FLUIDS AT REST

HYDROSTATIC

STABILITY

METACENTER

When a neutrally buoyant body is rotated, it will return to its original orientation if buoyancy and weight create a restoring moment. For a submerged body, this occurs when the center of gravity is below the center of buoyancy. The center of buoyancy will act like a pendulum pivot. For a floating body, a restoring moment is generated when the center of gravity is below a point known as the metacenter. In this case, the metacenter acts like a pendulum pivot. The moments of the wedge shaped volumes generated by rotation is equal to the moment due to the shift in the center of volume. These moments are:

$$M_W = K\ \theta \qquad\qquad M_V = V\ S$$

Equating the two moments gives

$$S = K\ \theta\ /\ V$$

The shift in the center of volume can also be related to rotation about the meta center:

$$S = BM\ \theta$$

Equating the two shifts gives

$$BM\ \theta = K\ \theta / V$$

$$BM = K / V$$

For a general case, the moment of the wedges is

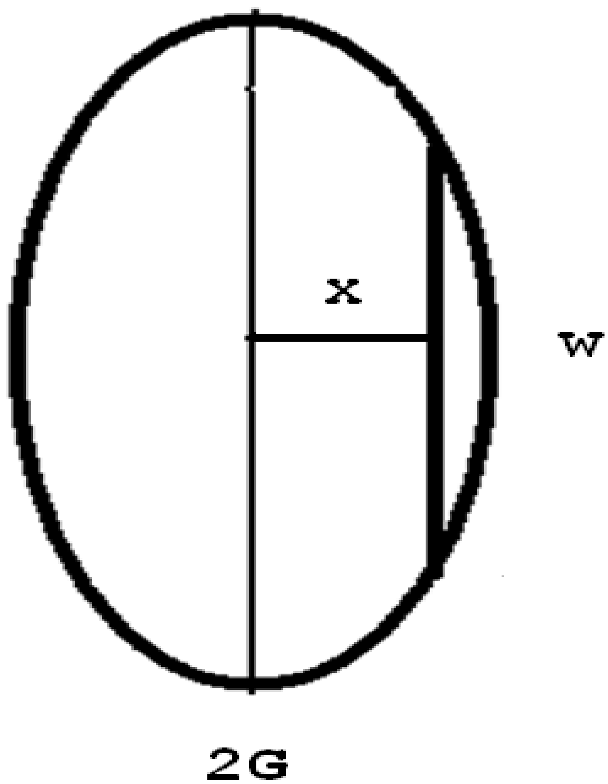
$$\int_{-G}^{+G} x\ x\theta\ w\ dx = K\ \theta$$

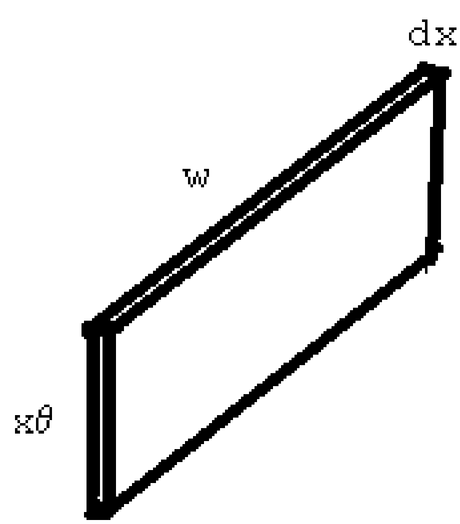
$$2 \int_{0}^{+G} x\ x\theta\ w\ dx = K\ \theta$$

