FLUIDS IN MOTION

PIPE

NETWORKS

PIPE NETWORKS

The basic element of a pipe network is a pipe.

Conservation of mass for a pipe gives

$$\dot{M}_{OUT} = \dot{M}_{IN}$$

$$[\rho CA]_{OUT} = [\rho CA]_{IN}$$

Since density and area are the same at inlet and outlet this equation implies that C is the same at inlet and outlet.

Conservation of energy gives

$$[\dot{M} gh]_{OUT} - [\dot{M} gh]_{IN} = \dot{M} gh_T - \dot{M} gh_L$$

$$h_{OUT} - h_{IN} = h_T - h_L$$

where at the inlet and outlet

$$h = C^2/2g + P/\rho g + z$$

and head loss is given by

$$h_L = (fL/D + \Sigma K)\ C^2/2g$$

For a pipe without a turbomachine this reduces to

$$[P/\rho g + z]_{OUT} - [P/\rho g + z]_{IN}$$

$$= [fL/D + \Sigma K]\ C^2/2g$$

$$= [fL/D + \Sigma K]/[2gA^2]\ \ Q^2$$

$$= R\ Q^2$$

Note that for pipes in series the net resistance is

$$R = \Sigma R$$

while for pipes in parallel the net resistance is

$$[\ 1\ /\ \Sigma[1/\sqrt{R}]\ ]^2$$

PRESSURE ITERATION METHOD FOR PIPE NETWORKS

In the pressure iteration method, one would first assume pressure at each node in the network where it is not known. Then for each node one would assume pressures at the surrounding nodes to be fixed. Next for each pipe connected to the node one balances head loss with pressure/gravity head: here pumps are treated as negative head losses while turbines are treated as positive head losses. This allows us to calculate the flow in each pipe and its direction. One then calculates the sum of the flows into the node treating flows in as positive and flows out as negative. If the $\Sigma Q>0$ then the node acts like a sink and the pressure there is too low and must be increased a bit. If the $\Sigma Q<0$ then the node acts like a source and the pressure there is too high and must be lowered a bit. Each node in the network is treated the same way. One sweeps through the network nodes again and again until the ΣQ for each node is approximately zero.

FLOW ITERATION METHOD FOR PIPE NETWORKS

In the flow iteration method, one assumes a distribution of flow which satisfies $\Sigma Q=0$ at each node in the network. The flow iteration method modifies flows throughout the network in a way which maintains $\Sigma Q=0$ at each node. In the method one identifies pipe loops in the network. Then for each loop one calculates the sum of the head losses as one moves around it in a clockwise sense. If flow in a pipe is clockwise head loss is taken to be positive whereas if flow is counterclockwise head loss is taken to be negative. For a loop if the $\Sigma h_L>0$ then there is too much clockwise flow: so flows must be reduced a bit in a clockwise sense. This decreases clockwise flows and increases counterclockwise flows. If the $\Sigma h_L<0$ then there is not enough clockwise flow: so flows must be increased a bit in a clockwise sense. This increases clockwise flows and decreases counterclockwise flows. Each loop in the network is treated the same way. One sweeps through the network loops again and again until the Σh_L for each loop is approximately zero. Special pseudo loops are used to connect reservoirs.

SYSTEM DEMAND

For a system where a pipe connects two reservoirs, the head H versus flow Q system demand equation has the form:

$$H = X + Y\,Q^2$$

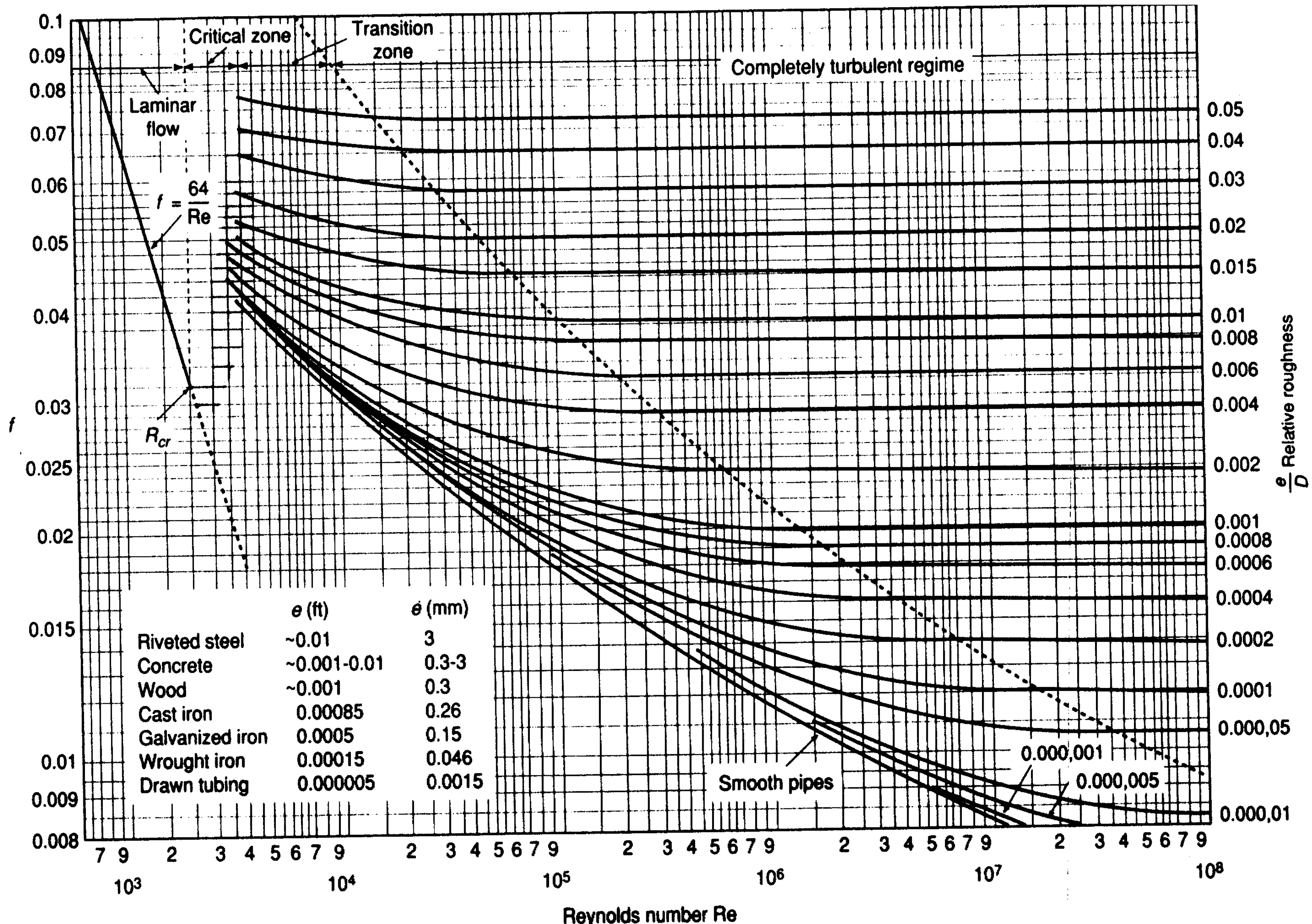
$$X = \Delta\,[P/\rho g + z] \qquad Y = [fL/D + \Sigma K]/[2gA^2]$$

X accounts for pressure and height changes between the reservoirs and Y accounts for losses along the pipe.

To pick a pump, one first calculates the specific speed $\mathbf{N}$ based on the system operating point. This is a nondimensional number which does not have pump size in it:

$$\mathbf{N} = [N\,\sqrt{Q}]/[H^{3/4}]$$

where N is the pump RPM, Q is its flow in GPM and H is its head in FEET. It allows one to pick the appropriate type of pump. Axial pumps have high Q but low H which gives them high $\mathbf{N}$. Radial pumps have lower Q but higher H which gives them lower $\mathbf{N}$. Positive Displacement pumps have the lowest Q but highest H which gives them the lowest $\mathbf{N}$. Next one scans pump catalogs of the type indicated by specific speed and picks the size of pump that will meet the system demand.



Moody diagram. (From L. F. Moody, *Trans. ASME*, Vol. 66, 1944.)

