

INCOMPRESSIBLE FLUID

CONSERVATION OF MASS

$$\partial U / \partial x + \partial V / \partial y + \partial W / \partial z = 0$$

CONSERVATION OF MOMENTUM

$$\begin{aligned} \rho (\partial U / \partial t + U \partial U / \partial x + V \partial U / \partial y + W \partial U / \partial z) = & - \partial P / \partial x \\ & + \mu (\partial^2 U / \partial x^2 + \partial^2 U / \partial y^2 + \partial^2 U / \partial z^2) \end{aligned}$$

$$\begin{aligned} \rho (\partial V / \partial t + U \partial V / \partial x + V \partial V / \partial y + W \partial V / \partial z) = & - \partial P / \partial y \\ & + \mu (\partial^2 V / \partial x^2 + \partial^2 V / \partial y^2 + \partial^2 V / \partial z^2) \end{aligned}$$

$$\begin{aligned} \rho (\partial W / \partial t + U \partial W / \partial x + V \partial W / \partial y + W \partial W / \partial z) = & - \partial P / \partial z - \rho g \\ & + \mu (\partial^2 W / \partial x^2 + \partial^2 W / \partial y^2 + \partial^2 W / \partial z^2) \end{aligned}$$

CONSERVATION OF ENERGY

$$\begin{aligned} \rho C (\partial T / \partial t + U \partial T / \partial x + V \partial T / \partial y + W \partial T / \partial z) = & \mu \Phi \\ + \partial / \partial x (k \partial T / \partial x) + \partial / \partial y (k \partial T / \partial y) + \partial / \partial z (k \partial T / \partial z) \end{aligned}$$

COMPRESSIBLE FLUID

CONSERVATION OF MASS

$$\begin{aligned} \frac{\partial \rho}{\partial t} + U \frac{\partial \rho}{\partial x} + V \frac{\partial \rho}{\partial y} + W \frac{\partial \rho}{\partial z} \\ + \rho \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right) = 0 \end{aligned}$$

CONSERVATION OF MOMENTUM

$$\begin{aligned} \rho \left(\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + W \frac{\partial U}{\partial z} \right) = - \frac{\partial P}{\partial x} \\ + \frac{\partial}{\partial x} \left(\lambda \left[\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right] \right) + \frac{\partial}{\partial x} \left(\mu \left[\frac{\partial U}{\partial x} + \frac{\partial U}{\partial x} \right] \right) \\ + \frac{\partial}{\partial y} \left(\mu \left[\frac{\partial V}{\partial x} + \frac{\partial U}{\partial y} \right] \right) + \frac{\partial}{\partial z} \left(\mu \left[\frac{\partial W}{\partial x} + \frac{\partial U}{\partial z} \right] \right) \end{aligned}$$

$$\begin{aligned} \rho \left(\frac{\partial V}{\partial t} + U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + W \frac{\partial V}{\partial z} \right) = - \frac{\partial P}{\partial y} \\ + \frac{\partial}{\partial y} \left(\lambda \left[\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right] \right) + \frac{\partial}{\partial x} \left(\mu \left[\frac{\partial V}{\partial x} + \frac{\partial U}{\partial y} \right] \right) \\ + \frac{\partial}{\partial y} \left(\mu \left[\frac{\partial V}{\partial y} + \frac{\partial V}{\partial y} \right] \right) + \frac{\partial}{\partial z} \left(\mu \left[\frac{\partial W}{\partial y} + \frac{\partial V}{\partial z} \right] \right) \end{aligned}$$

$$\begin{aligned} \rho \left(\frac{\partial W}{\partial t} + U \frac{\partial W}{\partial x} + V \frac{\partial W}{\partial y} + W \frac{\partial W}{\partial z} \right) = - \frac{\partial P}{\partial z} - \rho g \\ + \frac{\partial}{\partial z} \left(\lambda \left[\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right] \right) + \frac{\partial}{\partial x} \left(\mu \left[\frac{\partial W}{\partial x} + \frac{\partial U}{\partial z} \right] \right) \\ + \frac{\partial}{\partial y} \left(\mu \left[\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y} \right] \right) + \frac{\partial}{\partial z} \left(\mu \left[\frac{\partial W}{\partial z} + \frac{\partial W}{\partial z} \right] \right) \end{aligned}$$

CONSERVATION OF ENERGY

$$\begin{aligned} \rho C_v \left(\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} + W \frac{\partial T}{\partial z} \right) = \\ \mu \Phi - P \left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} \right) \\ + \frac{\partial}{\partial x} (k \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y} (k \frac{\partial T}{\partial y}) + \frac{\partial}{\partial z} (k \frac{\partial T}{\partial z}) \end{aligned}$$

CONSERVATION OF MASS

$$D/Dt \int_{V(t)} \rho \, dV = 0$$

$$\int_{V(t)} [\partial \rho / \partial t + \nabla \cdot (\rho \mathbf{v})] \, dV = 0$$

CONSERVATION OF MOMENTUM

$$D/Dt \int_{V(t)} \rho \mathbf{v} \, dV = \int_{S(t)} \boldsymbol{\sigma} \, dS + \int_{V(t)} \rho \mathbf{b} \, dV$$

$$\int_{V(t)} [\partial (\rho \mathbf{v}) / \partial t + \nabla \cdot (\rho \mathbf{v} \mathbf{v})] \, dV = \int_{S(t)} \boldsymbol{\sigma} \, dS + \int_{V(t)} \rho \mathbf{b} \, dV$$

CONSERVATION OF ENERGY

$$D/Dt \int_{V(t)} \rho e \, dV = - \int_{S(t)} \mathbf{q} \cdot \mathbf{n} \, dS + \int_{S(t)} \mathbf{v} \cdot \boldsymbol{\sigma} \, dS$$

$$e = u + \mathbf{v} \cdot \mathbf{v} / 2 + gz$$

$$\int_{V(t)} [\partial (\rho e) / \partial t + \nabla \cdot (\rho e \mathbf{v})] \, dV = - \int_{S(t)} \mathbf{q} \cdot \mathbf{n} \, dS + \int_{S(t)} \mathbf{v} \cdot \boldsymbol{\sigma} \, dS$$