FLUIDS IN MOTION

MOMENTUM

DEVICES

CONSERVATION LAWS

Conservation of Mass:

$$\Sigma\ [\rho CA]_{OUT} = \Sigma\ [\rho CA]_{IN}$$

$$\Sigma\ \dot{M}_{OUT} = \Sigma\ \dot{M}_{IN}$$

Conservation of Momentum:

$$\Sigma\ [\rho \mathbf{v} CA]_{OUT} - \Sigma\ [\rho \mathbf{v} CA]_{IN} = -\ \Sigma\ [PA\mathbf{n}]_{OUT} - \Sigma\ [PA\mathbf{n}]_{IN} + \mathbf{R}$$

$$\Sigma\ [\dot{M} U]_{OUT} - \Sigma\ [\dot{M} U]_{IN} = -\ \Sigma\ [PAn_x]_{OUT} - \Sigma\ [PAn_x]_{IN} + R_x$$

$$\Sigma\ [\dot{M} V]_{OUT} - \Sigma\ [\dot{M} V]_{IN} = -\ \Sigma\ [PAn_y]_{OUT} - \Sigma\ [PAn_y]_{IN} + R_y$$

$$\Sigma\ [\dot{M} W]_{OUT} - \Sigma\ [\dot{M} W]_{IN} = -\ \Sigma\ [PAn_z]_{OUT} - \Sigma\ [PAn_z]_{IN} + R_z$$

Conservation of Energy:

$$\Sigma\ [\dot{M} gh]_{OUT} - \Sigma\ [\dot{M} gh]_{IN} = +\ \Sigma\ \dot{T} - \Sigma\ \dot{L}$$

$$h = C^2/2g + P/\rho g + z$$

$$\dot{T} = \dot{M} gh_T \qquad \dot{L} = \dot{M} gh_L$$

$$h_L = (fL/D + \Sigma K)\ C^2/2g$$

ILLUSTRATION: FIRE HOSE NOZZLE

The knowns are the supply pressure and the inlet and outlet pipe diameters. The unknown is the force to hold the nozzle. Friction and gravity are insignificant.

Conservation of Energy gives

$$[C^2/2 + P/\rho + gz]_{OUT} = [C^2/2 + P/\rho + gz]_{IN}$$

$$[C^2/2]_{OUT} = [C^2/2 + P/\rho]_{IN}$$

Conservation of Mass gives

$$[\rho CA]_{OUT} = [\rho CA]_{IN}$$

$$[CA]_{OUT} = [CA]_{IN}$$

$$C_{IN} = A_{OUT}/A_{IN}\ C_{OUT}$$

$$C_{IN} = [D_{OUT}/D_{IN}]^2\ C_{OUT}$$

Mass into Energy gives

$$[C^2/2]_{OUT} = [D_{OUT}/D_{IN}]^4 \ [C^2/2]_{OUT} + [P/\rho]_{IN}$$

$$[\ 1 - [D_{OUT}/D_{IN}]^4\ ]\ [C^2/2]_{OUT} = [P/\rho]_{IN}$$

$$C_{OUT} = \sqrt{[2P_{IN}/\rho]} \ / \ \sqrt{[1-(D_{OUT}/D_{IN})^4]}$$

Conservation of Momentum gives

$$[\dot{M}\,U]_{OUT} - [\dot{M}\,U]_{IN} = -\ [PAn_x]_{OUT} - [PAn_x]_{IN} + R_x$$

$$[\rho CA\ C]_{OUT} - [\rho CA\ C]_{IN} = +\ [PA]_{IN} - F_x$$

$$[\rho CA]_{OUT}\ [C_{OUT} - C_{IN}] = +\ [PA]_{IN} - F_x$$

$$F_x = +\ [P\ \pi D^2/4]_{IN} - \rho\ C_{OUT}\ [\pi D^2/4]_{OUT}\ [C_{OUT} - C_{IN}]$$

$$F_x = +\ [P\ \pi D^2/4]_{IN} - \rho\ C_{OUT}\ [\pi D^2/4]_{OUT}\ [1 - [D_{OUT}/D_{IN}]^2\ ]\ C_{OUT}$$

$$C_{OUT} = \sqrt{[2P_{IN}/\rho]} \ / \ \sqrt{[1-(D_{OUT}/D_{IN})^4]}$$

ILLUSTRATION: PIPE BENDS

The knowns are the supply pressure, the pipe diameter and the bend angles. The unknown is the force to hold the bend. Friction and gravity are insignificant.

Conservation of Energy gives

$$[C^2/2 + P/\rho + gz]_{OUT} = [C^2/2 + P/\rho + gz]_{IN}$$

$$[C^2/2]_{OUT} = [C^2/2]_{IN}$$

Conservation of Mass gives

$$[\rho CA]_{OUT} = [\rho CA]_{IN}$$

These equations show that C is the same inlet and outlet.

