WATER WAVE INTERACTION
WITH STRUCTURES

PREAMBLE

Most water waves are generated by storms at sea. Many waves are present in a storm sea state: each has a different wavelength and period. Theory shows that the speed of propagation of a wave or its phase speed is a function of water depth. It travels faster in deeper water. Theory also shows that the speed of a wave is a function of its wavelength. Long wavelength waves travel faster than short wavelength waves. This explains why storm generated waves, which approach shore, are generally a single wavelength. Because waves travel at different speeds, they tend to separate or disperse. When waves approach shore, they are influenced by the seabed by a process known as refraction. This can focus or spread out wave energy onto a site. Close to shore water depth is not the same everywhere: so points on wave crests move at different speeds and crests become bent. This explains why crests which approach a shore line tend to line up with it: points in deep water travel faster than points in shallow water and overtake them.

Wave energy travels at a speed known as the group speed. This is generally not the same as the phase speed. However for shallow water both speeds are the same and they depend only on the water depth. A large low pressure system moving over shallow water would generate an enormous wave if the system speed and the wave energy speed were the same. Basically wave energy gets trapped in the system frame when the system speed matches the wave energy speed. Tides are basically shallow water waves. Here the pull of the Moon mimics a low pressure system. Theory shows that if water depth was 22km everywhere on Earth the Moon pull would produce gigantic tides. They would probably drain the oceans and swamp the continents everyday. Fortunately the average water depth is only 3km.

Water waves can interact with structures and cause them to move or experience loads. For wave structure interaction, an important parameter is $5D/\lambda$ where D is the characteristic dimension of the structure and λ is the wavelength. Structures are considered large if $5D/\lambda$ is much greater than unity: they are considered small if $5D/\lambda$ is much less than unity. Small structures are basically transparent to waves. Large structures scatter waves. There are two types of scattering: reflection and diffraction.

These notes start with an overview of water wave theory. Then interaction of waves with small structures is considered. Finally interaction with large structures is considered.

WATER WAVES

To calculate wave interactions with structures, one needs a detailed knowledge of the wave field. Water wave theory provides this. It will be assumed for much of what is given below that wave amplitudes are very small. It turns out that this is good even for waves not far from breaking. Water waves in deep water propagate for long distances with little loss of energy. They lose energy in shallow water due to interaction with the seabed. They also lose energy when they move pass small structures and when they break on beaches. Water wave theory ignores these energy losses. It assumes that water has zero viscosity and it is incompressible. It also assumes that its motion is irrotational. This means that water particles do not spin. With these assumptions, the conservation laws reduce to potential flow forms.

Conservation of mass considerations give:

$$\nabla . \mathbf{v} = 0 \qquad \nabla^2 \varphi = 0 \quad .$$

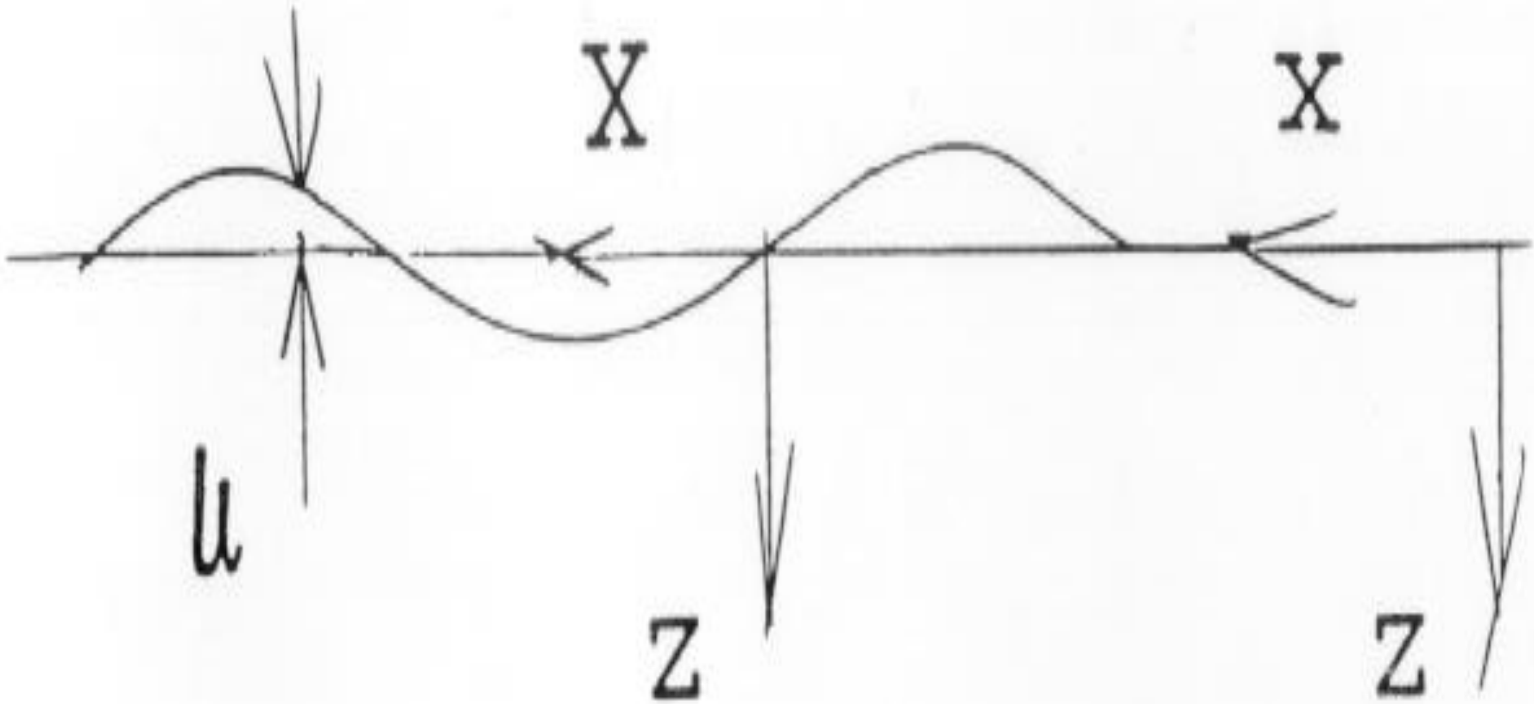
The velocity vector $\mathbf{v}$ in terms of the potential φ is

$$\mathbf{v} = \nabla\varphi = U\mathbf{i} + V\mathbf{j} + W\mathbf{k} \quad .$$

Conservation of momentum considerations give:

$$\rho\ \partial\mathbf{v}/\partial t + \rho\ \nabla\ \mathbf{v}.\mathbf{v}/2 + \nabla\ P + \nabla\ \rho g z = 0$$

$$\partial\varphi/\partial t + \nabla\varphi.\nabla\varphi/2 + P/\rho + gz = C \quad .$$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It turns out that, for water waves, mass is the main governing equation: momentum is used as a boundary condition. The kinematic or motion constraint at the seabed is:

$$\partial\varphi/\partial z = 0 \qquad \text{at } z = -h$$

where h is the water depth. The kinematic or motion constraint at the water surface is based on:

$$D\eta/Dt = Dz/Dt$$

where η is the vertical deflection of the water from the still water line. The η for a point on the water must follow the z for that point. The constraint gives:

$$\partial\eta/\partial t + \partial\varphi/\partial x \, \partial\eta/\partial x = \partial\varphi/\partial z \qquad \text{at } z = \eta \quad .$$

For small amplitude waves, this becomes:

$$\partial\eta/\partial t = \partial\varphi/\partial z \qquad \text{at } z = 0 \quad .$$

The dynamic or load constraint at the water surface is:

$$\partial\varphi/\partial t + \nabla\varphi.\nabla\varphi/2 + P/\rho + g\eta = 0 \qquad \text{at } z = \eta \quad .$$

For small amplitude waves, this becomes:

$$\partial\varphi/\partial t + g\eta = 0 \qquad \text{at } z = 0 \quad .$$

Manipulation of the water surface constraints allows one to eliminate η from the formulation. One gets:

$$\partial^2\varphi/\partial t^2 \;+\; g\partial\varphi/\partial z \;=\; 0 \qquad \text{at} \quad z = 0 \quad .$$

The Separation of Variables solution procedure gives:

$$\varphi = \varphi_O \, \text{Cosh}[k(z+h)]/\text{Cosh}[kh] \; \text{Cos}(kX)$$

where $kX = k(x - C_P t) = kx - \omega t$ where X is the horizontal coordinate of a wave fixed frame, x is the horizontal coordinate of an inertial frame, C_P is the wave phase speed, k is the wave number and ω is the wave frequency. The wave number k in term of the wave length λ is: $k = 2\pi/\lambda$.