This gives

$$K = \int_{-G}^{+G} x^2\ w\ dx = 2 \int_{0}^{+G} x^2\ w\ dx$$

$$K = \sum_{-G}^{+G} x^2\ w\ \Delta x = 2 \sum_{0}^{+G} x^2\ w\ \Delta x$$

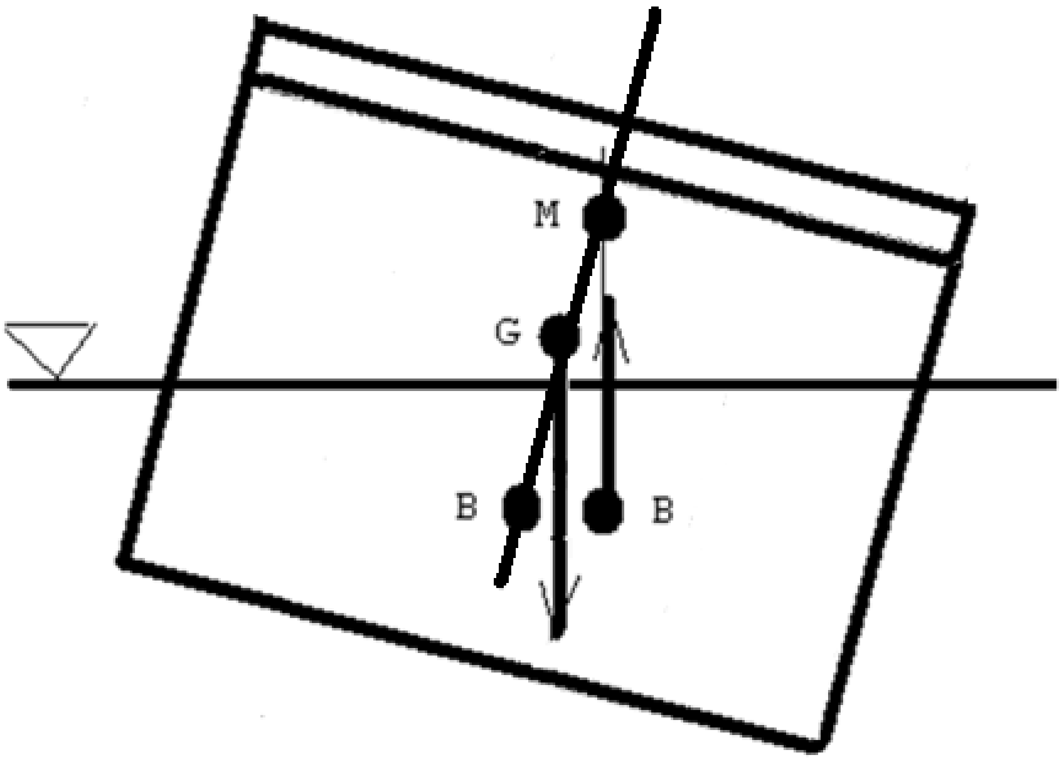

x
w
2G


dx
w
xθ

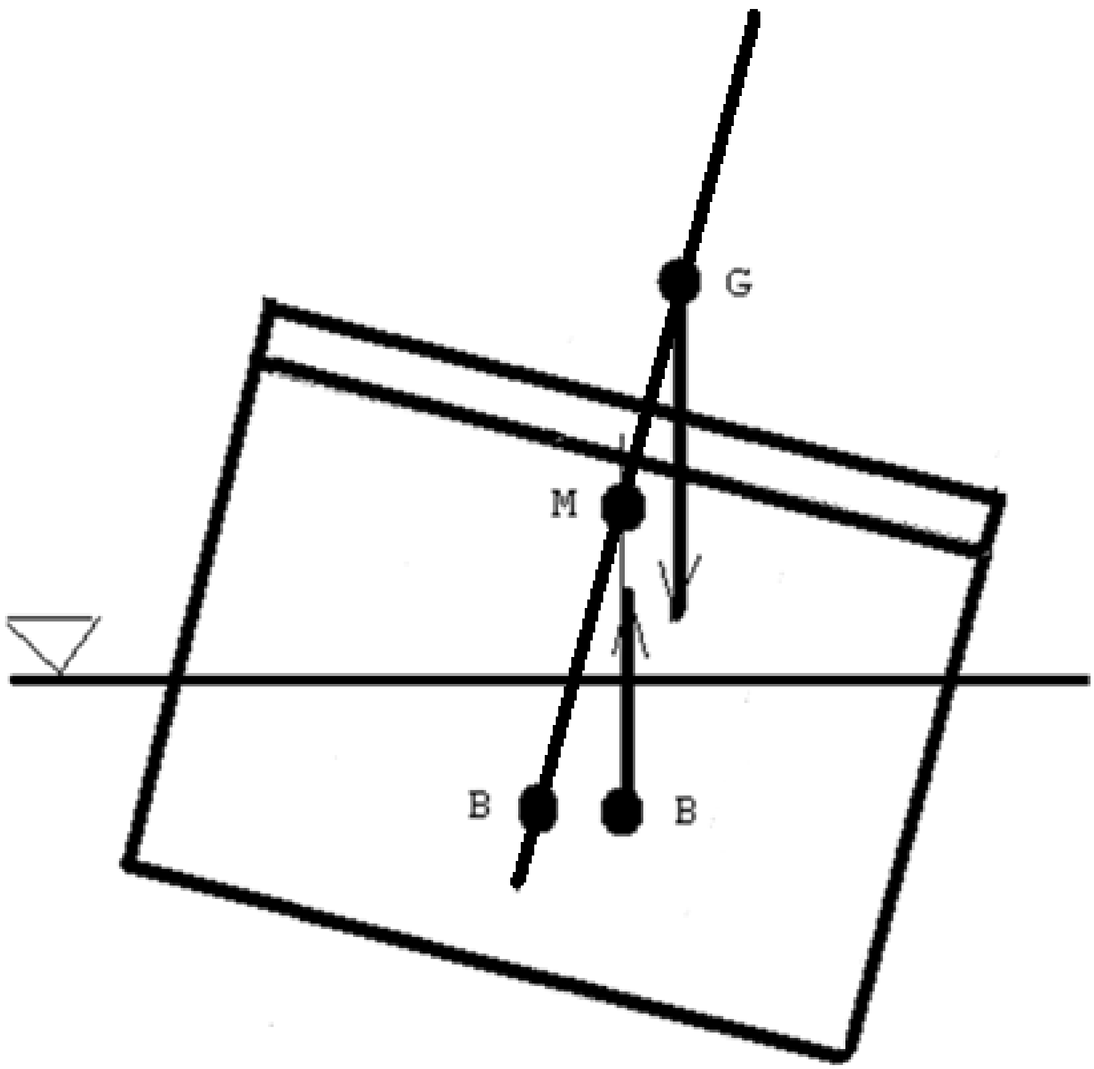
The metacenter M occurs at the intersection of two lines. One line passes through the center of gravity or G and the center of buoyancy or B of a floating body when it is not rotated: the other line is a vertical line through B when the body is rotated. Inspection of a sketch of these lines shows that, if M is above G, gravity and buoyancy generate a restoring moment, whereas if M is below G, gravity and buoyancy generate an overturning moment. One finds the location of M by finding the shift in the center of volume generated during rotation and noting that this shift could result from a rotation about an imaginary point which turns out to be the metacenter. The distance between B and the center of gravity G is BG. Geometry gives GM:

$$GM = BM - BG$$

If GM is positive, M is above G and the body is stable. If GM is negative, M is below G and the body is unstable.



STABLE



UNSTABLE

SINGLE HULL BODIES

$$S \rho g V = \int_{-G}^{+G} x \rho g x\Theta w \, dx$$

$$S \rho g V = 2 \int_{0}^{+G} x \rho g x\Theta w \, dx$$

Slice volume is: $dV = x\Theta w \, dx$

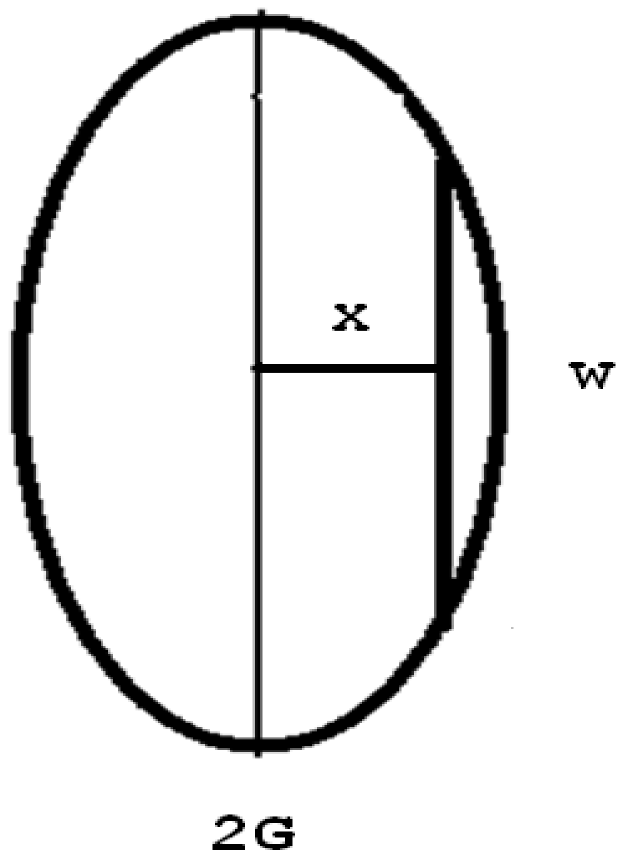
Slice Weight is: $dW = \rho g \, dV$

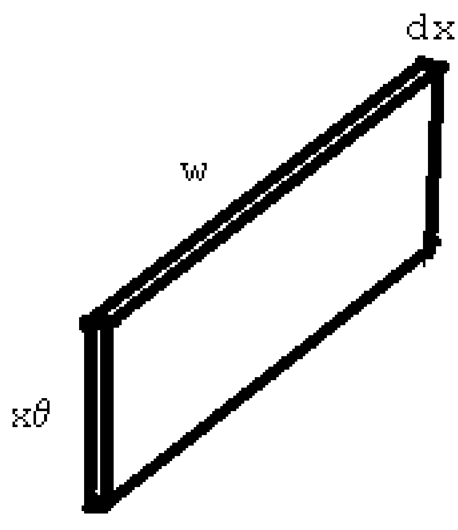
Slice Moment is: $x \, dW$

Integration gives: $\rho g K \Theta$

Manipulation gives: $S = K/V \Theta = R \Theta$

Metacentric Radius: R


x
w
2G


dx
w
xθ

SINGLE BOX RECTANGULAR BARGE

For roll of the barge the wedge factor is