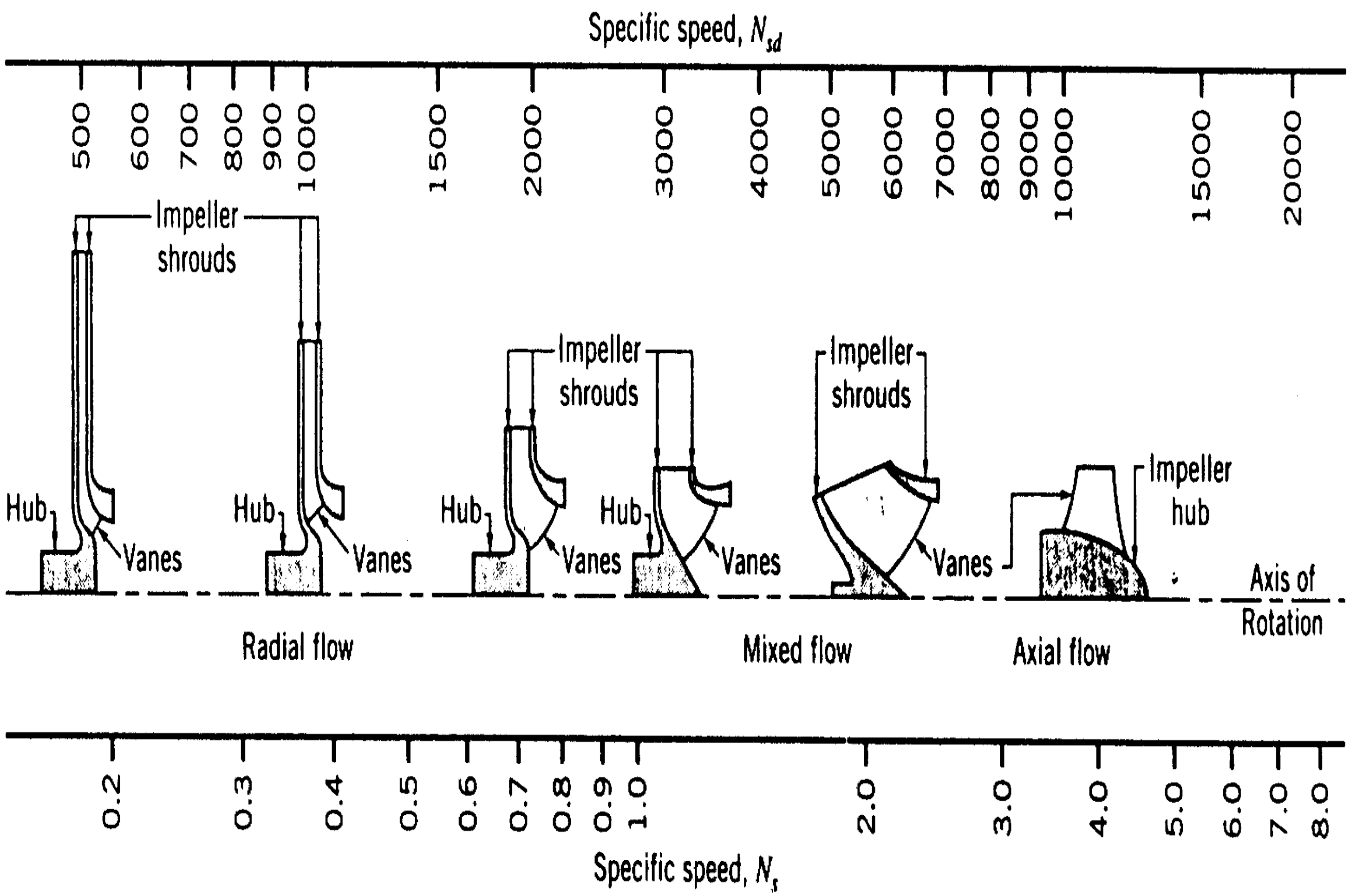
Nominal Loss Coefficients K (Turbulent Flow)[a]

