FLUIDS AT REST

CONCEPTS

PRESSURE DEPTH LAW

Consider an imaginary vertical cylinder of water extending down from an interface between air and water. A schematic is shown on the next page. Let the cross sectional area of the cylinder is A and let its height be h. Let the pressure at the top be P_o and the pressure at the bottom be P.

The weight of the cylinder is

$$W = \rho g V = \rho g A h$$

The pressure load on the cylinder is

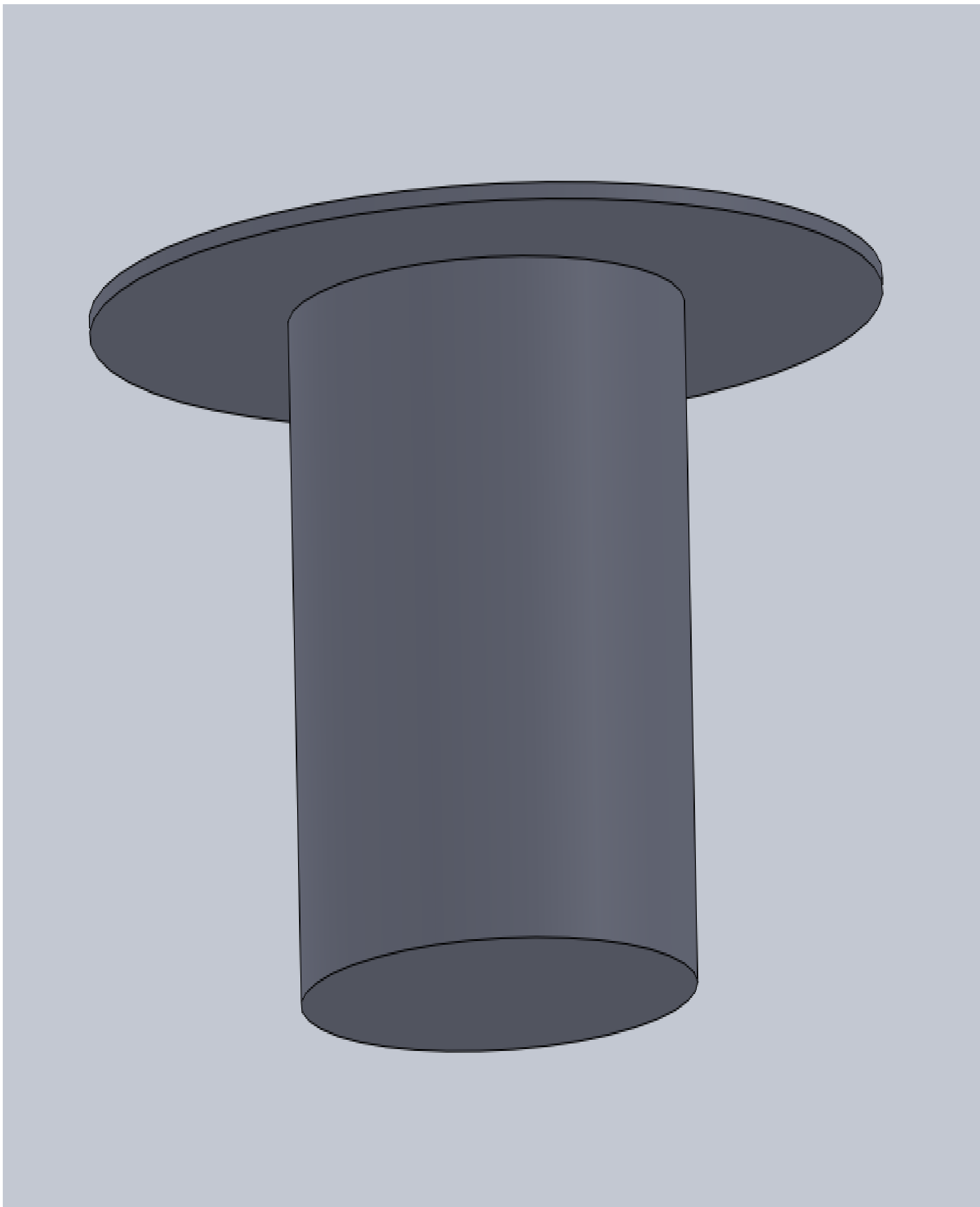
$$P A - P_o A$$

