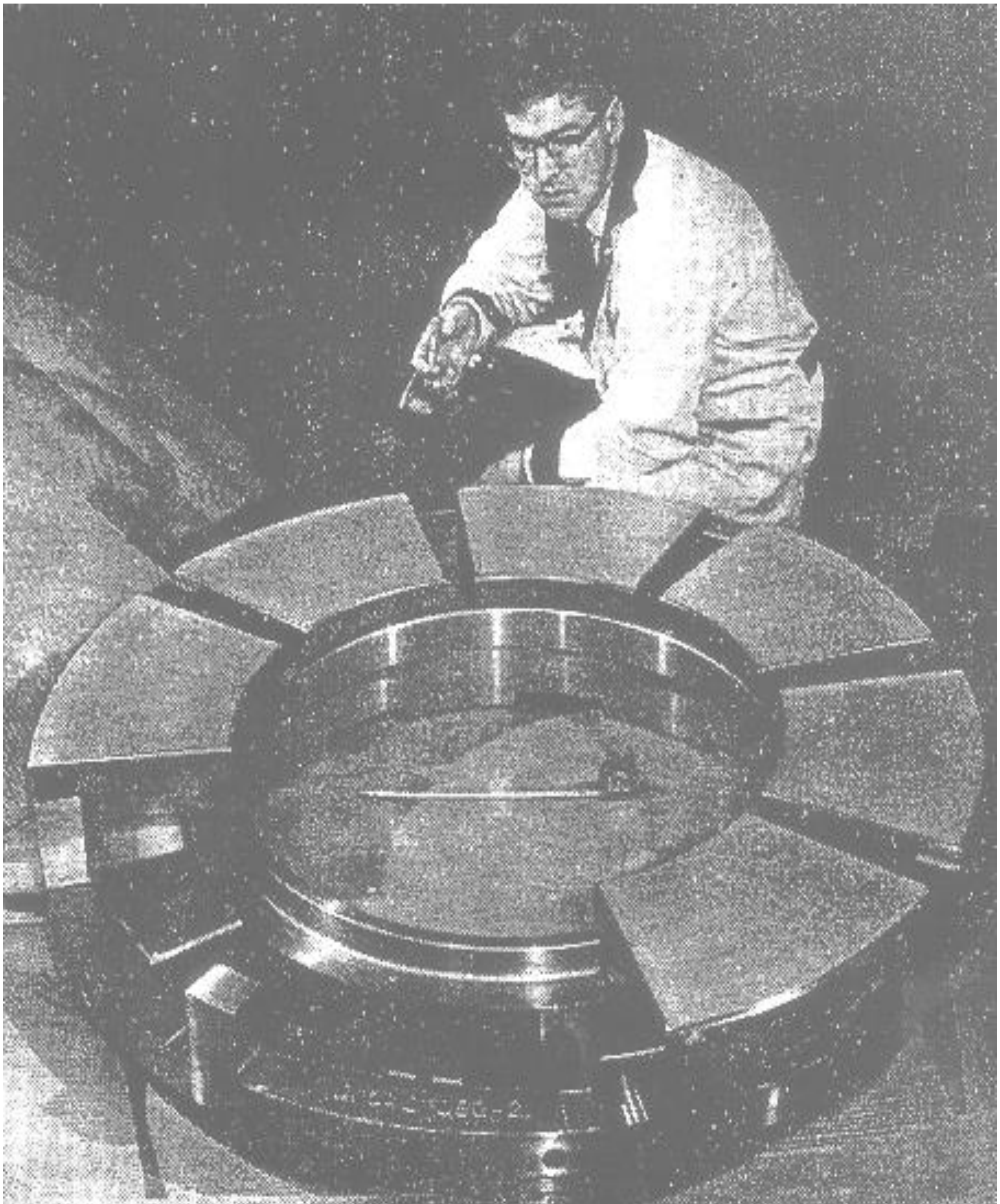
HYDRODYNAMIC LUBRICATION

PREAMBLE

When fluid moves through narrow spaces, the Reynolds Number of the flow is very low because the gap between the spaces, which is the characteristic dimension for such a flow, is very small. Low Reynolds Number means that viscous forces on the fluid are much greater than inertia forces. High pressures are generated when fluid moves through such spaces. Hydrodynamic lubrication thrust bearings on ships and submarines use these pressures to isolate the engine from the propeller shaft load. Such a bearing is shown in the photograph on the next page. Basically, the bearing consists of a series of pads which ride over a vertical disk. Each pad is inclined slightly relative to the disk, creating a wedge shaped converging passageway beneath it. Oil is dragged into these passageways as the pads ride over the disk, and this generates high pressures. One can calculate the pressures beneath the pads using Reynolds Equation for Pressure. This is basically a balance of viscous and pressure forces.