$$K = 2 \int_{0}^{+G} x^2 \, w \, dx$$

$$= 2 * 2L * G^3/3$$

The volume of the barge is

$$V = 2L * 2G * h$$

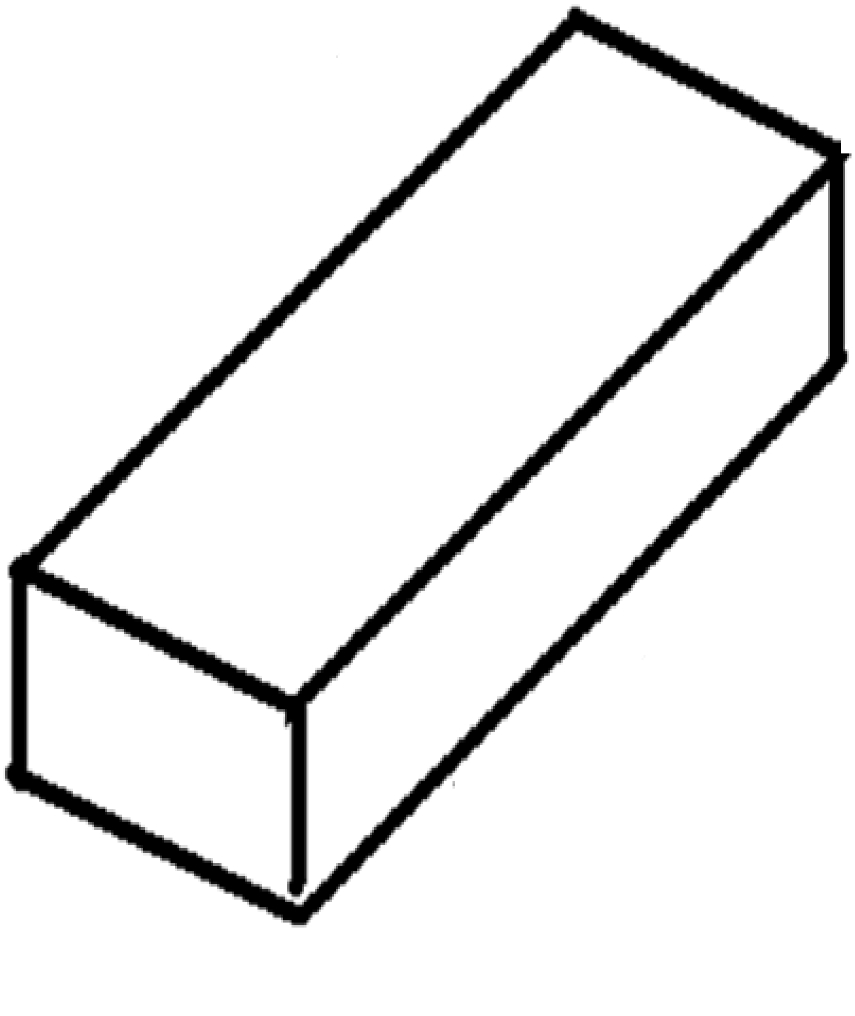
Manipulation gives

$$S = K/V \, \Theta = R \, \Theta$$

$$= G^2/[3h] \, \Theta$$

So the roll metacentric radius is

$$R = G^2/[3h]$$



GBS RIG

For roll of the GBS the wedge factor is

$$K = 2 \int_{0}^{+G} x^2 \, 2\sqrt{[G^2-x^2]} \, dx$$

$$= \pi G^4/4$$

The volume of the GBS rig is

$$V = \pi G^2 * h$$

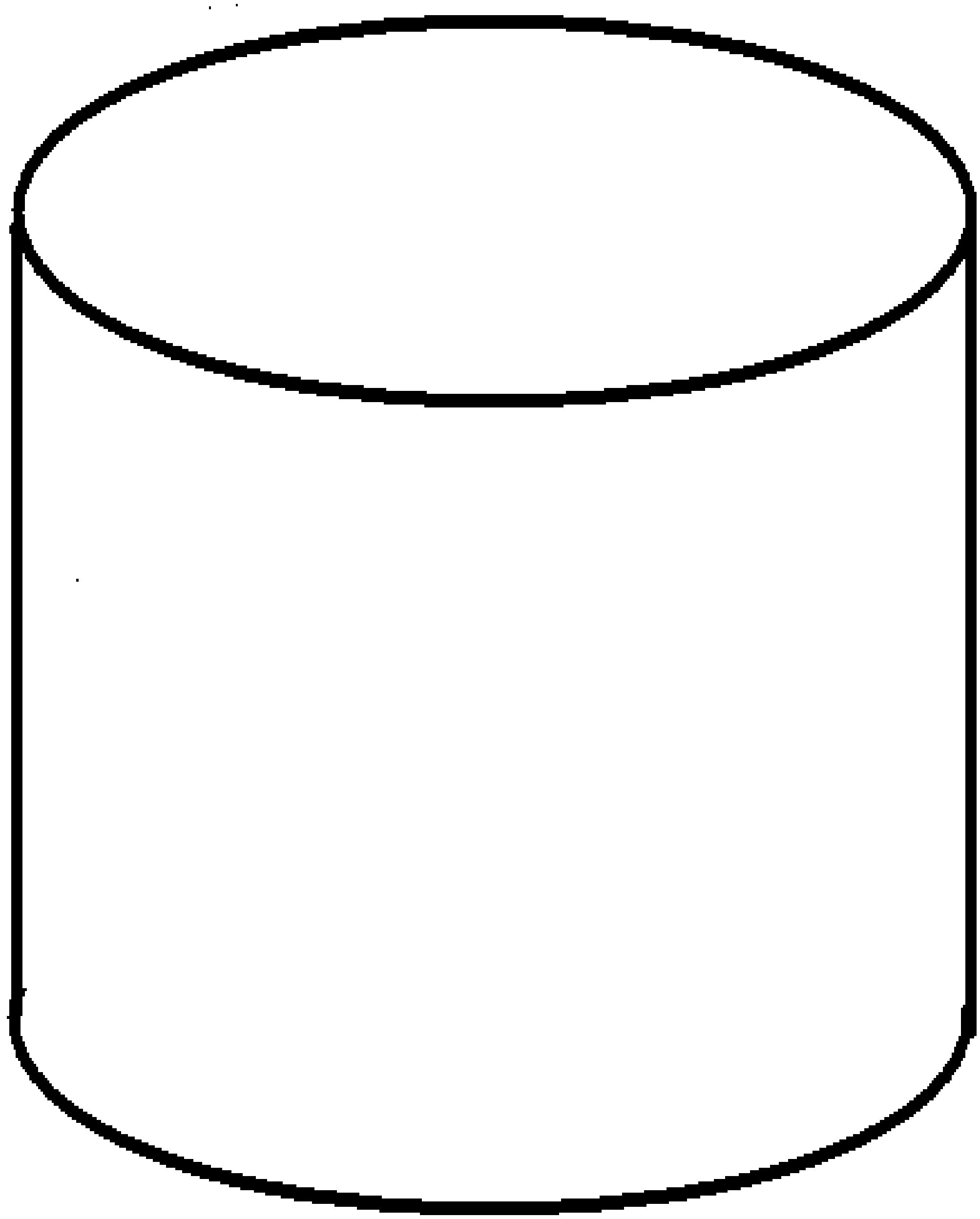
Manipulation gives

$$S = K/V \, \Theta = R \, \Theta$$

$$= G^2/4h \quad \Theta$$

So the roll metacentric radius is

$$R = G^2/4h$$



