FLUIDS AT REST

PRESSURE

DEPTH LAW

BUOYANCY

PRESSURE DEPTH LAW

Consider a small volume of fluid anywhere within a body of fluid at rest. The mass of the fluid within the volume is

$$\rho\ \Delta x\ \Delta y\ \Delta z$$

The weight of the fluid within the volume is

$$\rho\ g\ \Delta x\ \Delta y\ \Delta z$$

The pressure load on the bottom is

$$P\ \Delta x\ \Delta y$$

The pressure load on the top is

$$(P + \Delta P)\ \Delta x\ \Delta y$$

A summation of loads gives

$$- \rho\ g\ \Delta x\ \Delta y\ \Delta z\ +\ P\ \Delta x\ \Delta y\ -\ (P + \Delta P)\ \Delta x\ \Delta y\ =\ 0$$

Manipulation gives

$$\Delta P/\Delta z = - \rho\ g$$

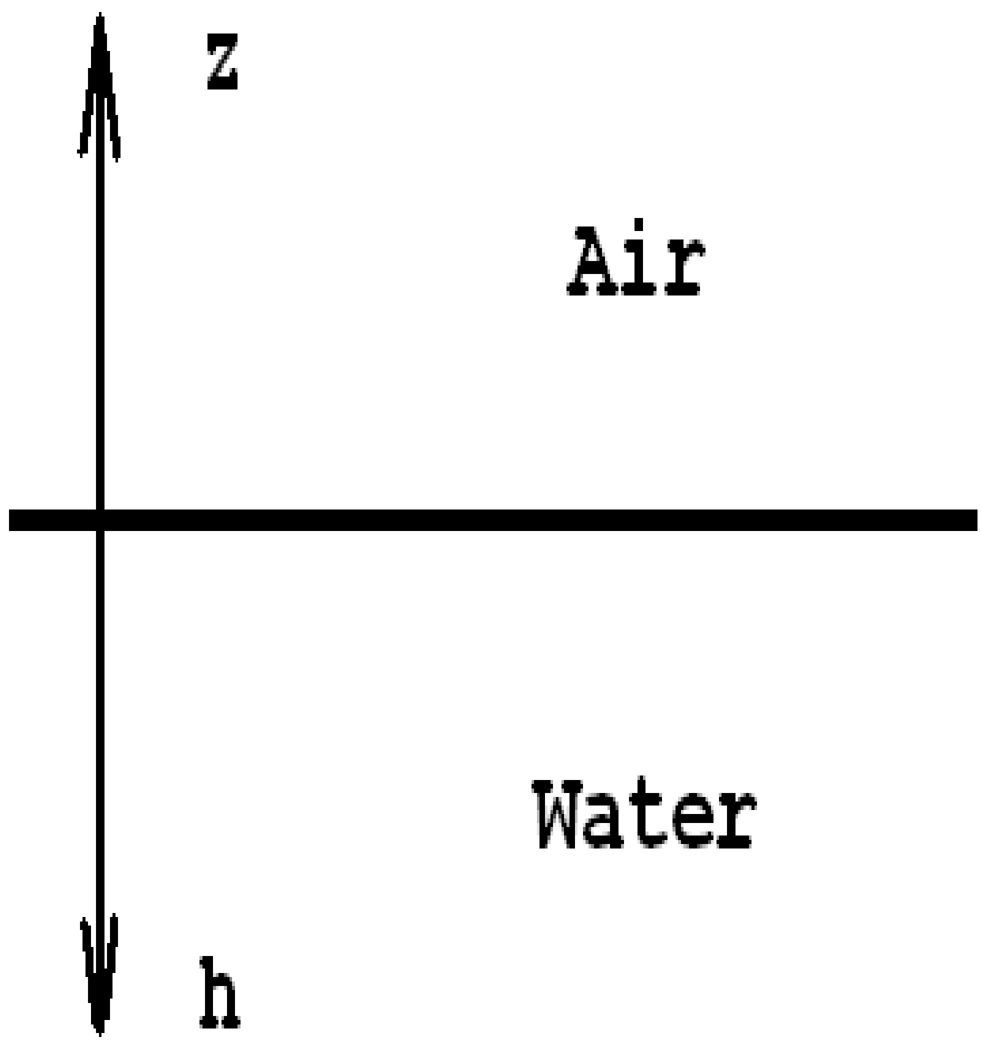
Because gravity only acts vertically, the horizontal pressure gradients are both zero:

$$\Delta P/\Delta x = 0 \qquad \Delta P/\Delta y = 0$$

Illustration: Constant Pressure Gas over a Constant Density Liquid. Let the gas pressure be P_o. Let the z axis point upwards from the gas liquid interface. Let h be the depth downwards from the interface. In this case, for the liquid

$$\Delta P/\Delta z = - \rho\ g \qquad \Delta P/\Delta h = + \rho\ g$$

Manipulation gives


z
Air
Water
h

$$\Delta P = - \rho \, g \, \Delta z \qquad \Delta P = + \rho \, g \, \Delta h$$

Substitution into these equations gives

$$P - P_o = - \rho \, g \, z = + \rho \, g \, h$$

Illustration: Ideal Gas Constant Temperature Atmosphere. Let the z axis point upwards from the ground. Let h be the distance up to a point in the atmosphere. In this case