Type of fitting	Screwed			Flanged		
Diameter	1 in.	2 in.	4 in.	2 in.	4 in.	8 in.
Globe valve (fully open)	8.2	6.9	5.7	8.5	6.0	5.8
(half open)	20	17	14	21	15	14
(one-quarter open)	57	48	40	60	42	41
Angle valve (fully open)	4.7	2.0	1.0	2.4	2.0	2.0
Swing check valve (fully open)	2.9	2.1	2.0	2.0	2.0	2.0
Gate valve (fully open)	0.24	0.16	0.11	0.35	0.16	0.07
Return bend	1.5	.95	.64	0.35	0.30	0.25
Tee (branch)	1.8	1.4	1.1	0.80	0.64	0.58
Tee (line)	0.9	0.9	0.9	0.19	0.14	0.10
Standard elbow	1.5	0.95	0.64	0.39	0.30	0.26
Long sweep elbow	0.72	0.41	0.23	0.30	0.19	0.15
45° elbow	0.32	0.30	0.29			
Square-edged entrance			0.5			
Reentrant entrance			0.8			
Well-rounded entrance			0.03			
Pipe exit			1.0			
Sudden contraction[b] — Area ratio 2:1			0.25			
Sudden contraction[b] — Area ratio 5:1			0.41			
Sudden contraction[b] — Area ratio 10:1			0.46			
Orifice plate — Area ratio A/A_0 1.5:1			0.85			
Orifice plate — Area ratio A/A_0 2:1			3.4			
Orifice plate — Area ratio A/A_0 4:1			29			
Orifice plate — Area ratio A/A_0 ≥ 6:1			$2.78\left(\frac{A}{A_0} - 0.6\right)^2$			
Sudden enlargement[c]			$\left(1 - \frac{A_1}{A_2}\right)^2$			
90° miter bend (without vanes)			1.1			
(with vanes)			0.2			
General contraction (30° included angle)			0.02			
General contraction (70° included angle)			0.07			

[a]Values for other geometries can be found in *Technical Paper 410*, The Crane Company, 1957.

[b]Based on exit velocity V_2.

[c]Based on entrance velocity V_1.


Specific speed, N_{sd}
500
600
700
800
900
1000
1500
2000
3000
4000
5000
6000
7000
8000
9000
10000
15000
20000
Impeller shrouds
Hub
Vanes
Hub
Vanes
Impeller shrouds
Hub
Vanes
Hub
Vanes
Impeller shrouds
Vanes
Impeller hub
Axis of Rotation
Radial flow
Mixed flow
Axial flow
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
Specific speed, N_s

CAVITATION PREVENTION

To prevent cavitation, the pump is located in the system at a point where it has the Net Positive Suction Head or NPSH recommended by the manufacturer:

$$NPSH = P_s/\rho g + C_s C_s/2g - P_v/\rho g$$

In this equation, P_v is the absolute vapor pressure of the fluid being pumped, and P_s and C_s are the absolute pressure and speed at the pump inlet. For a system where a pipe connects a low reservoir to a high reservoir, conservation of energy from the low reservoir to the pump inlet gives:

$$P_o/\rho g - [P_s/\rho g + C_s C_s/2g + d] = h_L$$

where P_o is the absolute pressure of the air above the low reservoir and d is the height of the pump above the surface of the low reservoir. Manipulation gives

$$d = (P_o - P_v)/\rho g - h_L - NPSH$$

This shows that d might have to be negative.