DOUBLE HULL BODIES

$$S\ \rho g\ V = 2 \int_{H-G}^{H+G} x\ \rho g\ x\Theta\ w\ dx$$

$$S\ \rho g\ V = 2 \int_{-G}^{+G} [H+r]\ \rho g\ [H+r]\ \Theta\ w\ dr$$

Slice volume is: $dV = x\Theta\ w\ dx$

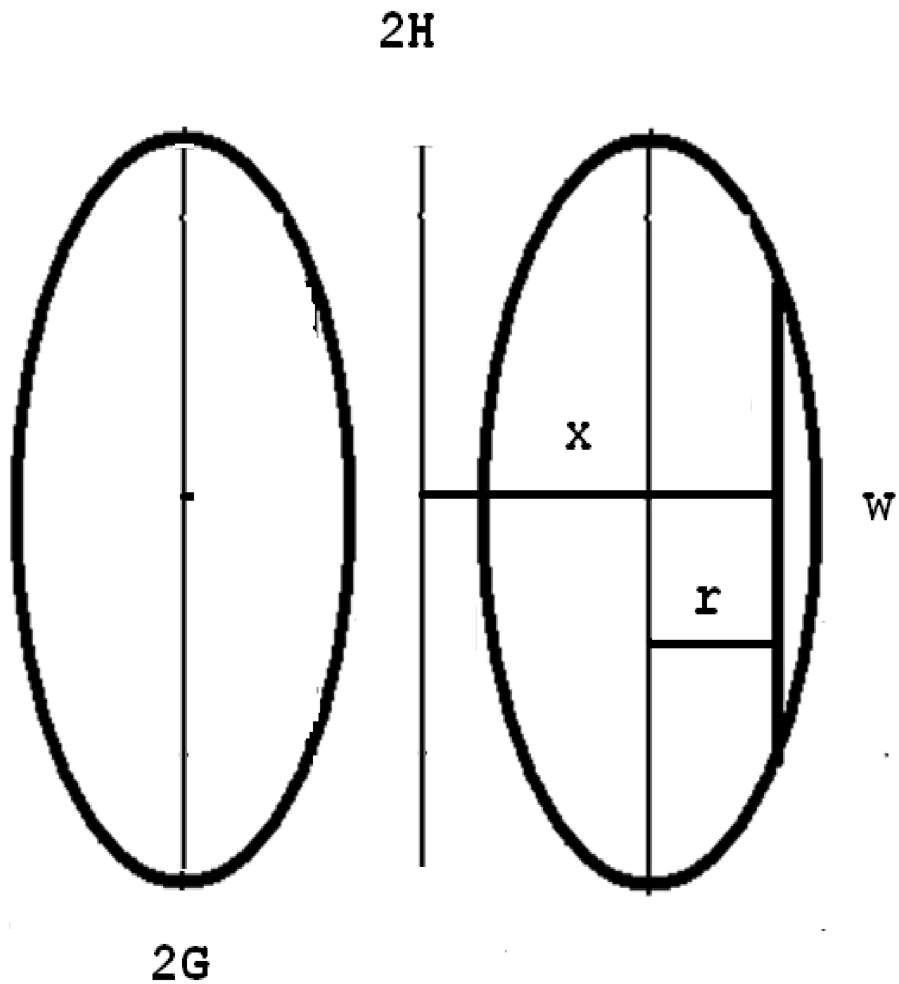
Slice Weight is: $dW = \rho g\ dV$

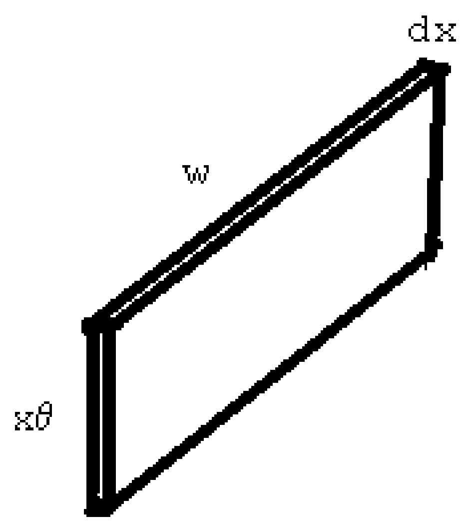
Slice Moment is: $x\ dW$

Integration gives: $\rho g\ K\ \Theta$

Manipulation gives: $S = K/V\ \Theta = R\ \Theta$

Metacentric Radius: R


2H
x
w
r
2G


dx
w
xθ

DOUBLE BOX RECTANGULAR BARGE

For roll of the barge the wedge factor is

$$K = 2 \int_{-G}^{+G} (H+r)^2 \, 2L \, dr$$

$$= 2L \, (4G^3/3 + 4H^2G)$$

The volume of the barge is