The wave profile equation has the form:

$$\eta = \eta_O \, \text{Sin}(kX) \quad .$$

Substitution into the combined water surface constraint gives the dispersion relationships:

$$C_P \;=\; \sqrt{(g/k \, \text{Tanh}[kh])}$$

$$\omega \;=\; \sqrt{(gk \, \text{Tanh}[kh])} \quad .$$

These show that deep water waves travel faster than shallow water waves. They also show that long wave length waves travel faster than short wave length waves.

Substitution into the water surface constraints gives the connection between potential amplitude and wave amplitude:

$$\varphi_O = - gT/[2\pi]\ H/2$$

where T is the wave period and H is the wave height.

Differentiation gives the water particle velocities:

$$U = \partial\varphi/\partial x = - \varphi_O\ k\ Cosh[k(z+h)]/Cosh[kh]\ Sin(kX)$$
$$= + H/2\ 2\pi/T\ Cosh[k(z+h)]/Sinh[kh]\ Sin(kX)$$

$$W = \partial\varphi/\partial z = + \varphi_O\ k\ Sinh[k(z+h)]/Cosh[kh]\ Cos(kX)$$
$$= - H/2\ 2\pi/T\ Sinh[k(z+h)]/Sinh[kh]\ Cos(kX) \quad .$$

These can be used to get drag loads on small structures.

The momentum equation gives the wave pressure

$$\Delta P = \rho g\ \eta\ Cosh[k(z+h)]/Cosh[kh] \quad .$$

This can be used to get pressure loads on structures.

For deep water, the solution becomes:

$$\varphi = \varphi_O\ e^{-kz}\ Cos(kX) \qquad \eta = \eta_O\ Sin(kX) \quad .$$

With this, the dispersion relationships become:

$$C_P = \sqrt{g/k} \qquad \omega = \sqrt{gk} \quad .$$

The velocities become:

$$U = + H/2 \; 2\pi/T \; e^{kz} \; Sin(kX)$$
$$W = - H/2 \; 2\pi/T \; e^{kz} \; Cos(kX) \; .$$

The wave pressure becomes:

$$\Delta P = \rho g \; \eta \; e^{kz} \quad .$$

Note that, at half a wave length down into the water:

$$e^{kz} = e^{[2\pi/\lambda][-\lambda/2]} = e^{-\pi} = 0.043 \quad .$$

This shows that, at the half wave length depth, wave motions are less than 5% of surface motions. It is customary to take water to be deep when the seabed is below the half wave length depth. This implies that, at Hibernia, where water depth is around 75m, the water can be assumed to be deep for wave lengths less than 150m. The dispersion relationship shows that the period of 150m waves is around 10 seconds.

Wave energy travels at a speed known as the group speed. This is generally not the same as the phase speed of a wave. One can show that the group speed is given by:

$$C_G = d\omega/dk = C_P \; (1/2 + [kh]/Sinh[2kh]) \quad .$$

The wave energy density is:

$$\mathbf{E} = 1/8 \; \rho g \; H^2 \quad .$$

One can show that wave energy flux is:

$$\mathbf{P} = C_G \; \mathbf{E} \quad .$$

Group speed is responsible for many important phenomena.

