BOUNDARY LAYER WALL AND WAKE DRAG

The development of wall drag formulas for long slender bodies makes use of velocity profile and wall shear data:

$$U/\mathbf{U} = (y/\delta)^{1/n} \qquad \tau = C\ \rho\mathbf{U}^2/(\mathbf{U}\delta/\nu)^{1/k}$$

Momentum considerations show that the wall drag is:

$$\mathbf{D}/b = \rho\ \mathbf{U}^2\Theta \qquad \Theta = \int U/\mathbf{U}(1-U/\mathbf{U})dy$$

The rate of change of drag along the wall is:

$$\tau = d[\mathbf{D}/b]/dx = \rho\mathbf{U}^2\ d\Theta/dx$$

Substitution into this followed by integration gives:

$$\delta = A\ x\ R_{EX}^{-1/a} \qquad \mathbf{D} = M\ bx\ R_{EX}^{-1/m}\ \rho\mathbf{U}^2$$

For a body with length L the drag is:

$$\mathbf{D} = M\ bL\ R_{EL}^{-1/m}\ \rho\mathbf{U}^2$$

Wake drag at the rear of the body would have the form:

$$\mathbf{W} = C_D\ B\ \rho\mathbf{U}^2/2$$

The power required to overcome these drags is:

$$\mathbf{P} = [\ \mathbf{D} + \mathbf{W}\ ]\ \mathbf{U}$$

DRAG ON A SUBMARINE

Consider a submarine with an outer diameter D equal to 10m and a length L equal to 100m. It is travelling through water at 10 knots. Calculate the drag on the submarine and the power required to push it through the water.

The drag is made up of two components: wall drag and wake drag. Wall drag is due to viscous shear stresses in the wall boundary layer. Wake drag is due to boundary layer separation at the rear. The drags are given by:

$$\text{Wall Drag :} \quad M\, \pi D\, L\, R_{EL}{}^{-1/m}\, \rho U^2$$

$$\text{Wake Drag :} \quad C_D\, \pi D^2/4\, \rho U^2/2$$

The Reynolds Number R_{EL} is UL/υ. The constants M and m depend on R_{EL}. One knot is approximately 0.5m/s. So speed U is approximately 5m/s. The viscosity of water is approximately 10^{-6}. So R_{EL} is approximately 5 times 10^{+8}. For R_{EL} greater than 10^{+8} M is 0.0152 and m is 7. The suction created by the propellor would tend to make the wake drag coefficient C_D low: let it be 0.05. With this the drags are:

$$\text{Wall Drag: } 0.0152 * \pi*10 * 100 * 0.061 * 1000 * 25 = 72864 \text{ N}$$

$$\text{Wake Drag: } 0.05 * \pi*10*10/4 * 1000 * 25 / 2 = 49088 \text{ N}$$

The power needed to overcome the drag is drag times speed:

$$(72864 + 49088) * 5 = 0.61 \text{ MW}$$