$$V = 2 * 2L * 2G * h$$

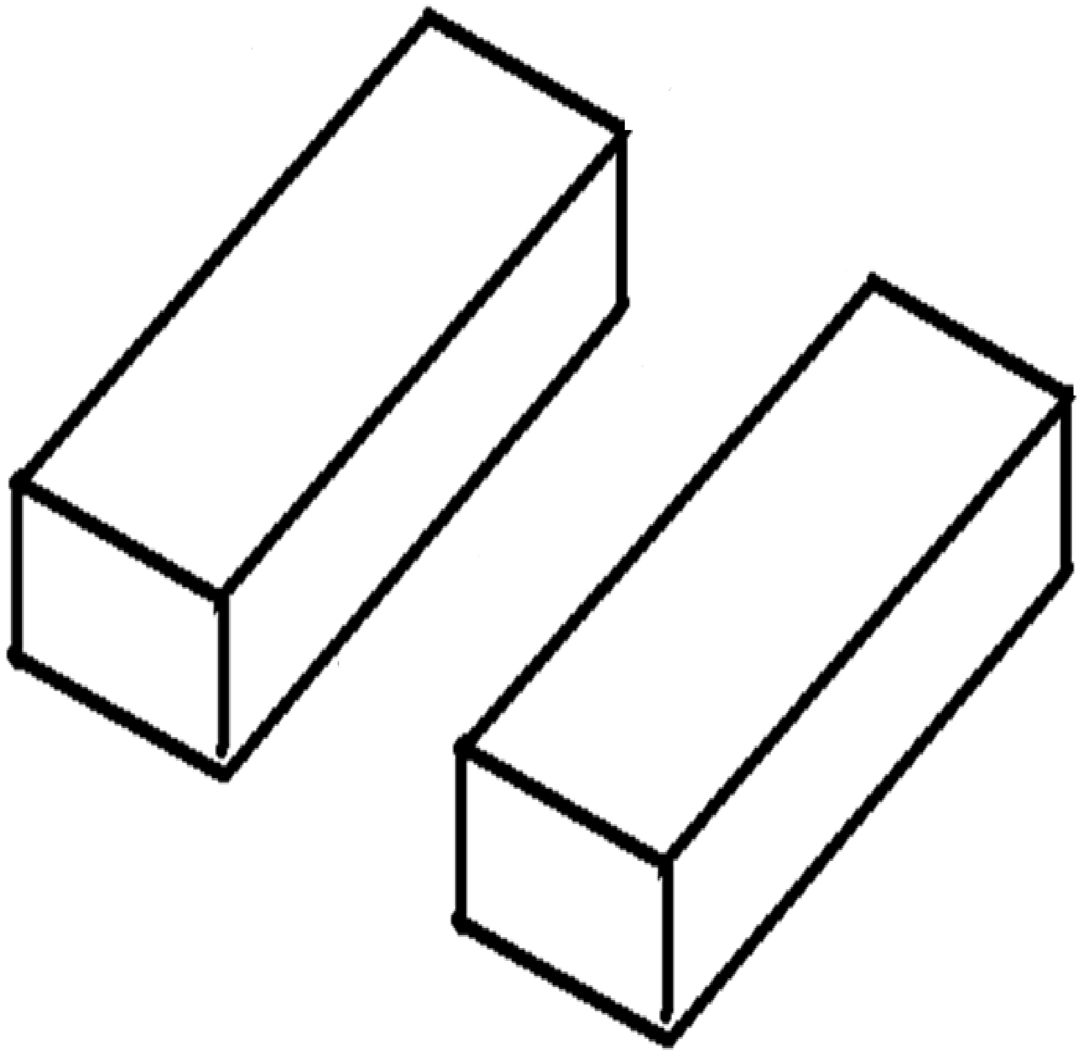
Manipulation gives

$$S = K/V \, \Theta = R \, \Theta$$

$$= (G^2/3h + H^2/h) \quad \Theta$$

So the roll metacentric radius is

$$R = G^2/3h + H^2/h$$



OIL RIG

For roll of the rig the wedge factor is

$$K = 4 \int_{-G}^{+G} (H+r)^2 \; 2\sqrt{[G^2-r^2]} \;\; dr$$

$$= \pi G^4 + 4\pi G^2 H^2$$

The volume of the rig is

$$V = 4 * \pi G^2 * h$$

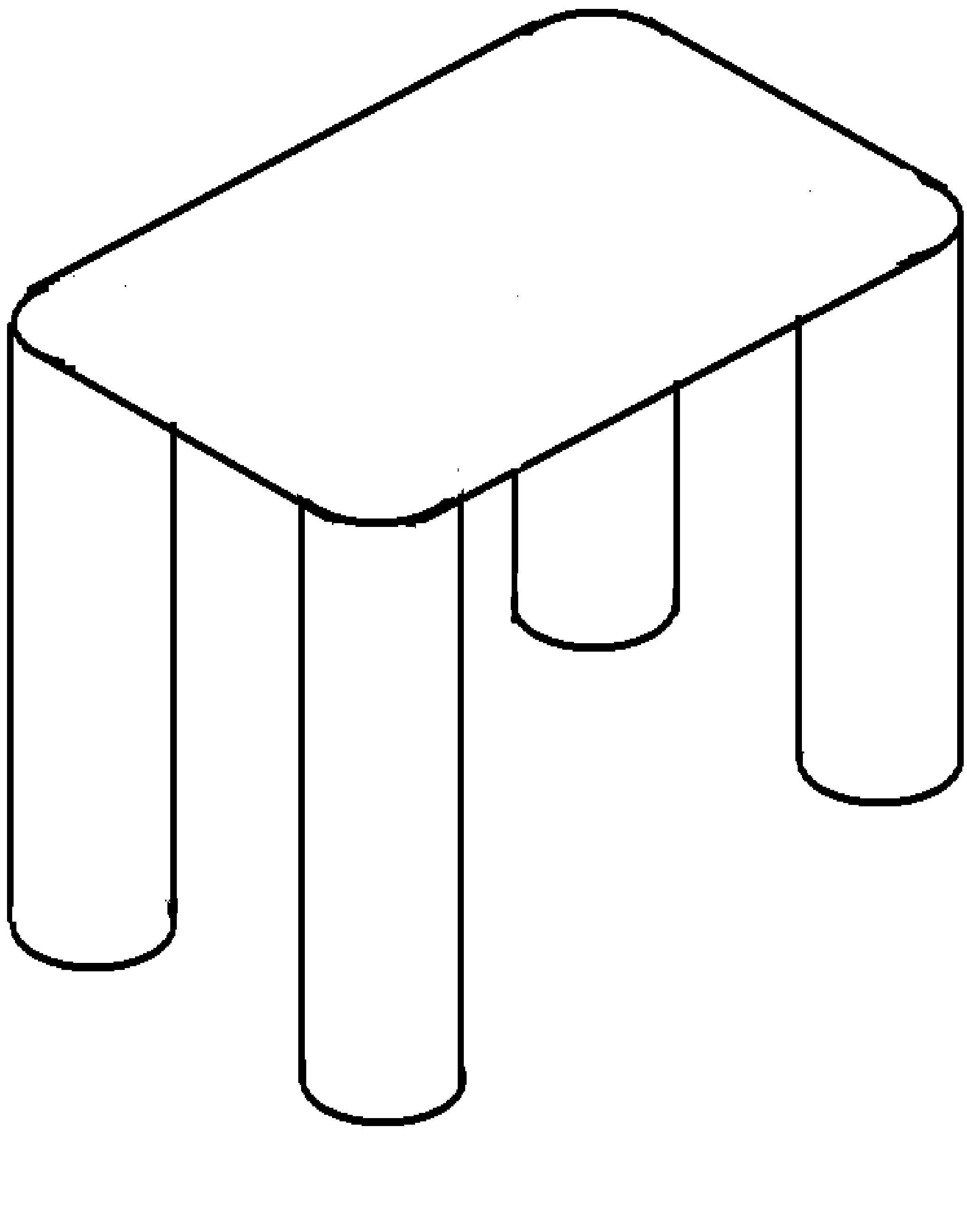
Manipulation gives

$$S = K/V \; \Theta \;\; = \;\; R \; \Theta$$

$$= (G^2/4h + H^2/h) \;\; \Theta$$

So the roll metacentric radius is

$$R \;\; = \;\; G^2/4h + H^2/h$$