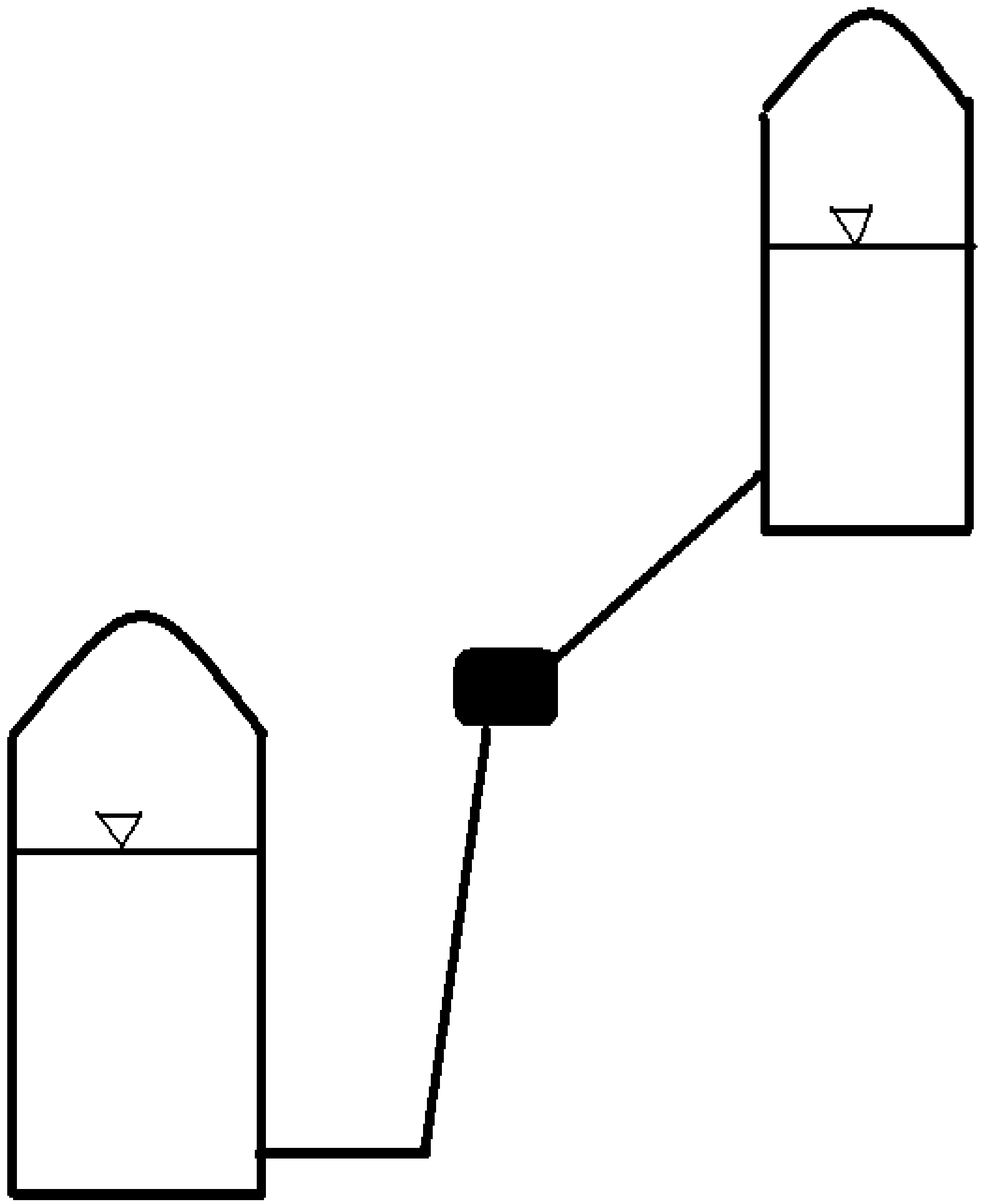


ILLUSTRATION: TANK PIPE SYSTEM

A pump is used to draw 0.01 m^3/s of water from a reservoir and deposit it in a holding tank open to atmosphere. The vertical distance from the surface of the reservoir up to the water surface in the holding tank is 100m. The diameter of the pipe that connects the reservoir to the tank is 0.25m and its length is 1000m. The pipe has roughness bumps that are 0.025mm high. How much pump power would be required for this task? [35] How would you determine the type of pump appropriate for this task? [5] Say the pump manufacturer said it needed an NPSH of 5m. What does that mean? [5] Imagine that the pump is replaced by a 50m head turbine. How much power would the turbine produce? [15]

For a single pipe Conservation of Energy gives

$$h_{OUT} - h_{IN} = h_T - h_L$$

$$h = C^2/2g + P/\rho g + z$$

$$h_L = (fL/D + \Sigma K)\ C^2/2g$$

For the pump case : h_{OUT}=100m and h_{IN}=0. Take $\Sigma K=0$. We get flow speed C from Q=CA. We get Reynolds Number Re from $Re=CD/\upsilon$. We get relative roughness ε from $\varepsilon=e/D$.

$$C = Q/A = 0.01/[\pi*0.25^2/4] = 0.01/0.0491 = 0.204m/s$$

$$Re = CD/\upsilon = 0.204*0.25/0.000001 = 51000$$

$$\varepsilon = e/D = 0.025/250.0 = 0.0001$$

The Moody diagram gives the friction factor: f=0.02. Substitution into energy gives the pump head:

$$h_P = h_{OUT} - h_{IN} + (fL/D + \Sigma K)\ C^2/2g$$

$$= 100 - 0 + (0.02*1000/0.25)\ 0.204^2/[2*9.81]$$

$$= 100\text{m (approximately)}$$

Substitution into $\rho g h_P Q$ gives power: **P** = 9.8kW.

The Specific Speed of a pump is equal to $N\sqrt{Q}/H^{3/4}$ where N is the rotor speed in RPM, Q is flow rate in GPM and H is pump head in FEET. A Specific Speed chart gives the pump type.