A load balance gives

$$P A - P_o A - \rho g A h = 0$$

$$\Delta P = P - P_o = \rho g h$$

This is the pressure depth law for water.



BUOYANCY

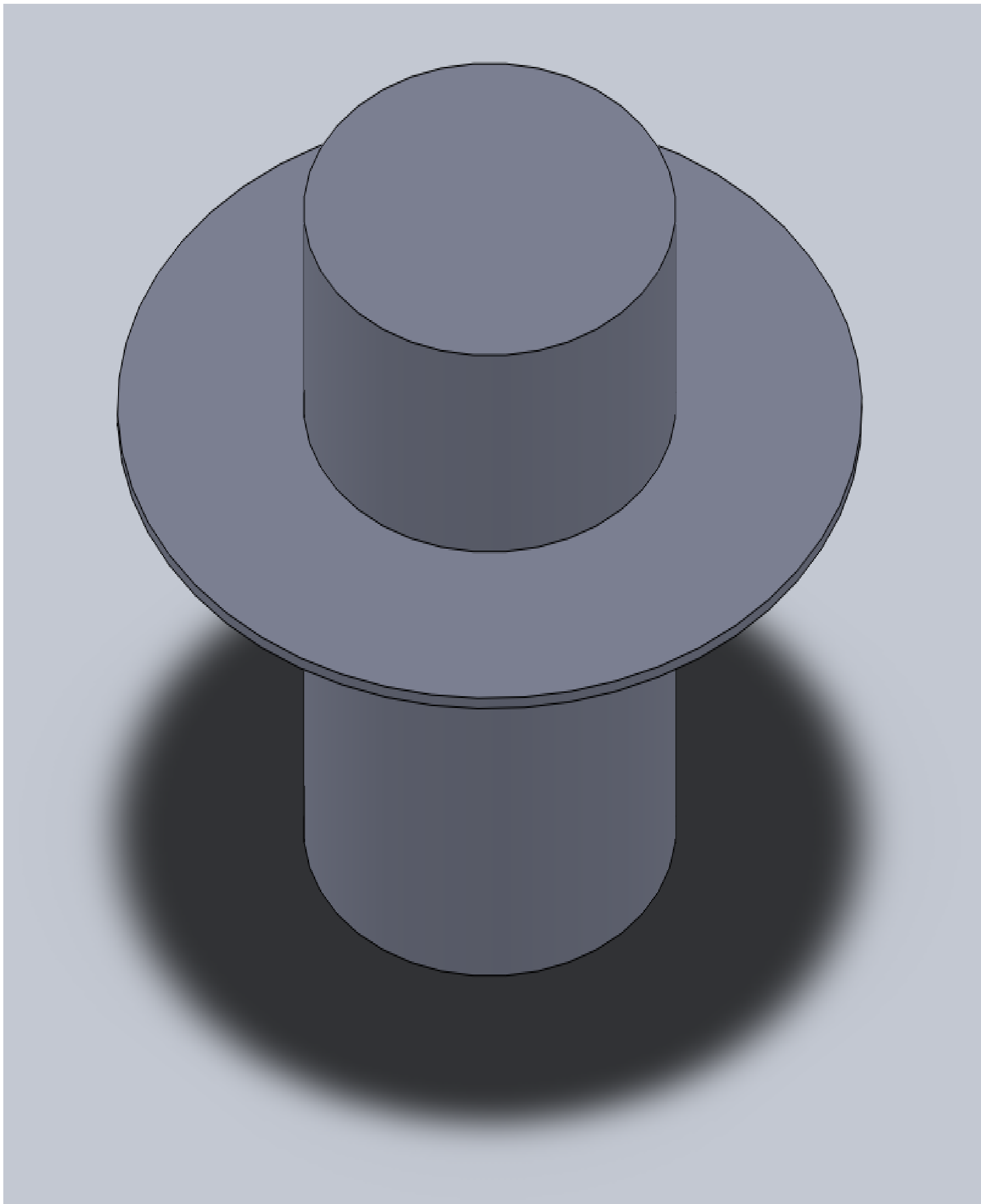
Consider an imaginary vertical cylinder of water extending down from an interface between air and water. A schematic is shown on the previous page. Let the cross sectional area of the cylinder be A and let its height be h. Let the pressure at the top be P_o and the pressure at the bottom be P.