REYNOLDS EQUATION FOR PRESSURE

For a Cartesian geometry, derivation of Reynolds Equation starts with the following simplified form of the Conservation Laws

$$\partial U/\partial x + \partial V/\partial y + \partial W/\partial z = 0$$

$$\partial P/\partial x = \mu\, \partial^2 U/\partial z^2 \qquad \partial P/\partial y = \mu\, \partial^2 V/\partial z^2 \qquad 0 = \mu\, \partial^2 W/\partial z^2$$

where U V W are velocities in the x y z directions respectively, P is pressure and μ is the fluid viscosity.

Integration of Mass across the gap gives

$$\int [\partial U/\partial x + \partial V/\partial y + \partial W/\partial z]\, dz = 0$$

Manipulation gives

$$\partial I/\partial x + \partial J/\partial y + \partial K/\partial z = 0$$

$$I = \int U dz \qquad J = \int V dz \qquad K = \int W dz$$

Integration of Momentum twice across the gap gives

$$U = \partial P/\partial x\ (z^2-zh)/2\mu + (U_T-U_B) z/h + U_B$$

$$V = \partial P/\partial y\ (z^2-zh)/2\mu + (V_T-V_B) z/h + V_B$$

$$W = (W_T-W_B) z/h + W_B$$

where h is the local gap and the subscripts T and B indicate the velocities of the bearing surfaces. Substitution into the integrated mass equation gives

$$\partial/\partial x\ (h^3\ \partial P/\partial x) + \partial/\partial y\ (h^3\ \partial P/\partial y) = 6\mu\, S\ \partial h/\partial x$$

where S is the speed of the bearing surface in the x direction. This is Reynolds Equation for Pressure. For a cylindrical geometry, Reynolds Equation is

$$\partial/\partial r\ (rh^3\ \partial P/\partial r) + r\ \partial/\partial c(h^3\ \partial P/\partial c) = 6\mu S\ \ \partial h/\partial \Theta$$

where r is the radial direction, c is the circumferential direction and Θ is the circumferential angle.

Analytical solutions to Reynolds Equation are possible only for simple geometries. For example, for a Cartesian geometry with blocked sides, Reynolds Equation reduces to

$$d/dx\ (h^3\ dP/dx) = 6\mu\ S\ dh/dx = H\ dh/dx$$

Integration of this equation gives

$$h^3\ dP/dx = H\ h + A$$

where A is a constant of integration. Manipulation gives

$$dP/dx = H/h^2 + A/h^3$$

For a linear wedge gap variation

$$h = s\ x + b \qquad s = (b-a)/d$$

where a is the back gap, b is the front gap, d is the bearing width and s is the bearing slope. Substitution into the pressure gradient equation gives

$$dP/dx = H/(sx+b)^2 + A/(sx+b)^3$$

Another integration gives

$$P = -H/[s(sx+b)] - A/[2s(sx+b)^2] + B$$

where B is another constant of integration. At the front and back edges of the bearing, pressure is atmospheric pressure **P**. Application of these boundary conditions gives

$$A = 2H\ [a^2b-b^2a]\ /\ [b^2-a^2] \qquad B = \mathbf{P} + H\ /\ [s(b+a)]$$

For general bearings, one must use computational fluid dynamics or CFD to solve Reynolds Equation. This connects each point in the bearing to its neighbours to the north south east west. For a thrust bearing, one gets the template:

$$P_P = \frac{(A\ P_E + B\ P_W + C\ P_N + D\ P_S + H)}{(A + B + C + D)}$$

$$A = [(h_E+h_P)/2]^3\ \ r_P\ /\ [\Delta c^2]$$

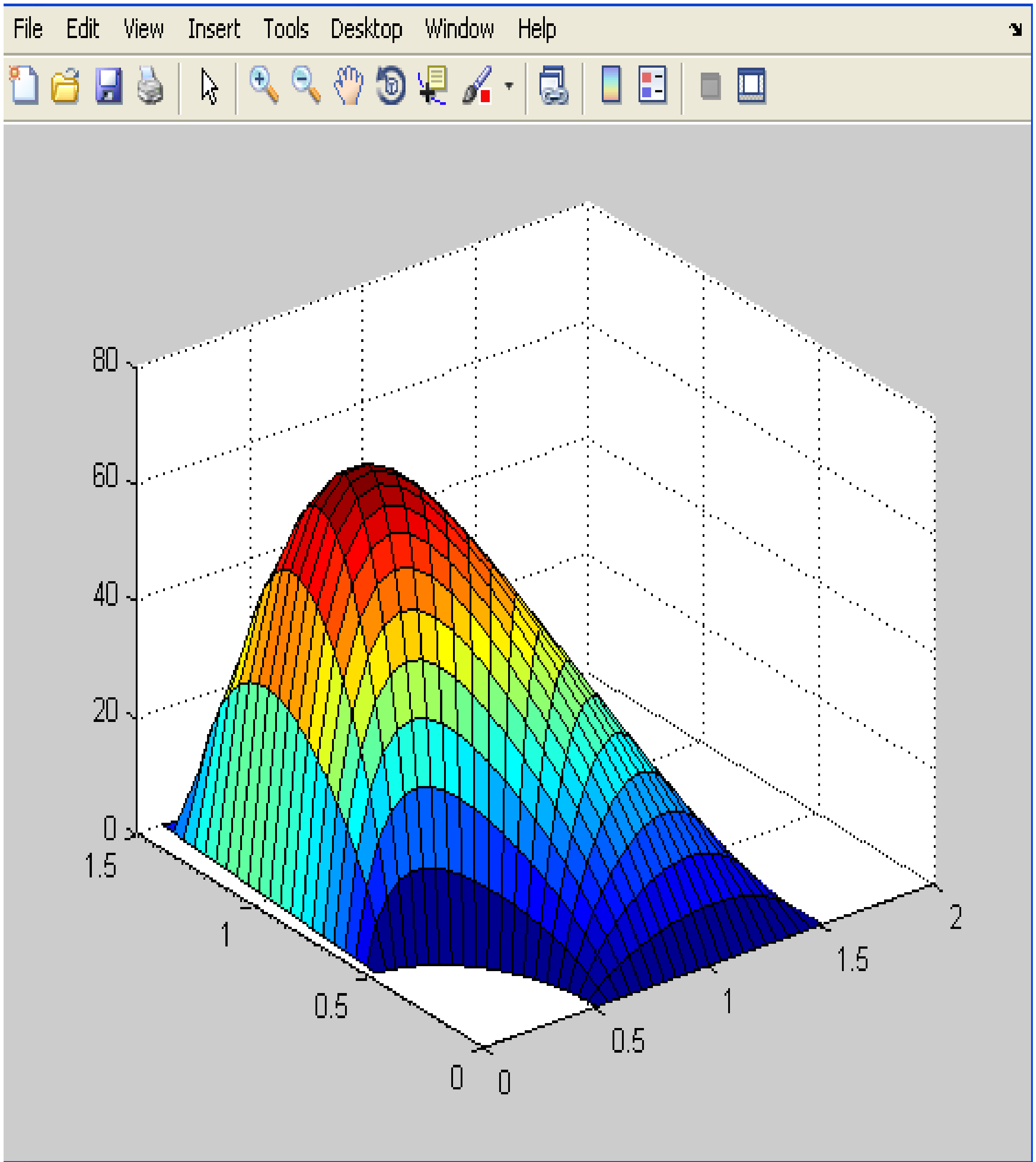
$$B = [(h_W+h_P)/2]^3\ \ r_P\ /\ [\Delta c^2]$$

$$C = [(h_N+h_P)/2]^3\ \ [(r_N+r_P)/2]\ /\ [\Delta r^2]$$

$$D = [(h_S+h_P)/2]^3\ \ [(r_S+r_P)/2]\ /\ [\Delta r^2]$$

$$H = -\ 6\mu\ r_P\omega\ (h_E-h_W)/[2\Delta\Theta]$$

The details of this procedure are beyond the scope of this note. A pressure plot obtained by applying CFD to a thrust bearing is shown on the next page. Integration of pressure over the bearing area gives the load supported by the bearing.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