The NPSH of a pump is equal to $P_s/\rho g + U_s U_s/2g - P_v/\rho g$. To avoid cavitation within the pump, the manufacturer recommends a certain value of NPSH: the inlet stagnation head should be greater than the vapor head by a specific amount.

The head of the turbine is 50m. The reservoir head is 100m. This gives 50m of head that must be taken away by losses. Take $\Sigma K=0$. Let f be 0.02. Substitution into energy gives C. Should use Moody Diagram to fine tune f. Substitution into Q=CA gives flow. Substitution into **P**=$\rho g h_t Q$ gives power.

$$h_{OUT} - h_{IN} = h_T - h_L$$

$$0 - 100 = -50 - (fL/D + \Sigma K)\ C^2/2g$$

$$C = \sqrt{[50*2*9.81]/[0.02*1000/0.25]} = 3.5\text{ m/s}$$

$Q = CA = 0.17m^3/s$ **P** $= \rho g h_t Q = 83kW$

ILLUSTRATION : HEAT EXCHANGER COIL

A coil in a certain heat exchanger is 20m long and forms a closed loop. It has forty 180° bends each with K=0.5. The coil is made of copper pipe with ID equal to 1cm and roughness 0.0015mm. Derive the system demand equation for the exchanger. Let the water flow rate in the coil be 5gpm. What would be the pump power required to run the exchanger?

Imagine that the coil is broken at some point: in this case the heat exchanger becomes like a regular pipe system with an inlet and an outlet. Because the inlet and outlet are at the same point, h_{OUT} equals h_{IN}. This implies that h_P is equal to h_L. The system demand equation is just

$$h_S = h_L = (fL/D + \Sigma K)\ C^2/2g \qquad C=Q/A$$

The flow speed C is 4m/s. The Reynolds Number Re is 40000. The pipe roughness is 0.00015. Knowing the Reynolds Number and the pipe roughness we can get the friction factor from the Moody Chart: it is 0.022. The ΣK is 20. Substitution into the system demand equation gives an h_S of 52m.

At the operating point the system demand head h_S is equal to the pump head h_P. Knowing h_P we can get the pump power from the flow power equation

$$\mathbf{P} = PQ = \rho g\ h_P\ Q$$

Substitution into this gives the power 161W.

ILLUSTRATION : SIPHON

A siphon is used to draw water from a small pond 20m above a cabin in the woods. The overall length of the siphon is 100m and its diameter is 2.5cm. It is made from plastic which can be taken to be smooth. Determine the flow rate through the basic siphon for the case where there is no friction. Determine the flow rate through the basic siphon for the case where there is friction. Determine the flow rate when a 20m head pump is added to help pump water downhill. What type of pump is appropriate for this system?

When there is no friction conservation of energy reduces to

$$h_{OUT} = h_{IN}$$

$$h_{OUT} = C^2/2g \qquad h_{IN} = H$$

This gives

$$C = \sqrt{[2gH]} = 19.8 \text{ m/s}$$

$$Q = C\ A = C\ \pi D^2/4$$
$$= 0.010 \text{ m}^3/\text{s} = 10.0 \text{ L/s}$$

The power in the flow $\mathbf{P}$ is

$$\mathbf{P} = P\ Q = \rho\ C^2/2\ C\ A$$
$$= 196020\ 0.010 = 19602 \text{ Watts}$$

When there is friction conservation of energy reduces to

$$h_{OUT} = h_{IN} - h_L$$

$$h_{OUT} = C^2/2g \qquad h_{IN} = H$$

$$h_L = (fL/D + \Sigma K)\ C^2/2g$$

Manipulation of energy gives

$$C^2/2g = H - (fL/D + \Sigma K)\ C^2/2g$$

$$H = C^2/2g + (fL/D + \Sigma K)\ C^2/2g$$

$$H = (1 + fL/D + \Sigma K)\ C^2/2g$$

Assume that the entrance K is 1.0 and the friction factor f is 0.01. In this case