The pressure load on the cylinder is

$$B = P\ A - P_o\ A = \Delta P\ A$$

$$= \rho g h\ A = \rho g\ V = W$$

The pressure load is known as the buoyancy. It is equal to the weight of the displaced volume of water. A floating cylinder would have a part below water and a part above water. A schematic is shown on the next page. The part below water would displace a volume of water with a buoyancy force equal to the body weight.



BUOYANCY SPRING

The buoyancy force can be written as

$$B = \rho g A \ h$$

This resembles a spring

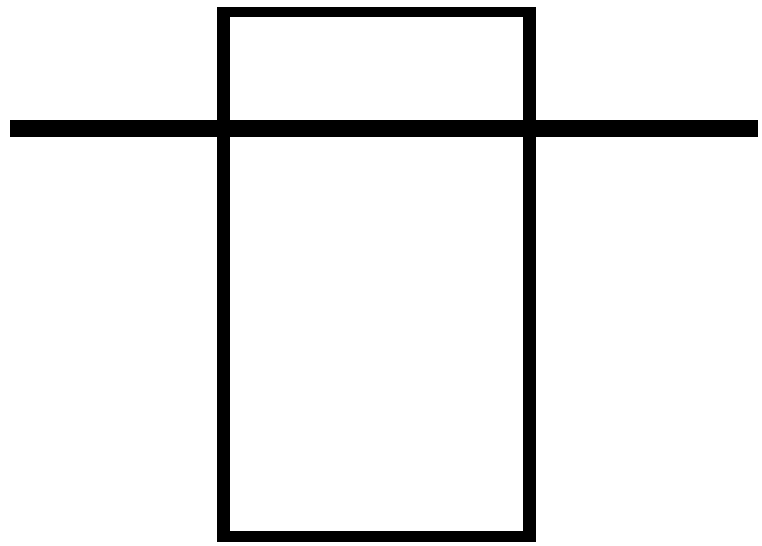
$$F = K \ x$$

The buoyancy spring constant is

$$K = \rho g A$$

A schematic of the analogy is on the next page.

Pushing the cylinder downwards increases the buoyancy force. It compresses the buoyancy spring. It pushes the bottom down to a level where the pressure is higher and that creates an increase in the force upwards.



STABILITY

To study the concept of stability, consider a rig with 4 legs that are spaced far apart. A schematic is shown on the next page. Let the spacing of the legs be 2H. Let the area of each leg be A and let the depth of submergence be h. Because H is large, when the rig rolls an angle θ, the displaced volume at the bottom of each leg is approximately a cylinder with area A and height $H\theta$. The torque due to buoyancy is