Conservation of Momentum gives

$$\Sigma\ [\dot{M}\,U]_{OUT} - \Sigma\ [\dot{M}\,U]_{IN} = -\ \Sigma\ [PAn_x]_{OUT} - \Sigma\ [PAn_x]_{IN} + R_x$$

$$\Sigma\ [\dot{M}\,V]_{OUT} - \Sigma\ [\dot{M}\,V]_{IN} = -\ \Sigma\ [PAn_y]_{OUT} - \Sigma\ [PAn_y]_{IN} + R_y$$

$$\Sigma\ [\dot{M}\,W]_{OUT} - \Sigma\ [\dot{M}\,W]_{IN} = -\ \Sigma\ [PAn_z]_{OUT} - \Sigma\ [PAn_z]_{IN} + R_z$$

Solving for the loads on the bend gives

$$F_x = -\ \Sigma\ [\dot{M}\,U]_{OUT} + \Sigma\ [\dot{M}\,U]_{IN} - \Sigma\ [PAn_x]_{OUT} - \Sigma\ [PAn_x]_{IN}$$

$$F_y = -\ \Sigma\ [\dot{M}\,V]_{OUT} + \Sigma\ [\dot{M}\,V]_{IN} - \Sigma\ [PAn_y]_{OUT} - \Sigma\ [PAn_y]_{IN}$$

$$F_z = -\ \Sigma\ [\dot{M}\,W]_{OUT} + \Sigma\ [\dot{M}\,W]_{IN} - \Sigma\ [PAn_z]_{OUT} - \Sigma\ [PAn_z]_{IN}$$

where the velocity components are

$$U_{IN} = -\ C\ n_x \qquad V_{IN} = -\ C\ n_y \qquad W_{IN} = -\ C\ n_z$$

$$U_{OUT} = +\ C\ n_x \qquad V_{OUT} = +\ C\ n_y \qquad W_{OUT} = +\ C\ n_z$$

ILLUSTRATION : WALL IMPACT BY JET

The knowns are the jet velocity, the jet mass flow rate and the wall angle. The unknowns are the force on the wall and the mass flow rates. Friction and gravity are insignificant. The jet is surrounded by atmospheric pressure.

One can imagine the jet to be split into upper and lower sections. Conservation of Energy for each section gives

$$C^2/2 = K$$

This implies C is the same at the inlet and the outlets

Conservation of Mass gives

$$\dot{M}_1 = \dot{M}_2 + \dot{M}_3$$

where 1 indicates the inlet, 2 indicates up the wall and 3 indicates down the wall.

Conservation of Normal Momentum gives

$$F_N = \dot{M}_1 \ C \ \mathrm{Sin}\theta$$

Conservation of Tangential Momentum gives

$$\dot{M}_1 \ C \ \mathrm{Cos}\theta = \dot{M}_2 \ C - \dot{M}_3 \ C$$

Manipulation gives

$$\dot{M}_1 \ \mathrm{Cos}\theta = \dot{M}_2 - \dot{M}_3$$

Conservation of Mass gives

$$\dot{M}_1 = \dot{M}_2 + \dot{M}_3$$

Addition gives

$$\dot{M}_2 = \dot{M}_1 \ [1 + \mathrm{Cos}\theta] / 2$$

$$\dot{M}_3 = \dot{M}_1 - \dot{M}_2$$

ILLUSTRATION : WATER BOMBER

A water bomber uses a scoop to pick up a load of water. Let speed of the bomber be C and the scoop area be A. What is the force on the bomber as it picks up a load of water? What power is required to overcome this force? How long would it take to fill a tank with volume V?

It is best to work this problem in the frame of reference of the bomber. Conservation of Momentum gives:

$$\rho \, Q \; [U_{OUT} - U_{IN}] = + R = - F$$

where F is the force on the bomber. In the bomber reference frame $U_{OUT} = 0$ and $U_{IN} = - C$. Also $Q = AC$. Substitution into momentum gives the force:

$$F = - \rho \, Q \, C = - \rho \, AC \, C = - \rho \, A \, C^2$$

The power is just $\mathbf{P} = FC$. The fill time is $T = V/Q$.

ILLUSTRATION : SNOW PLOW

A snow plow picks up a horizontal layer of snow which is 2.5m wide and 0.25m high and turns it so that it is moving perpendicular to the direction of motion of the plow. Calculate the snow load on the plow when it is moving at 50km/hr. Assume that the density of snow is 900 kg/m^3.

The snow has two force components acting on it: both in the horizontal plane. These are

$$\sum \dot{M}\,(U_{OUT}-U_{IN}) = R_x = -\,F_x \qquad \sum \dot{M}\,(V_{OUT}-V_{IN}) = R_y = -\,F_y$$

Let the speed of the plow be C. In this case, U_{OUT} is equal to zero and U_{IN} is equal to - C, while V_{OUT} is equal to - C and V_{IN} is equal to zero.

The mass flow rate is

$$\dot{M} = \rho C A \qquad A = wd$$

Substitution gives

$$\dot{M} = 8.68 m^3/s$$

$$F_x = -\ 108 kN$$

$$F_y = +\ 108 kN$$

ILLUSTRATION : VEHICLE DECELERATION

A vehicle is decelerated by a scoop stuck in water. The scoop turns the water horizontally through 180°. Let the mass of the vehicle be M and its speed be C. Conservation of Momentum for the scoop gives:

$$\rho\ Q\ [U_{OUT} - U_{IN}] = -F$$

Here U_{OUT} = +C and U_{IN} = -C. Also Q = C bh where b is the width of the scoop and h is the thickness of the sheet of water picked up by the scoop. One gets:

$$F = -2\ \rho\ bh\ C^2$$

The equation of motion for the vehicle is:

$$M\ d^2C/dt^2 = F = -2\ \rho\ bh\ C^2$$

Integration of this equation gives C as a function of time. Integration of C over time gives the distance S travelled by the vehicle. One gets:

$$S = \int C\ dt$$