\partial y) + \partial/\partial z\ (\mu/a\ \partial k/\partial z) \right]$$

DISSIPATION RATE OF TURBULENCE

$$\partial \varepsilon/\partial t + U\partial \varepsilon/\partial x + V\partial \varepsilon/\partial y + W\partial \varepsilon/\partial z = D_P - D_D$$

$$+ \left[\partial/\partial x\ (\mu/b\ \partial \varepsilon/\partial x) + \partial/\partial y\ (\mu/b\ \partial \varepsilon/\partial y) + \partial/\partial z\ (\mu/b\ \partial \varepsilon/\partial z) \right]$$

PRODUCTION AND DISSIPATION FUNCTIONS

$$T_P = G\ \mu_t / \rho \qquad D_P = T_P\ C_1\ \varepsilon / k$$

$$T_D = C_D\ \varepsilon \qquad D_D = C_2\ \varepsilon^2 / k$$

VISCOSITIES AND CONSTANTS

$$\mu_t = C_3\ k^2 / \varepsilon \qquad \mu = \mu_t + \mu_1$$

$C_D = 1.0$ $\qquad C_1 = 1.44$

$C_2 = 1.92$ $\qquad C_3 = 0.9$

$a = 1.0$ $\qquad b = 1.3$

TIME STEPPING

$$\partial M/\partial t = N \qquad M_{NEW} = M_{OLD} + \Delta t\ N_{OLD}$$