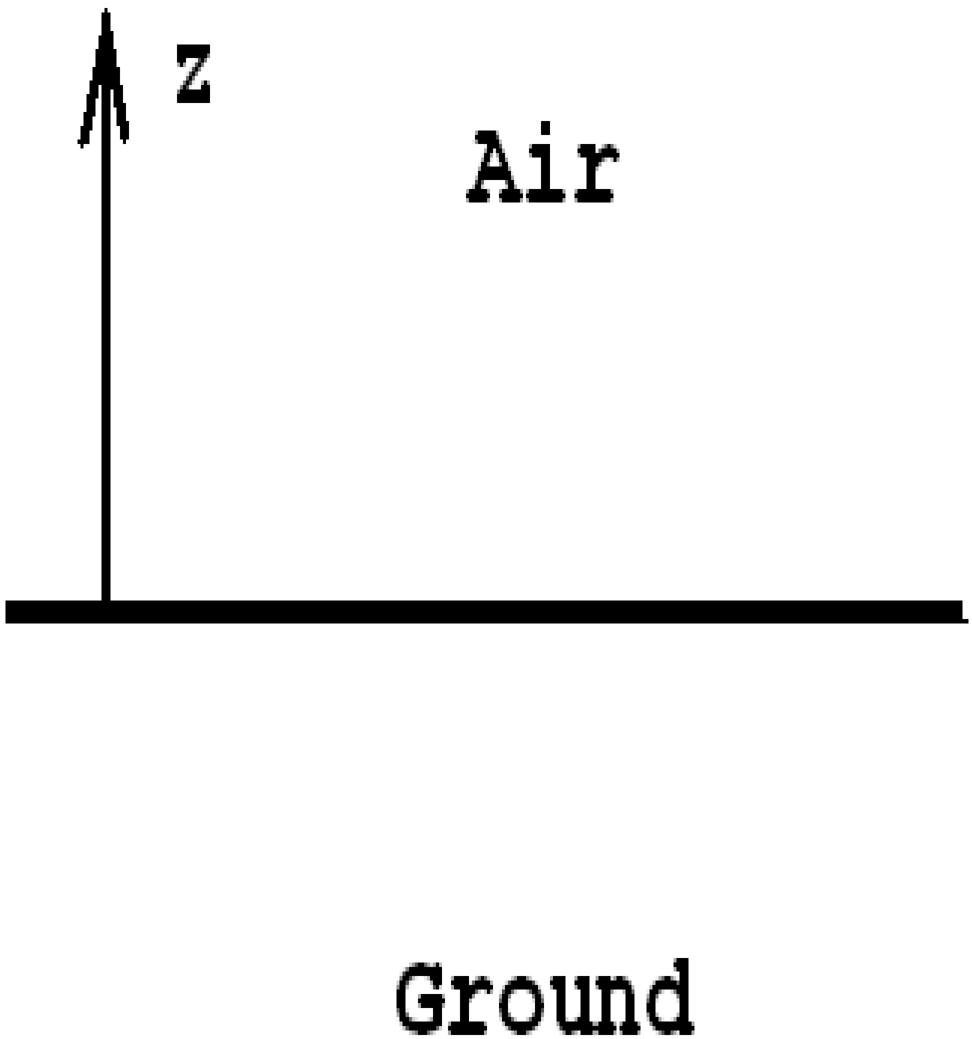
$$\Delta P/\Delta z = - \rho \, g = - P/[RT] \, g$$

Manipulation gives

$$\Delta P/P = - g/[RT] \, \Delta z \qquad dP/P = - g/[RT] \, dz$$

Integration gives

$$\ln (P/P_o) = - [gh]/[RT] \qquad P = P_o \, e^{-[gh]/[RT]}$$


z
Air
Ground

BUOYANCY

Consider a floating body. The submerged volume can be approximated by an infinite number of infinitesimal vertical tubes. Let the height of a tube be d and its cross sectional area be Δs. The volume of the tube is:

$$\Delta V = d \, \Delta s$$

The pressure load on each tube is:

$$\Delta B = \Delta P \, \Delta s$$

$$= \rho g d \, \Delta s = \rho g \, \Delta V = \Delta W$$

The total load is

$$B = \Sigma \, \Delta B = \Sigma \, \rho g \, \Delta V = \rho g \, V$$

One can see that the pressure load is equal to the weight of the displaced volume of fluid. The load B is known as the buoyancy. Simple moment calculations give the location of the center of the buoyancy load.