$$\Delta T = 4\ H\ \rho g\ A\ H\theta$$

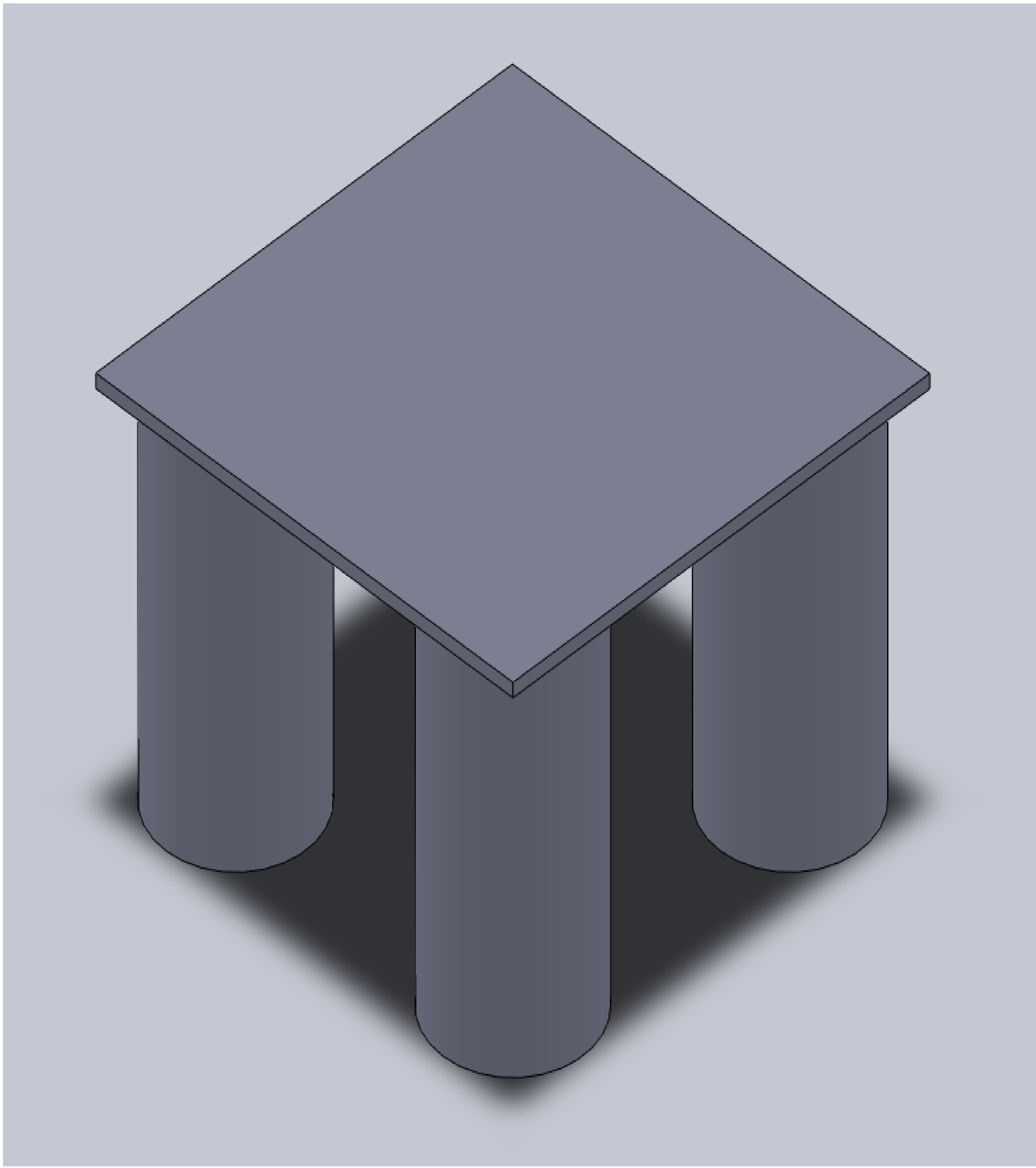
The rotation causes a shift S in the center of the displaced volume and the torque due to this is