$$C = \sqrt{[2g\mathbf{H}]}$$

$$\mathbf{H} = H / (1 + fL/D + \Sigma K)$$
$$= 20 / (1 + 40 + 1) = 0.476$$

$$C = \sqrt{[2g\mathbf{H}]} = 3.06 \text{ m/s}$$

$$Q = C\ A$$
$$= 0.00154 \text{ m}^3\text{/s} = 1.54 \text{ L/s}$$

The Reynolds Number based on the above flow is

$$Re = \rho\ C\ D\ /\ \mu$$

$$= 1000\ 3.06\ 0.025\ /\ 0.001 = 76500$$

The Moody Chart gives f around 0.019. This gives

$$C = \sqrt{[2g\mathbf{H}]}$$

$$\mathbf{H} = H\ /\ (1 + fL/D + \Sigma K)$$

$$= 20\ /\ (1 + 76 + 1) = 0.256$$

$$C = \sqrt{[2g\mathbf{H}]} = 2.25\ m/s$$

$$Q = C\ A$$

$$= 0.00114\ m^3/s = 1.14\ L/s$$

The power in the flow $\mathbf{P}$ is

$$\mathbf{P} = P\ Q = \rho\ C^2/2\ C\ A$$

$$= 2531\ 0.00114 = 2.9\ Watts$$

Friction has consumed most of the available power.

A curve fit to the Moody Chart for smooth pipes is

$$f = 1.325 / [\ \ln [5.74/Re^{0.9}\ ]\ ]^2$$

Substitution into this gives

$$f = 1.325 / [\ \ln [5.74/[76500]^{0.9}\ ]^2 = 0.019$$

This agrees with the Moody chart value.

