VEHICLE DECELERATION

The schematic on the next page shows a vehicle that is being decelerated by a scoop stuck in water. Let the mass of the vehicle be M and its speed be C. Conservation of Momentum for the scoop gives:

$$\rho\ Q\ [U_{OUT} - U_{IN}] = -\ R$$

Here $U_{OUT} = +C$ and $U_{IN} = -C$. Also $Q = C\ bh$ where b is the width of the scoop and h is the thickness of the sheet of water picked up by the scoop. One gets:

$$R = -\ 2\ \rho\ bh\ C^2$$

The equation of motion for the vehicle is:

$$M\ d^2C/dt^2 = R = -\ 2\ \rho\ bh\ C^2$$

Integration of this equation gives C as a function of time. Integration of C over time gives the distance S travelled by the vehicle. One gets:

$$S = \int C\ dt$$


Vehicle

WATER SPRINKLER TURBINE

A certain water sprinkler is used as a turbine. The 4 arms of the sprinkler have a radius of 0.5m. The diameter of each sprinkler pipe is 2cm. Each pipe has a 90° bend at its outlet. This bend makes an angle of 45° up from the horizontal. The overall flow rate is 8 L/s. Plot the power output of the turbine versus RPM. Comment on the plot.

There are two ways to work this problem. One is based on Conservation of Rotational Momentum for fluid moving in a rotating reference frame. This is:

$$\Sigma \mathbf{M} - \mathbf{M}_o = \partial/\partial t \int_V \mathbf{r} x (\rho \mathbf{V})\ dV + \int_S \mathbf{r} x (\rho \mathbf{V})\ (\mathbf{V}.\mathbf{n})\ dS$$

$$\mathbf{M}_o = \int_V \mathbf{r} x \mathbf{A}\ \rho dV$$

$$\mathbf{A} = d^2\mathbf{R}/dt^2 + 2\mathbf{\Omega} x \mathbf{V} + \mathbf{\Omega} x (\mathbf{\Omega} x \mathbf{r}) + (d\mathbf{\Omega}/dt) x \mathbf{r}$$

For the sprinkler problem $\Sigma \mathbf{M}$ is the brake torque.

A simpler way to work the problem is based on Conservation of Rotational Momentum for fluid moving in an inertial reference frame. This is:

$$\Delta\ [\rho Q\ V_t\ R\ ] = T$$

Here this reduces to

$$\rho Q\ [\ Q/A\ \text{Cos}(\theta)\ -\ R\omega\ ]\ R\ =\ T$$

The power is

$$\mathbf{P} = T\omega = \rho Q\ [\ Q/A\ \text{Cos}(\theta)\ -\ R\omega\ ]\ R\omega$$

Differentiation shows that the power peaks when the tip speed $R\omega$ is half the tangential component of the jet speed $Q/A\ \text{Cos}(\theta)$. For the given geometry and flow parameters, one gets approximately a peak power of 40 watts at a speed of 40 RPM. A power versus rpm plot for the sprinkler is given on the next page.