When the spacing of the legs is large relative to the diameter of the legs, the wedge shaped volumes can be taken to be cylinders with total volume

$$4 \ \pi G^2 \ H\Theta$$

The moment of these volumes is

$$H \ 4 \ \pi G^2 \ H\Theta \ = K \ \Theta$$

Manipulation gives

$$S = K/V \ \Theta \ = \ R \ \Theta$$

$$= H^2/h \ \ \Theta$$

So the roll metacentric radius is

$$R \ = \ H^2/h$$

INTEGRALS

$$\int_{-G}^{+G} \sqrt{[G^2-r^2]}\ dr = r/2\ \sqrt{[G^2-r^2]} + G^2/2\ \mathrm{Sin}^{-1}[r/G]$$

$$\int_{-G}^{+G} r\ \sqrt{[G^2-r^2]}\ dr = -\ [G^2-r^2]^{3/2}\ /\ 3$$

$$\int_{-G}^{+G} r^2\ \sqrt{[G^2-r^2]}\ dr = -\ r\ [G^2-r^2]^{3/2}\ /\ 4$$

$$+\ r\ G^2/8\ \sqrt{[G^2-r^2]}\ +\ G^4/8\ \mathrm{Sin}^{-1}\ [r/G]$$

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%
%   OIL RIG ROLL STABILITY
%
    PANELS=10000;PI=3.14159;
    RADIUS=5.0;DEPTH=5.0;
    CHANGE=2.0*RADIUS/PANELS;
    GRAVITY=9.81;DENSITY=1000.0;
    VOLUME=4.0*DEPTH*PI*RADIUS^2;
    CENTROID=10.0;
%
%   APPROXIMATE METACENTER
    BM=CENTROID^2/DEPTH
%   EXACT METACENTER
    BM=CENTROID^2/DEPTH ....
     +RADIUS^2/DEPTH/4.0
%
%   PANEL METHOD
    WEDGE=0.0;
    LOCATION=-RADIUS+CHANGE/2.0;
    for STEPS=1:PANELS
    ARM=CENTROID+LOCATION;
    WIDTH=2.0*sqrt(RADIUS^2-LOCATION^2);
    WEDGE=WEDGE+4.0*ARM^2*WIDTH*CHANGE;
    LOCATION=LOCATION+CHANGE;
    end
    BM=WEDGE/VOLUME
```