

ENGINEERING 4020  
MARINE FLUID DYNAMICS

HOME WORK #1

A certain submarine has a length  $L$  equal to 25m and a diameter  $D$  equal to 5m. Its cruising speed is 25 knots. Data from a 1:10 scale model suggests that when the submarine is operating well below the water surface its drag coefficient is 0.1. What would be the drag on the prototype? What should be the speed of the model submarine in this case? Is it realistic? What should be the speed of the model submarine when it is operating close to the water surface? Is it realistic?

A tube shaped streamlined instrumentation pod is observed to free fall through the water. Its diameter is 0.5m. Its drag coefficient is 0.1. The difference between the weight of the pod and its buoyancy is 1000N. What would be the terminal speed of the pod?

A certain oil rig sits in waves with period  $T$ . The characteristic dimension of the rig is  $D$ . Derive a wave period coefficient for the rig. [Hint:  $g$ ] Deep water wave theory gives the following connection between wave period  $T$  and wavelength  $\lambda$ :  $T = \sqrt{2\pi\lambda/g}$ . What does this suggest about the ratio  $D/\lambda$ ?

## SOLUTION #1      OUTLINE

The drag coefficient is:  $C_D = \mathbf{D} / [[\rho U^2/2] A]$ . Solving for drag gives:  $\mathbf{D} = C_D * [[\rho U^2/2] A]$ . The profile area  $A$  is  $\pi D^2/4$ . The speed  $U$  is given. The Reynolds Number is  $\rho U D / \mu$ . If the model was tested in water, its speed would be 10 times the prototype speed. This speed is too fast for a wave tank. The Froude Number is  $U/\sqrt{[gD]}$ . In this case, the model speed would be  $1/\sqrt{10}$  times that of the prototype. This speed is a bit fast for a wave tank.

The drag coefficient is:  $C_D = \mathbf{D} / [[\rho U^2/2] A]$ . Solving for speed gives:  $U = \sqrt{[[2\mathbf{D}]/[\rho A C_D]]}$ . The profile area  $A$  is  $\pi D^2/4$ :  $D$  is given. At the terminal speed, the drag  $\mathbf{D}$  is equal to the difference between the weight of the pod and its buoyancy. This allows us to calculate  $U$ .

A reference period is:  $\mathbf{T} = \sqrt{[D/g]}$ . So the period coefficient is:  $C_T = T/\mathbf{T} = T/\sqrt{[D/g]}$ . Substitution of  $T = \sqrt{[2\pi\lambda/g]}$  into the  $C_T$  equation gives:  $C_T = \sqrt{[2\pi]} \sqrt{[\lambda/D]}$ . This equation shows that if  $C_T$  is fixed the ratio  $\lambda/D$  is also fixed. This implies geometric similitude. A photo of a rig in a wave would look same at any scale.