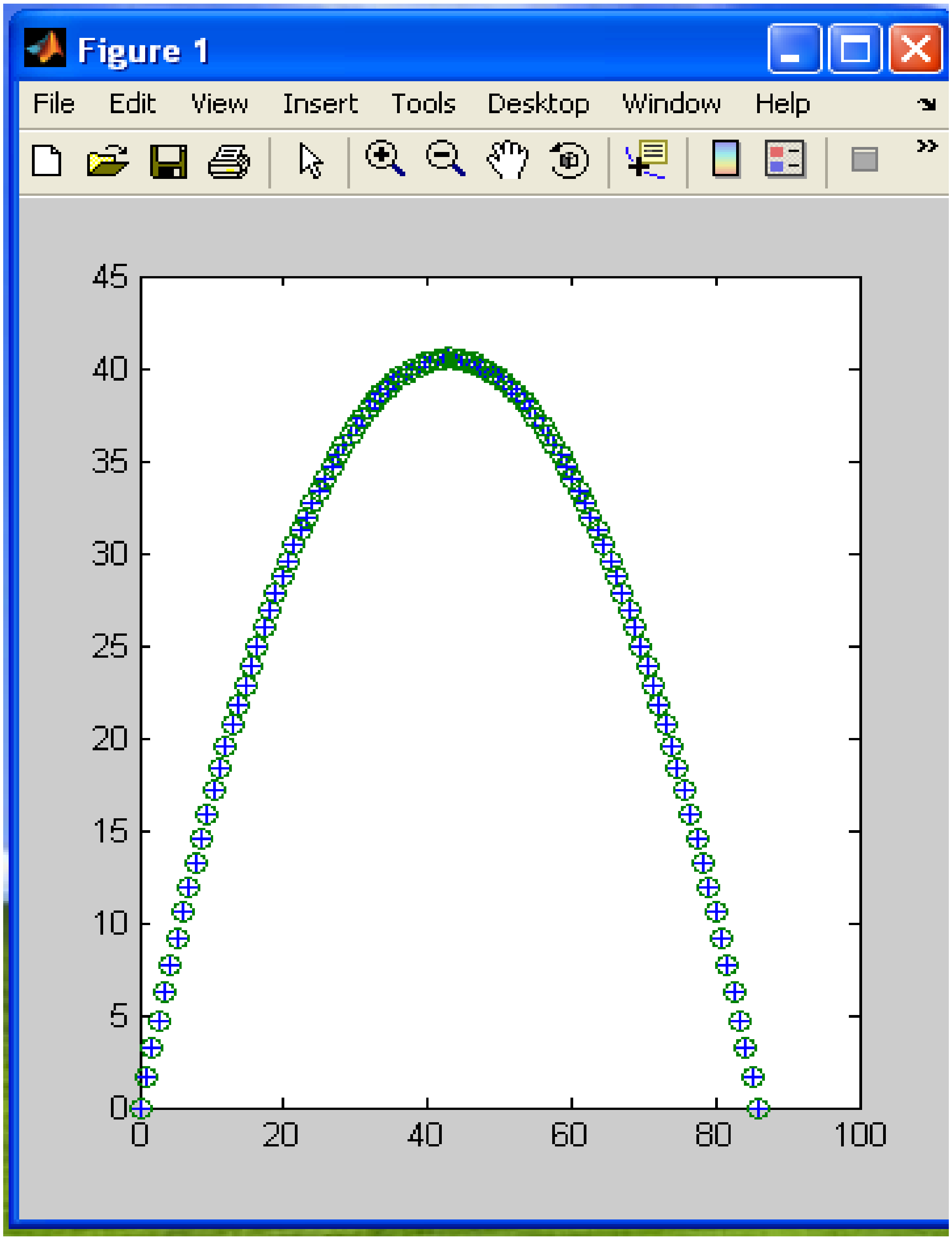

Figure 1
File Edit View Insert Tools Desktop Window Help
45
40
35
30
25
20
15
10
5
0
0 20 40 60 80 100

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% SPRINKLER TURBINE POWER
pi=3.14159; steps=100;
gravity=9.81; density=1000.0;
diameter=0.02; arm=0.5;
angle=45.0*pi/180.0; flow=0.002;
area=pi/4.0*diameter^2;
mass=density*flow;
speed=flow/area;
speed=speed*cos(angle);
free=speed/arm;
change=free/steps;
steps=steps+1;
spin=0.0;
for cycle=1:steps
 nozzle=arm*spin;
 vjet=speed-nozzle;
 watts(cycle)=mass*vjet*arm*spin;
 rpm(cycle)=spin/(2.0*pi)*60.0;
 a=+mass*speed*arm; b=-mass*spin*arm^2;
 torque=a+b; power(cycle)=torque*spin;
 spin=spin+change;
end
plot(rpm,watts*4,'+',rpm,power*4,'O')
```