Group speed can cause energy trap phenomena. For example, a 2D Moon moving over a body of shallow water would create the following water surface deflection:

$$\eta/h = [P/\rho] / [U^2-gh]$$

where U is the speed of the Moon. This equation shows that infinite deflections would be generated if the Moon speed matched the group speed. When something like a truck moves slowly over a floating ice sheet, it generates a bowl shaped depression directly beneath itself. However, as it picks up speed, at some point bow and stern waves suddenly appear in the sheet. It turns out that at the speed where waves appear the group speed of the waves is equal to the load speed. This means energy cannot propagate away from the load. It becomes trapped in the load frame, and wave amplitudes increase to absorb it. This can cause the sheet to break. To prevent this, high speed loads travel through the critical speed as fast as possible. For a 1m thick sheet, the critical speed is around 50km/hr.

Group speed also explains the wave pattern behind a ship. When a ship is at a certain location, it puts a certain amount of energy into the water. As the ship moves forward, this energy propagates away from the generation site. Someone flying overhead sees a stationary or fixed pattern in the water relative to the ship. Only so much of the wave energy put into the water contributes to such a stationary pattern. The rest interacts with energy put in at other sites and is cancelled out. If wave energy propagated at the phase speed C_P, wave theory shows that the wave energy which contributes to the stationary pattern would be found on a circle passing through the site and the ship. However, wave energy travels at the group speed C_G not the phase speed. In deep water, $C_G = C_P/2$. This means wave energy would be found on a circle half the size of that based on phase speed. All of the circles based on group speed fall inside a wedge known as the Kelvin Wedge. Wave theory and observations also show that there are two types of waves within the wedge: transverse and diverging waves.

Waves at sea after a storm are random. They are made up of an infinite number of frequencies. A spectrum shows how the energy in a wave field is spread out over a range of frequencies. A popular 2 parameter fit to a wave amplitude spectrum is the ITTC fit:

$$S_\eta = A/\omega^5 \ e^{-B/\omega 4}$$

$$A=346H^2/T^4 \qquad B=691/T^4$$

where H is the significant wave height and T is the significant wave period. The Joint North Sea Wave Energy Project or JONSWAP fit is popular 3 parameter fit.

A Response Amplitude Operator or RAO can be used to connect a wave spectrum to a structure motion or load response spectrum

$$S_R = RAO^2 \ S_\eta \quad .$$

An RAO is basically a Magnitude Ratio. For a specific wave period, it is the amplitude of structure response divided by the wave amplitude. For small structures, Morisons Equation can be used to get RAOs. For large structures, they can be obtained using the CFD procedure known as the Panel Method. One can also get RAOs from experiments.

All sorts of statistical and probabilistic information can be obtained from spectra. For structures, the analysis makes use of the following moments of the spectrum:

$$M_n = 1/2 \int_0^{\infty} S_R(\omega)\ \omega^n\ d\omega \quad .$$

One can show that the significant response height and period of a structure motion or load are:

$$2\ R_S = 4\ \sqrt{M_0} \qquad T_S = 2\pi\ M_0/M_1\ .$$

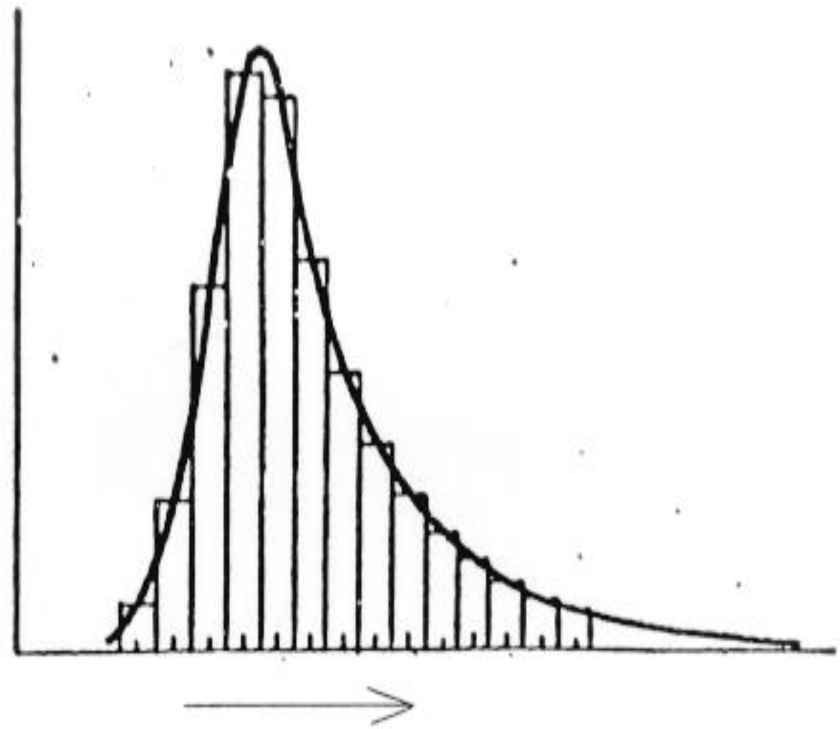
The probability of a response exceeding a certain level is:

$$P(R_o > R_\bullet) = e^{-X} \qquad X = R_\bullet R_\bullet/[2M_0] \quad .$$

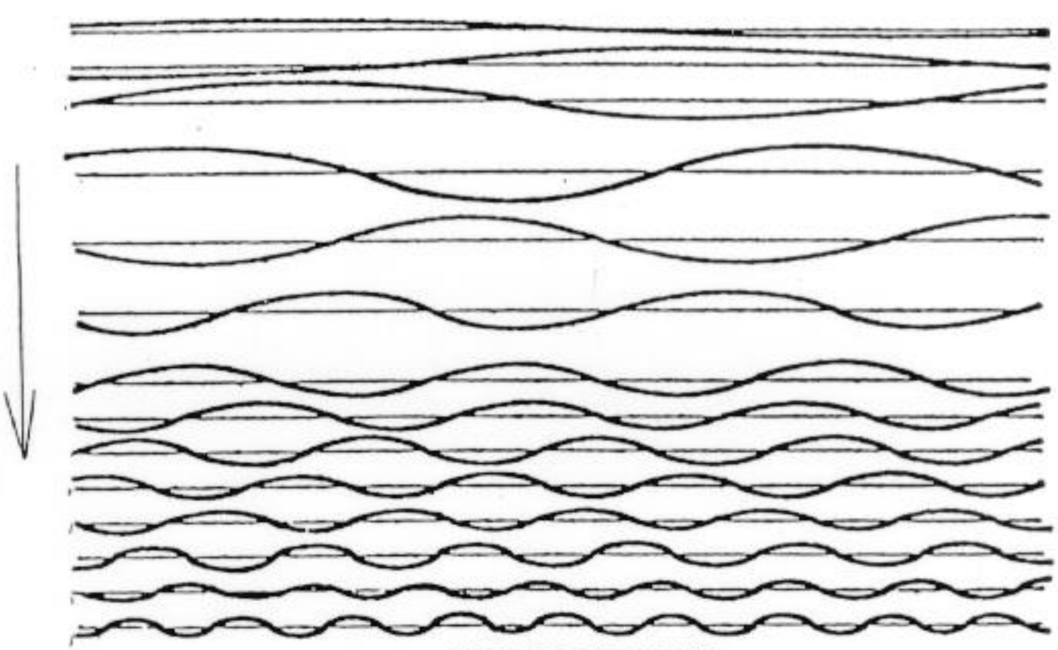
The theory assumes that spectra follow a Rayleigh Distribution. Actual spectra deviate from this and predictions must often be corrected. A correction factor based on moments of the spectrum is:

$$CF = \sqrt{1-\varepsilon\varepsilon} \qquad \varepsilon = [M_0M_4-M_2M_2]/[M_0M_4]$$

where ε is known as the broadness parameter.