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HOME WORK #2

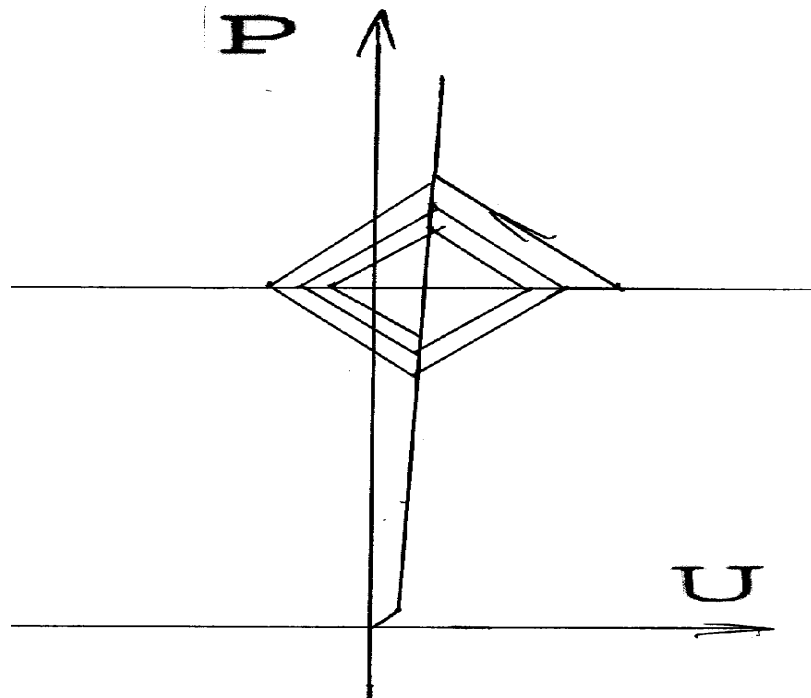
In the hydraulic transients lab, a pressure sensor was placed flush with the pipe wall just upstream of the valve. Imagine a setup where the sensor was placed at the end of a small tube attached to the pipe. Using wave reflection concepts, describe what happens in the tube when the valve in the pipe is suddenly shut. What would be the maximum pressure generated inside the tube? Assume that the wave speed for the pipe is 1000m/s and the wave speed for the tube is 500m/s. Also assume that, for the pipe, the initial pressure is 1BAR absolute and the initial flow speed is 0.5m/s. The initial pressure in the tube is also 1BAR.

Using wave reflection concepts, describe what happens when a stable leaky valve is suddenly shut. Using wave reflection concepts, describe what happens when an unstable leaky valve is suddenly shut.

## SOLUTION #2      OUTLINE

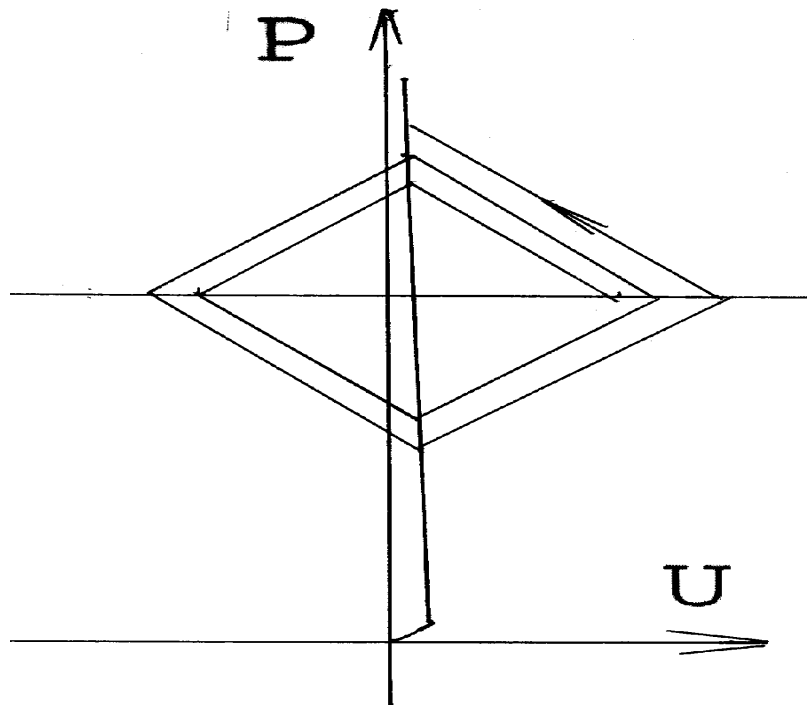
The small tube is so small that flow in or out of it has no influence on what happens in the pipe. When the valve in the pipe is shut, a positive surge wave of 5BAR (6BAR TOTAL) travels up the pipe. When that wave reaches the inlet to the tube, it creates a pressure imbalance there. This causes a flow of 1m/s from the pipe into the tube. This propagates as an inflow wave with the pipe pressure (6BAR) behind it. When it hits the sensor, it creates a flow imbalance. The pressure suddenly rises an extra 5BAR (11BAR TOTAL) at the sensor to stop the inflow. A surge pressure wave of 5BAR (11BAR TOTAL) propagates back up the tube stopping inflow as it goes. When the surge wave reaches the pipe, it creates a pressure imbalance. This causes a back flow of 1m/s out of the tube into the pipe. This propagates back down the tube to the sensor returning the pressure to 6BAR as it goes. When the back flow reaches the sensor, it creates a flow imbalance. The pressure suddenly drops at the sensor by 5BAR from 6BAR to 1BAR to stop the back flow. A suction pressure wave of minus 5BAR travels back up the tube stopping the back flow as it goes. When this reaches the tube inlet, conditions in the tube are back to where they started. From then on, the cycle repeats, and friction causes the transients to gradually decay. By the time the return wave in the pipe reaches the tube, they are all gone.

