POTENTIAL FLOWS

The theory of lifting bodies such as foils and wings is based on potential or ideal flow theory. This in turn is based on two major assumptions. First, the fluid is taken to be inviscid, which means it has zero viscosity. Second, the fluid motion is taken to be irrotational, which means each fluid particle does not spin on its own internal axis. Particles move through space like the carts on a Ferris Wheel. The fluid is said to be ideal. It can be compressible or incompressible. For hydrodynamic flows, the fluid can be taken to be incompressible.

When a body moves at steady speed through an ideal fluid, theory shows that the net load acting on the body is zero. This includes bodies that in reality have lift and drag forces acting on them. This is known as D'Alemberts Paradox. So it appears that ideal fluid theory is of little practical value.

Ideal fluid theory predicts that for a body shaped like a foil the fluid is able to turn the sharp corner at the trailing edge and move back over the top of the foil to join with fluid that moved around the leading edge and over the top. The two bits of fluid would pass through two stagnation or zero velocity points:

one on the bottom and one on the top. In reality, the fluid cannot turn the sharp corner at the rear. The fluid has to undergo infinite deceleration and acceleration to turn such a corner. Associated with this is an infinite suction pressure. As a real fluid tries to moves away from this into a higher pressure region on top of the foil, it moves inside a boundary layer. Within it, energy is taken from the fluid by viscous drag forces. The low to high pressure is known as an adverse pressure gradient. It turns out that fluid in a boundary layer would not be able to move into such a strong gradient and would be stopped at the trailing edge. The fluid is said to separate. The trailing edge becomes a stagnation point and a separation point. The fluid can be seen to leave the trailing edge smoothly. It turns out that the loads on the foil in this case are not zero. Note that this happens because of the behavior of a boundary layer, which is a mainly viscous phenomenon. This suggests that without viscosity wings would not work and present day airplanes would not be able to fly!

One can use a potential vortex to force the ideal flow over a foil to mimic a real flow. The vortex drags the stagnation point normally on top of the foil back to the trailing edge. When this is done, loads are no longer zero. A fundamental theorem of potential flow theory is that the net circulation or rotation in the flow must be constant. For a foil which started from rest, this would be zero. When a foil starts to move, a circulation is set up to make the flow leave the trailing edge smoothly. Theory suggests that an equal amount in opposite direction must be shed in a vortex sheet to keep the net circulation zero. Every time the circulation changes around the foil a vortex must be shed. These vortices are carried back from the foil by the flow. Each causes an upwash or a downwash on the foil depending on how it is rotating: its effect gets smaller as it is carried downstream by the flow.

Unsteady foil theory tries to account for the shed vortices. For certain special motions, such as a foil undergoing pure heave or pure rotation, analytical solutions have been developed. An analytical solution also exists for a foil moving at a steady speed through a sinusoidal gust. For foils with arbitrary motions, analytical solutions are impossible. For such cases, one must resort to numerical methods.

Probably the biggest effect of foil motion is it changes the apparent angle of attack of the foil. A heaving and pitching foil creates a flow onto itself. It also creates shed vortices. Load is always perpendicular to the apparent angle of attack so it can lean forward or backward. If it leans backward, it gives rise to a drag on the foil, whereas if it leans forward, it gives rise to a thrust on the foil.

An important parameter for oscillating foils is the ratio of the fluid transit time **T** divided by the oscillation period T. The transit time **T** is the speed of the flow S divided into the chord C. It gives a measure of how fast flow over a foil sets up. Common sense would suggest that if the time parameter is much less than unity then the flow should behave as a steady flow whereas if it is much greater than unity the flow would be very unsteady. In the literature, the time parameter is known as the Strouhal Number.

Vortices can only end at a wall or form loops. For steady flow over a wing, horseshoe shaped vortices are shed along the span of the wing because circulation varies along the span. These vortices are strongest at the tips of the wing. They are carried downstream by the passing stream. If there was no viscosity, the vortices would complete a loop back where the wing first began to move. However, they are dissipated by friction before they can do this. The horseshoe vortices create an upwash or downwash depending on which way they are rotating, and this changes the angle of attack of the wing along its span. This in turn changes the lift and drag of the wing. The strength of the horseshoe vortices is determined by the circulation on the wing, which depends on the geometry of the wing and its speed. Details of this are beyond the scope of this note.

Potential or ideal flows around bodies are usually obtained by superposition of certain basic or elemental flows. Superposition produces in the flow a stream surface that separates inner and outer flows. The stream surface mimics a thin shell body in the flow that deflects inner and outer flows. We are usually interested in the outer flow. The most elemental flow is a stream. This is usually uniform, meaning that all fluid particles are moving in the same direction at the same speed. Another elemental flow is a source. Here all fluid particles are moving outwards from a center. The center is a line in 2D and a point in 3D. At the center the fluid is moving at infinite speed! The inverse of a source is a sink. Here all fluid particles are moving inwards to a center. Superposition of a strong source and a strong sink of equal strength very close together produces the elemental flow known as a doublet. The final elemental flow is known as a potential vortex. Here all fluid particles are moving along circular streamlines. The speed of the particles is inversely proportional to the streamline radius, so particles at the center of the vortex move at infinite speed! Points in a flow where fluid particles are moving at infinite speed are known as singularities. Such points do not exist in reality!