$$\Delta T = S\ 4\ \rho g\ Ah$$

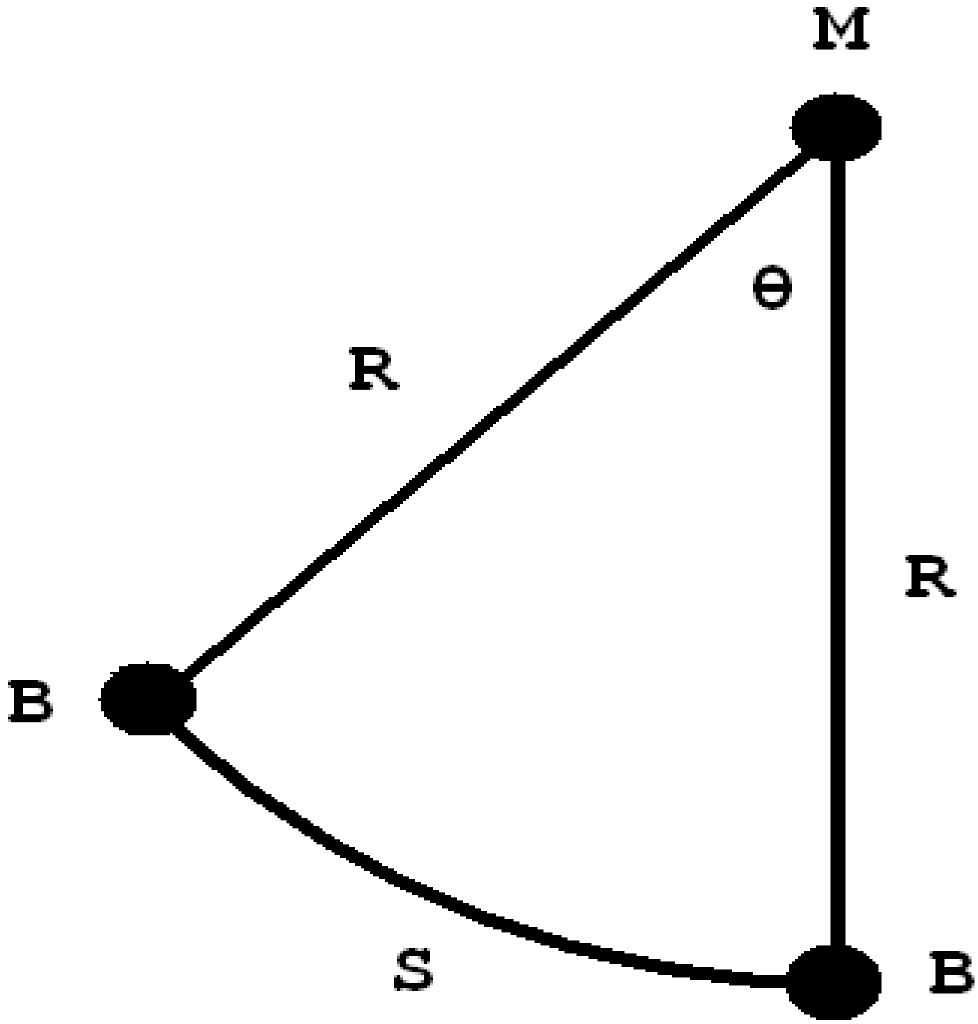
A torque balance gives

$$S\ 4\ \rho g\ Ah = 4\ H\ \rho g\ A\ H\theta$$

$$S = H^2/h\ \theta = R\ \theta$$



This suggests that the center of buoyancy has moved along a circular arc with radius R. A schematic of this shift is shown in the sketch on the next page. The radius is known as the metacentric radius. The center of rotation is known as the metacenter. The line of action of the buoyancy force always passes through this point. Knowing the location of the metacenter allows us to locate the line of action of the buoyancy force. If the center of gravity is below the metacenter, the weight and buoyancy forces create a restoring moment and the rig is stable. If the center of gravity is above the metacenter, the weight and buoyancy forces create an overturning moment and the rig is unstable.


M
θ
R
R
B
S
B