For a stable leaky valve, when the valve is suddenly shut, pressure suddenly rises at the valve. This causes a pressure surge wave which travels up the pipe to the storage tank. When it reaches the tank, it causes a back flow relative to the flow at the valve. This travels as a wave back to the valve. When it reaches the valve, the pressure suddenly drops. Because of the positive slope of its characteristic, the flow requirements of a stable valve are lower at lower pressure. This means that the pressure drop at the valve must be smaller than the initial pressure rise (only some of the initial backflow needs to be removed). As the cycle continues, all changes keep getting smaller and transient dies away.



STABLE LEAKY VALVE

For an unstable leaky valve, when the valve is suddenly shut, pressure suddenly rises at the valve. This causes a pressure surge wave which travels up the pipe to the storage tank. When it reaches the tank, it causes a back flow relative to the flow at the valve. This travels as a wave back to the valve. When it reaches the valve, the pressure suddenly drops. Because of the negative slope of its characteristic, the flow requirements of an unstable valve are higher at lower pressure. This means that the pressure drop at the valve must be bigger than the initial pressure rise (flow has to be turned around more than the initial backflow). As the cycle continues, all changes keep getting bigger and transients grow.



UNSTABLE LEAKY VALVE

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HOME WORK #3

A steel brace tube on a certain exploration rig has a length  $L=10.0\text{m}$  and an  $OD=1.0\text{m}$  and an  $ID=0.98\text{m}$ . What current speed would cause the tube to undergo 1<sup>st</sup> mode resonance due to vortex shedding? Assume that the tube has clamped-clamped supports and has air inside. Imagine now that the tube had a square cross section. What current speed would cause galloping? For this, assume that the damping factor for the tube is 0.05.

A certain marine riser tube bundle has steel tubes with an  $OD=0.15\text{m}$  and an  $ID=0.14\text{m}$ . The distance between the tube supports is  $5.0\text{m}$ . There is oil inside the tubes. The damping factor for the bundle is 0.15 and the bundle factor is 5. What current speed would cause the tubes to vibrate? Assume that the tubes have pivot supports. Repeat the calculation for the case where the tubes have clamped-clamped supports.

What internal oil flow speed would cause a tube in the marine riser to buckle? Assume pivot supports and zero gage pressure and tension. What internal oil flow speed would cause a tube in the riser to undergo whip? Assume the tube is a cantilever beam  $5\text{m}$  long.

The certain sail is made from nylon  $2\text{mm}$  thick. It has a tension of  $500\text{N}$  per meter width. What wind speed on one side of it would cause the sail to flutter?

### SOLUTION #3      OUTLINE

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The critical speed for vortex shedding is:

$$S = D/[\mathbf{ST}] \qquad \mathbf{T} = \mathbf{T}$$

For clamped-clamped supports, the periods are:

$$\mathbf{T}_n = 2\pi L^2/K_n \sqrt{m/EI}$$

The 1<sup>st</sup> period has K equal to 22.4. All geometry is given so the critical speed S can be calculated.

The galloping critical speed is:

$$S = S_o \, M/M_o \, \zeta \, a$$
$$S_o = D/\mathbf{T} \qquad M_o = \rho D^2$$

All geometry is given so S can be calculated.