Superposition of a 2D stream and a 2D doublet with a potential vortex gives approximately the flow pattern around a spinning cylinder. It turns out that the flow around the cylinder can be mapped into flow around a foil shape. The lift on the foil per unit span is

$$\rho \Gamma S$$

where S is the stream speed, Γ is the vortex strength or circulation and ρ is the fluid density. Note that lift is zero when the vortex strength is zero.

Joukowsky foils are obtained by mapping a circle into a foil shape using the mapping function

$$\alpha = x + [xa^2/(x^2+y^2)]$$

$$\beta = y - [ya^2/(x^2+y^2)]$$

Geometry gives

$$x = \mathbf{X} - n \qquad y = \mathbf{Y} + m$$

$$\mathbf{X} = - R \cos\Upsilon \qquad \mathbf{Y} = + R \sin\Upsilon$$

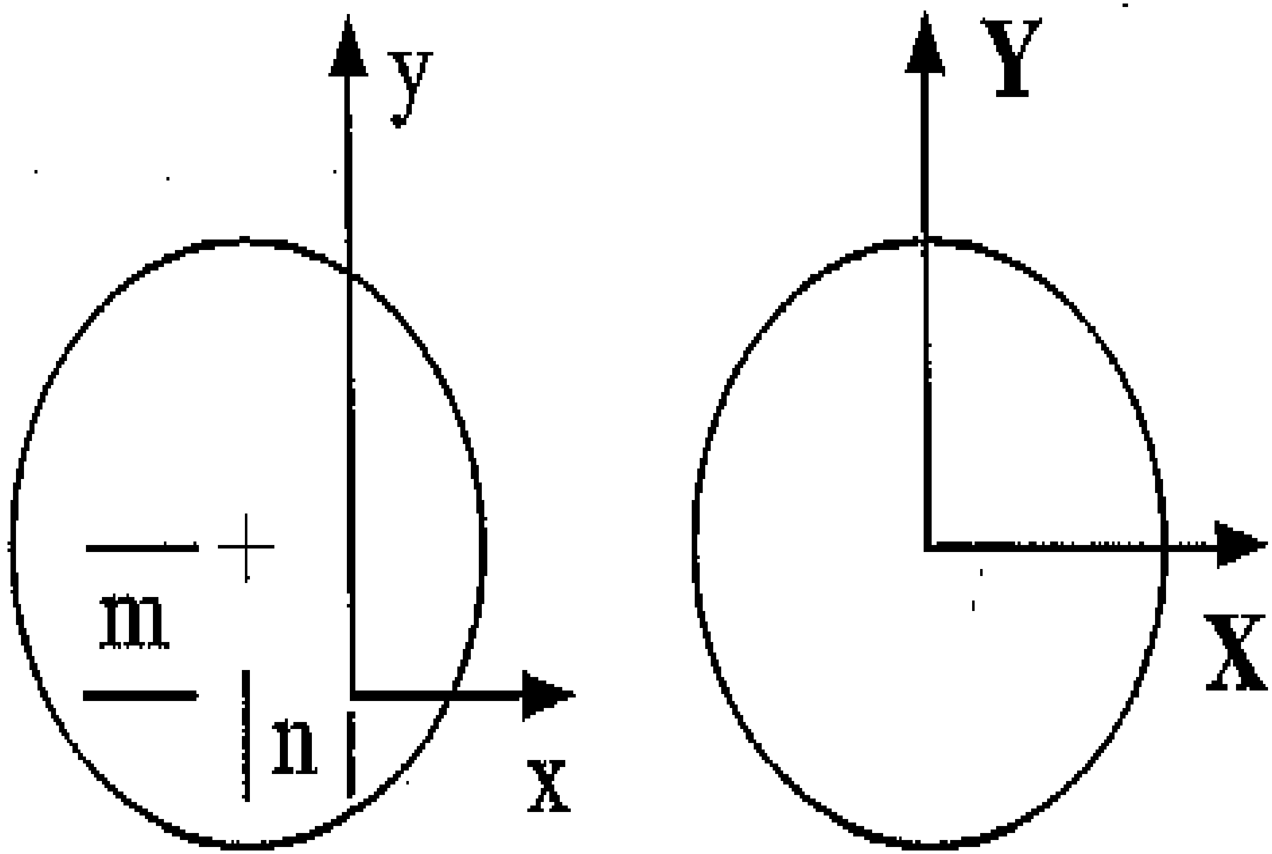
$$a = \sqrt{[R^2-m^2]} - n$$

where R is circle radius. The offsets n and m determine the shape of the foil. Some Joukowsky foils are shown on the next few pages.

To make the flow look realistic around a foil, the trailing edge must be a stagnation point. It turns out that the point where the x axis hits the circle in the circle plane maps to the trailing edge of the foil in the foil plane, and this point is a stagnation point in both planes. Setting the speed to zero there in the circle plane shows that the circulation must be:

$$\Gamma = 4\pi SR \sin\kappa$$

$$\kappa = \Theta + \varepsilon \qquad \varepsilon = \tan^{-1} [m/(n+a)]$$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