SPECTRUM

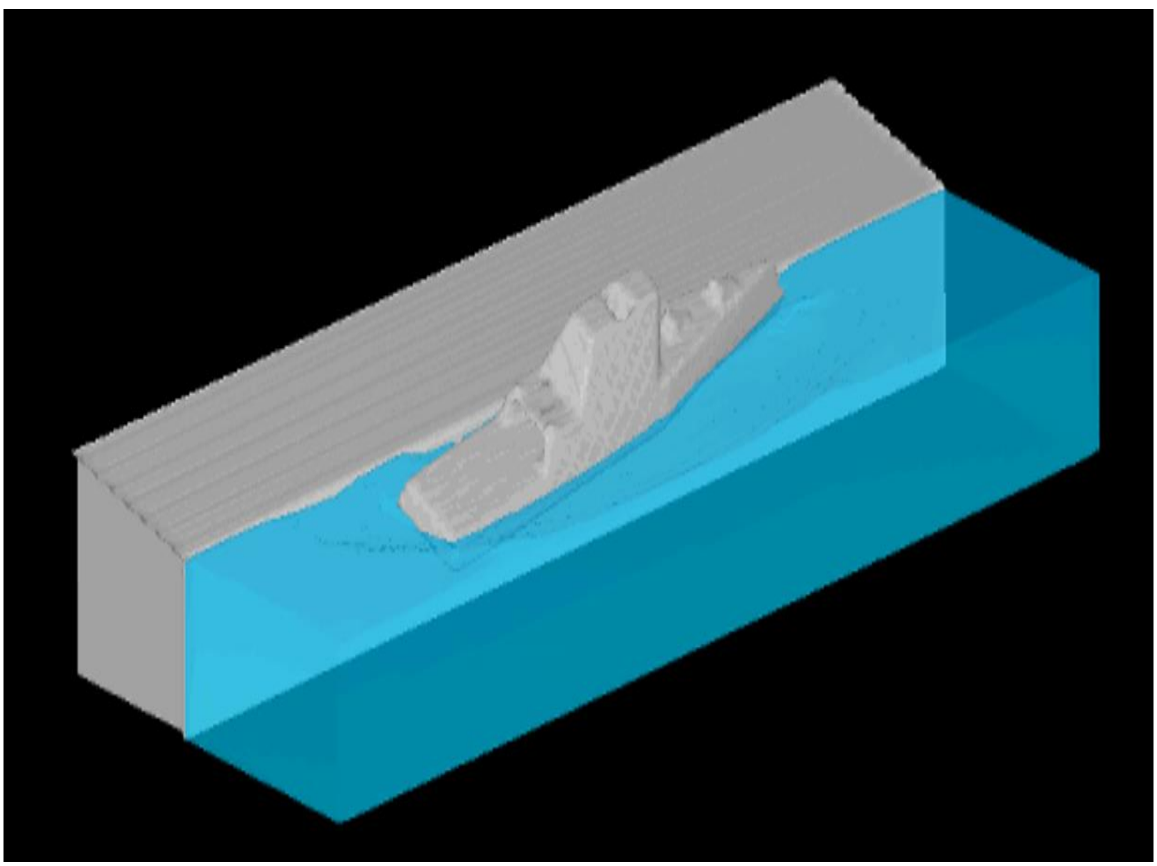


TIME OR DISTANCE

COMPONENT WAVES

STRUCTURES IN WAVES

Full scale tests at sea or model tests in a wave tank are the most accurate ways to study wave interaction with structures. One can also use real fluid CFD packages like FLOW-3D, which can often mimic the real world very closely. The image below shows a hydrodynamics flow produced by FLOW-3D. It shows the side launch of a ship step by step in time. For small structures, one can also use the Morison Equation. For large structures, one can use a Panel Method CFD procedure.



There can be two kinds of wave loads on a small structure: wake load due to the formation of wakes back of the structure and inertia load due to pressures in the water caused by acceleration and deceleration of water particles in the wave. In deep water, water particles move in circular orbits. In finite depth water, the orbits are ellipses. Let the orbit dimension normal to the structure be d and let the characteristic dimension of the structure be D. When 5D<<d, a well defined wake forms behind the structure. When 5D>>d, such a wake does not form. When 5D is approximately equal to d, flows are extremely complex. Let T be the wave period and let **T** be the time it takes a water particle to move pass the structure. It turns out that 5**T**<<T corresponds to 5D<<d while 5**T**>>T corresponds to 5D>>d. When 5D<<d, wakes form because transit time is short relative to wave period. So, water is moving sufficiently long in one direction to pass the structure. When 5D>>d, wakes do not form because transit time is long relative to wave period. So, before water particles can pass the structure, they reverse direction.