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The tube bundle critical speed is:

$$S = \beta/\mathbf{T} \sqrt{M\delta/\rho} = \beta S_o \sqrt{\delta M/M_o}$$

For pivot supports, the periods are

$$\mathbf{T}_n = [L/n]^2 [2/\pi] \sqrt{m/EI}$$



For clamped-clamped supports, the periods are:

$$\mathbf{T}_n = 2\pi L^2 / K_n \sqrt{m/EI}$$

All geometry is given so the speeds can be calculated.

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The pipe buckling critical speed is:

$$S = \sqrt{[EI/[\rho A] \pi^2/L^2 + T/[\rho A] - P/\rho]}$$

The pipe whip critical speed is:

$$S = [4 + 14 M_o/M] S_o$$

$$S_o = \sqrt{[EI]/[M_o L^2]} \quad M_o = \rho A$$

All geometry is given so the speeds can be calculated.

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The panel flutter critical speed is:

$$S = \sqrt{[Tk^2 + Dk^4 + K/w + \rho_B g - \rho_T g] * [ \rho_T/k + \rho_B/k + \sigma ] / [ \rho_B \rho_T + \sigma \rho_T k ]}$$

This gives a critical speed of zero with wavelength of infinity. Need to add some side support stiffness K.

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HOME WORK #4

A certain supertanker has straight sides which are 250m long. The draft of the tanker is 25m and its beam is 50m. Its cruising speed is 25 knots. Calculate the side wall drag on the tanker. The wake drag coefficient is 0.05. Calculate the wake drag on the tanker. Calculate the power to overcome these drags. What is the wave drag on a typical supertanker?

A certain hydrodynamic lubrication bearing for a ship has 4 pads. The pads are narrow and the outer and inner edges are blocked. Develop an equation for the circumferential variation of pressure in each pad. Develop an equation for the total load supported by the pads. Let the wedge angle of the pads be 60 degrees. Let the outer radius be 1.5m, the inner radius be 1.0m, the front gap be 1.5mm and the back gap be 0.5mm. Let the density of the bearing oil be  $880 \text{ kg/m}^3$  and its viscosity be  $0.15 \text{ Ns/m}^2$ . Let the RPM of the propeller shaft be 100. What is the load?

Imagine that you have a propeller that has 4 wedge shaped flat blades. Let the wedge angle be 60 degrees and let the angle of attack of the blades be 10 degrees. Let the hub radius of each blade be 0.5m and let the tip radius be 1m. The RPM of the propeller is 250. Estimate the total thrust of the propeller.

## SOLUTION #4      OUTLINE

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For a ship with length  $L$ , the wall drag is:

$$D = M b L R_{EL}^{-1/m} \rho U^2$$

where the width  $b$  is 2 times the draft plus the beam.  
The constants  $M$  and  $m$  depend on the Reynolds Number  $R_{EL}$ . Wake drag at the rear would have the form:

$$W = C_D B \rho U^2 / 2$$

The power required to overcome these drags is:

$$P = [ D + W ] U$$

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Reynolds Equation for a thrust bearing is:

$$\frac{\partial}{\partial r} (r h^3 \frac{\partial P}{\partial r}) + r \frac{\partial}{\partial c} (h^3 \frac{\partial P}{\partial c}) = 6 \mu S \frac{\partial h}{\partial \Theta}$$

where  $P$  is pressure,  $h$  is the bearing gap,  $r$  is the radial coordinate,  $c$  is the circumferential distance,  $\Theta$  is the circumferential angle and  $S$  equal to  $r\omega$  is

the bearing speed. For a narrow thrust bearing with blocked edges, this reduces to

$$d/dc (h^3 dP/dc) = 6\mu S dh/dc = H dh/dc$$

Integration of this equation gives

$$h^3 dP/dc = H h + A$$

where A is a constant. Manipulation gives

$$dP/dc = H/h^2 + A/h^3$$

For a linear wedge gap variation

$$h = s c + b \quad s = (b-a)/d$$

where a is the back gap, b is the front gap, d is the bearing width and s is the bearing slope. Substitution into the pressure gradient equation gives