When a pump is added conservation of energy becomes

$$h_{OUT} = h_{IN} + h_P - h_L$$

where

$$h_{OUT} = C^2/2g \qquad H = h_{IN} + h_P$$

$$h_L = (fL/D + \Sigma K)\ C^2/2g$$

Manipulation of energy gives

$$C^2/2g = H - (fL/D + \Sigma K)\ C^2/2g$$

$$H = C^2/2g + (fL/D + \Sigma K)\ C^2/2g$$

$$H = (1 + fL/D + \Sigma K)\ C^2/2g$$

Assume that K is 1.0 and f is 0.01. In this case

$$C = \sqrt{[2g\mathbf{H}]}$$

$$\mathbf{H} = H / (1 + fL/D + \Sigma K)$$
$$= 40 / (1 + 40 + 1) = 0.952$$

$$C = \sqrt{[2g\mathbf{H}]} = 4.32 \text{ m/s}$$

$$Q = C\ A$$
$$= 0.002.18 \text{ m}^3\text{/s} = 2.18 \text{ L/s}$$

The Reynolds Number based on the above flow is

$$Re = \rho\ C\ D\ /\ \mu$$

$$= 1000\ 4.32\ 0.025\ /\ 0.001$$

$$= 108000$$

The Moody Chart gives f around 0.018. This gives

$$C = \sqrt{[2g\mathbf{H}]}$$

$$\mathbf{H} = H / (1 + fL/D + \Sigma K)$$
$$= 40 / (1 + 72 + 1) = 0.541$$

$$C = \sqrt{[2g\mathbf{H}]} = 3.26 \text{ m/s}$$

$$Q = C\ A$$
$$= 0.00165 \text{ m}^3\text{/s} = 1.65 \text{ L/s}$$

The power in the flow $\mathbf{P}$ is

$$\mathbf{P} = P\ Q = \rho\ C^2/2\ C\ A$$

$$= 5314\ 0.00165 = 8.8 \text{ Watts}$$

The specific speed of a pump is

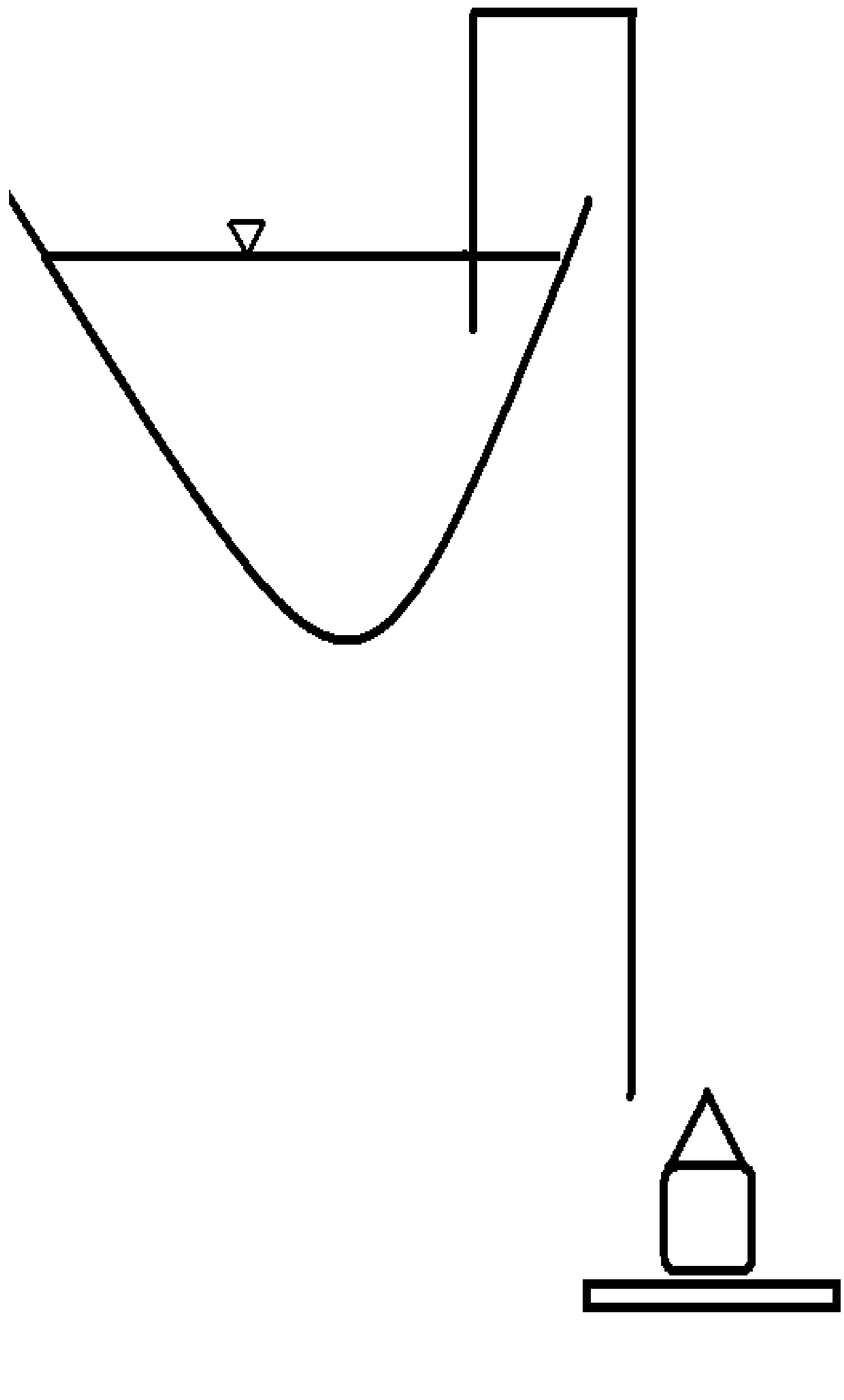
$$\mathbf{N} = [N\ \sqrt{Q}]/[H^{3/4}]$$

Here H is 65.6 feet and Q is 26.1 GPM. Assume that N is 1800 RPM. In the case the specific speed is

$$\mathbf{N} = 1800\ 5.10\ /\ 23.1 = 397$$

So a positive displacement pump is appropriate.

If there was a hill surrounding the pond that the siphon had to climb over before going down to the cabin, then cavitation could occur if the hill was high enough. Imagine that the siphon has a U shape as shown in the sketch on the next page.



As the water moves up the left leg of the siphon, pressure will fall because of friction and also because of the drop in hydrostatic pressure because we are moving vertically through a column of water. As the water moves through the horizontal leg of the siphon, pressure will continue to fall because of friction. However, when the water starts moving down the right leg, pressure will increase because the pressure rise due to movement down through a column of water will be greater than the pressure drop due to friction. So, the minimum pressure will occur at the upper right hand corner.

Conservation of Energy from the surface of the pond to the upper right hand corner of the siphon gives

$$h_{OUT} = h_{IN} - h_L$$

$$h_{OUT} = P/\rho g + C^2/2g + d \qquad h_{IN} = 0$$

$$h_L = (fL/D + \Sigma K)\ C^2/2g \qquad L = n + d + m$$

When P is set to the vapour pressure of water, the only unknown in this equation is d. One finds that for low flow, it is approximately 10m. For high flow, it is less than 10m.