For a small structure like a float, the drag load is

$$C_D \; A \; \rho \; \mathbf{S}.\mathbf{S}/2 \; \mathbf{s}$$

while the inertia load is

$$C_M \; \rho \; B \; d\mathbf{S}/dt$$

where **S** is the water particle velocity and d**S**/dt is the water particle acceleration. The submerged frontal area of the structure is A and its displaced volume is B. The drag and inertia loads can be combined to get Morisons equation:

$$\mathbf{F} = C_D A \rho \mathbf{S.S}/2 \mathbf{s} + C_M \rho B d\mathbf{S}/dt .$$

The drag and inertia coefficients depend on the shape of the structure. For 5D much less than d, the drag coefficient C_D for a sphere is around 0.5. For 5D much greater than d, the inertia coefficient C_M for a sphere is around 0.5. In the reverse limits, each coefficient is approximately zero.

Generally, one would look for the maximum values of **S** and d**S**/dt to get upper limits on loads. Assume that you know the wave height H and the wave period T. At Hibernia following a storm, H would be around 5m while T would be around 10s. How do you find maximum values of **S** and d**S**/dt? How do you get the orbit size d? Wave theory gives the particle velocities:

$$U = \partial\varphi/\partial x = + H/2\ 2\pi/T\ \mathrm{Cosh}[k(z+h)]/\mathrm{Sinh}[kh]\ \mathrm{Sin}(kX)$$

$$W = \partial\varphi/\partial z = - H/2\ 2\pi/T\ \mathrm{Sinh}[k(z+h)]/\mathrm{Sinh}[kh]\ \mathrm{Cos}(kX) .$$

Differentiation gives the particle accelerations

$$dU/dt = - H/2\ (2\pi/T)^2\ \mathrm{Cosh}[k(z+h)]/\mathrm{Sinh}[kh]\ \mathrm{Cos}(kX)$$

$$dW/dt = - H/2\ (2\pi/T)^2\ \mathrm{Sinh}[k(z+h)]/\mathrm{Sinh}[kh]\ \mathrm{Sin}(kX) .$$

The particle positions are:

$$x_p = x_o + H/2 \, \text{Cosh}[k(z+h)]/\text{Sinh}[kh] \, \text{Cos}(kX)$$
$$Z_p = z_o + H/2 \, \text{Sinh}[k(z+h)]/\text{Sinh}[kh] \, \text{Sin}(kX) \quad .$$

Wave theory also gives the dispersion relationship

$$C_P = \sqrt{(g/k \, \text{Tanh}[kh])}$$
$$\omega = \sqrt{(gk \, \text{Tanh}[kh])} \quad .$$

The dispersion relationship allows us to find the wave number k given a wave period T. This in turn allows us to find the particle velocities and accelerations. The particle position equations allow us to determine the orbit size d.

For a large structure such as a GBS or an FPSO, one can use a Panel Method CFD procedure to get motion or load responses. The simplest Panel Method is based on the following integral:

$$\varphi_P = 1/[2\pi] \int_S [\, 1/r \, \partial\varphi_Q/\partial n - \varphi_Q \, \partial(1/r)/\partial n \,] \, dS$$

where P and Q are points on the surface of the fluid. The boundary conditions allow us to replace all of the $\partial\varphi_Q/\partial n$ terms in the integral. The boundary conditions are: (1) the seabed kinematic constraint (2) the water surface dynamic and kinematic constraints (3) the structure kinematic constraint (4) the radiation constraint on outer surface.

When the surface of the fluid is divided into panels, the integral can be replaced by the following sum:

$$\varphi_P = 1/[2\pi] \sum_S [\ 1/r\ \partial\varphi_Q\ /\partial n - \varphi_Q\ \Delta(1/r)/\Delta n\ ]\ \Delta S \quad .$$

Together with boundary conditions, this equation gives φ_P. The unsteady Bernoulli equation then gives pressure:

$$\partial\varphi/\partial t + P/\rho + gz = 0 \quad .$$

Integration of pressure then gives loads:

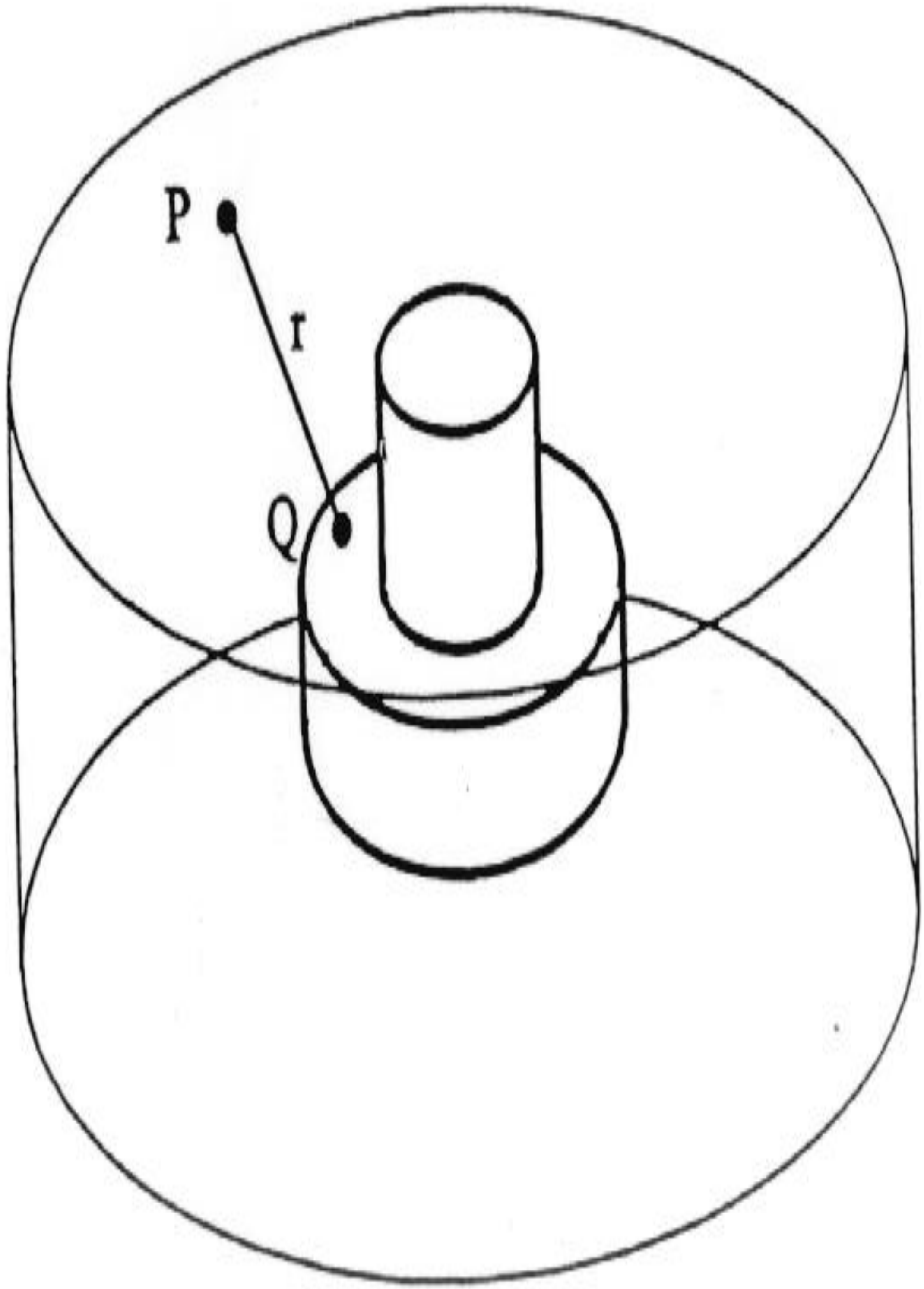
$$\mathbf{F} = -\int_S P\ \mathbf{n}\ dS \qquad \mathbf{M} = -\int_S P\ (\mathbf{r} \times \mathbf{n})\ dS \quad .$$

Substitution into Newton's Second Law then gives motions.

Another Panel Method is based on the following integral:

$$\varphi(P) = 1/[4\pi] \int_S f(Q)\ G(P,Q)\ dS \quad .$$

The strengths f of the complex sources G are adjusted so that there is no flow into the structure due to the wave. The details of this are beyond the scope of this note.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