$$dP/dc = H/(sc+b)^2 + A/(sc+b)^3$$

Another integration gives

$$P = -H/[s(sc+b)] - A/[2s(sc+b)^2] + B$$

where B is a constant. At the front and back edges of the bearing, pressure is atmospheric pressure **P**. Application of these boundary conditions gives

$$A = 2H [a^2b - b^2a] / [b^2 - a^2] \quad B = P + H / [s(b+a)]$$

The total thrust is

$$T = 4 \int P \Delta r \, dc$$

where  $\Delta r$  is the width of the bearing. Substitution into this followed by integration gives:

$$T = 4 \Delta r \left[ -H/s^2 \ln(sc+b) + A/2s^2 / (sc+b) + B c \right]$$

The limits are  $c$  equal to 0 and  $c$  equal to  $r\Psi$  where  $r$  is the average value of  $r$  and  $\Psi$  is the wedge angle.

The thrust for a propeller with 4 flat wedge shaped blades is:  $4 \int \rho \Gamma S \, dr$  where  $\Gamma = C \pi S \sin\theta$ . The chord  $C$  is  $r\Psi$  and the speed  $S$  is  $r\omega$ . Substitution into the thrust equation followed by integration gives:

$$\rho \pi \Psi \omega^2 \sin\theta (A^4 - B^4)$$

where  $A$  is the outer or tip radius of the propeller and  $B$  is the inner or hub radius.

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HOME WORK #5

A closed loop of copper pipe is used to heat a room in the winter and cool it in the summer. A pump in the loop moves water through the loop. The total length of pipe in the loop is 20m. The diameter of the pipe is 1cm. There are forty  $180^\circ$  bends in the loop. The flow rate is 5 GPM. Sketch the system demand curve. Determine the pump power. What type of pump should be used in the loop? Repeat the calculations for the case where the pipe diameter is doubled to 2cm. Repeat the calculations for the case where the pipe diameter is cut in half to 0.5cm. Comment on the results.

A certain jet propulsion unit for a boat takes in water at the speed of the boat and throws it out at twice the speed of the boat. The volumetric flow rate through the unit is 60 GPM. The outlet pipe diameter is 5cm. Determine the speed of the boat. Determine the thrust generated by the unit.

## SOLUTION #5      OUTLINE

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The system demand for a pipe is

$$H_s = X + Y Q^2$$
$$Q = C A \quad A = \pi D^2/4$$

$$X = \Delta [P/\rho g + z]$$
$$Y = [fL/D + \Sigma K]/[2gA^2]$$

All of the information needed to evaluate this is given.  
At the system operating point, the pump head  $H_p$  is equal to the system head  $H_s$ . The pump power is

$$P_p = \rho g H_p Q$$

The type of pump is determined by the specific speed

$$\mathbf{N} = [N \sqrt{Q}] / [H^{3/4}]$$

Doubling the pipe diameter decreases losses and reduces pump power. Halving the diameter does the opposite.

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Knowing the volumetric flow rate and the outlet pipe diameter allows one to calculate the outlet flow speed:

$$Q = C A \quad A = \pi D^2/4$$

Conservation of mass gives

$$\sum [\rho C A]_{\text{OUT}} - \sum [\rho C A]_{\text{IN}} = 0$$

The outlet flow speed is twice the inlet flow speed. This shows that the inlet pipe diameter is  $\sqrt{2}$  times the outlet pipe diameter. Conservation of momentum gives

$$\begin{aligned} & \sum [\rho \mathbf{v} C A]_{\text{OUT}} - \sum [\rho \mathbf{v} C A]_{\text{IN}} \\ &= - \sum [P A \mathbf{n}]_{\text{OUT}} - \sum [P A \mathbf{n}]_{\text{IN}} + \mathbf{R} \end{aligned}$$

For the horizontal direction this reduces to

$$\sum [\dot{M} U]_{\text{OUT}} - \sum [\dot{M} U]_{\text{IN}} = - \sum P A n_x + R_x$$

The inlet and outlet are at the same level and open to the surrounding pressure. So the pressure terms can be dropped. The force on the boat is minus  $R_x$ .

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