FLUIDS AT REST

SUBMERGED

SURFACES

HYDRAULIC GATES

To get loads on hydraulic gates one can break its surface up into an infinite number of infinitesimal bits of surface. The force on an infinitesimal bit of surface is:

$$d\mathbf{F} = P\ ds\ \mathbf{n}$$

where $\mathbf{n}$ is the inward normal on the surface and P is the pressure acting on it. The normal $\mathbf{n}$ is:

$$\mathbf{n} = n_x\ \mathbf{i} + n_y\ \mathbf{j} + n_z\ \mathbf{k}$$

where **ijk** indicates unit normal vectors. The force can be broken down into xyz components

$$d\mathbf{F} = dF_x\ \mathbf{i} + dF_y\ \mathbf{j} + dF_z\ \mathbf{k}$$
$$= P\ ds\ n_x\ \mathbf{i} + P\ ds\ n_y\ \mathbf{j} + P\ ds\ n_z\ \mathbf{k}$$

The pressure depth law gives

$$P = \rho g\ h$$

The total force can be obtained by integration of the component forces over the total surface:

$$F_x = \int P\ n_x\ ds \qquad F_y = \int P\ n_y\ ds \qquad F_z = \int P\ n_z\ ds$$

The total force is

$$\mathbf{F} = F_x\ \mathbf{i} + F_y\ \mathbf{j} + F_z\ \mathbf{k}$$

$$|\mathbf{F}| = \sqrt{\ [\ [F_x]^2 + [F_x]^2 + [F_x]^2\ ]}$$

Moment balances give the location of the forces.

The panel method for hydraulic gates starts by subdividing the surface of the gate into a finite number of finite size flat panels. The pressure depth law gives the pressure at the centroid of each panel. Pressure times panel area gives the force at the centroid. The unit normal pointing at the panel allows one to break the force into components. Summation gives the total force on the gate in each direction.

$$F_x = \sum P \; n_x \; \Delta s \qquad F_y = \sum P \; n_y \; \Delta s \qquad F_z = \sum P \; n_z \; \Delta s$$

The total force is

$$\mathbf{F} = F_x \, \mathbf{i} + F_y \, \mathbf{j} + F_z \, \mathbf{k}$$

$$|\mathbf{F}| = \sqrt{\;[\, [F_x]^2 + [F_x]^2 + [F_x]^2 \,]}$$

Moment balances give the location of the forces.

The pressure/weight method for hydraulic gates starts by boxing the gate with vertical and horizontal surfaces. The fluid within these surfaces is considered frozen to the gate. Then the horizontal and vertical pressure forces on the box surfaces are calculated. Force balances, which subtract the weight frozen to the gate, then give the horizontal and vertical forces on the gate. The total force is

$$\mathbf{F} = F_x \, \mathbf{i} + F_y \, \mathbf{j} + F_z \, \mathbf{k}$$

$$|\mathbf{F}| = \sqrt{\;[\, [F_x]^2 + [F_x]^2 + [F_x]^2 \,]}$$

Moment balances give the location of the forces.

HORIZONTAL FLAT GATE

The pressure acting on the gate is

$$\rho g\ H$$

The total force on the gate is:

$$\rho g\ H\ A$$

VERTICAL RECTANGULAR FLAT GATE

For a horizontal slice of the gate, the pressure is

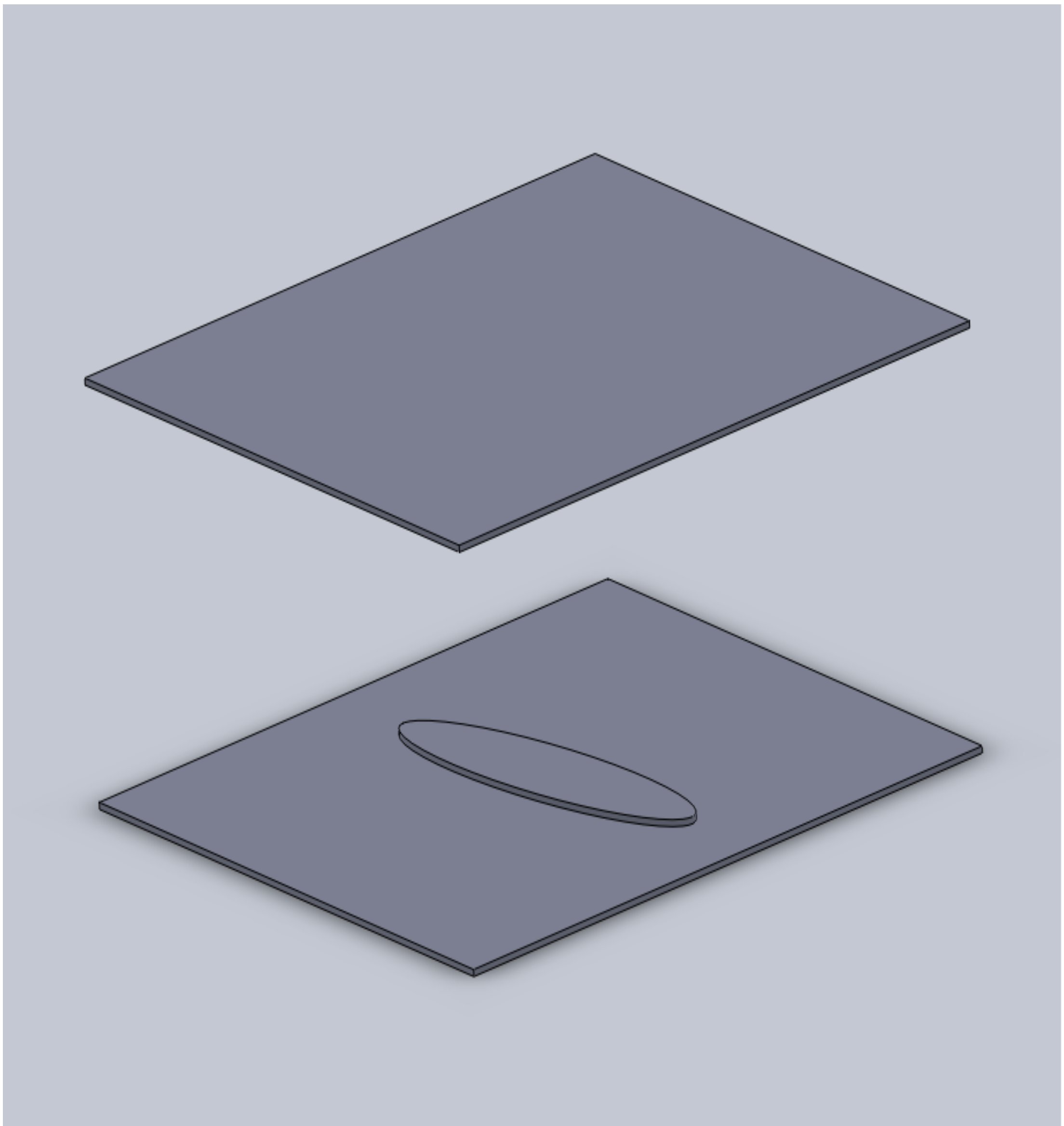
$$\rho g\ (H + r)$$

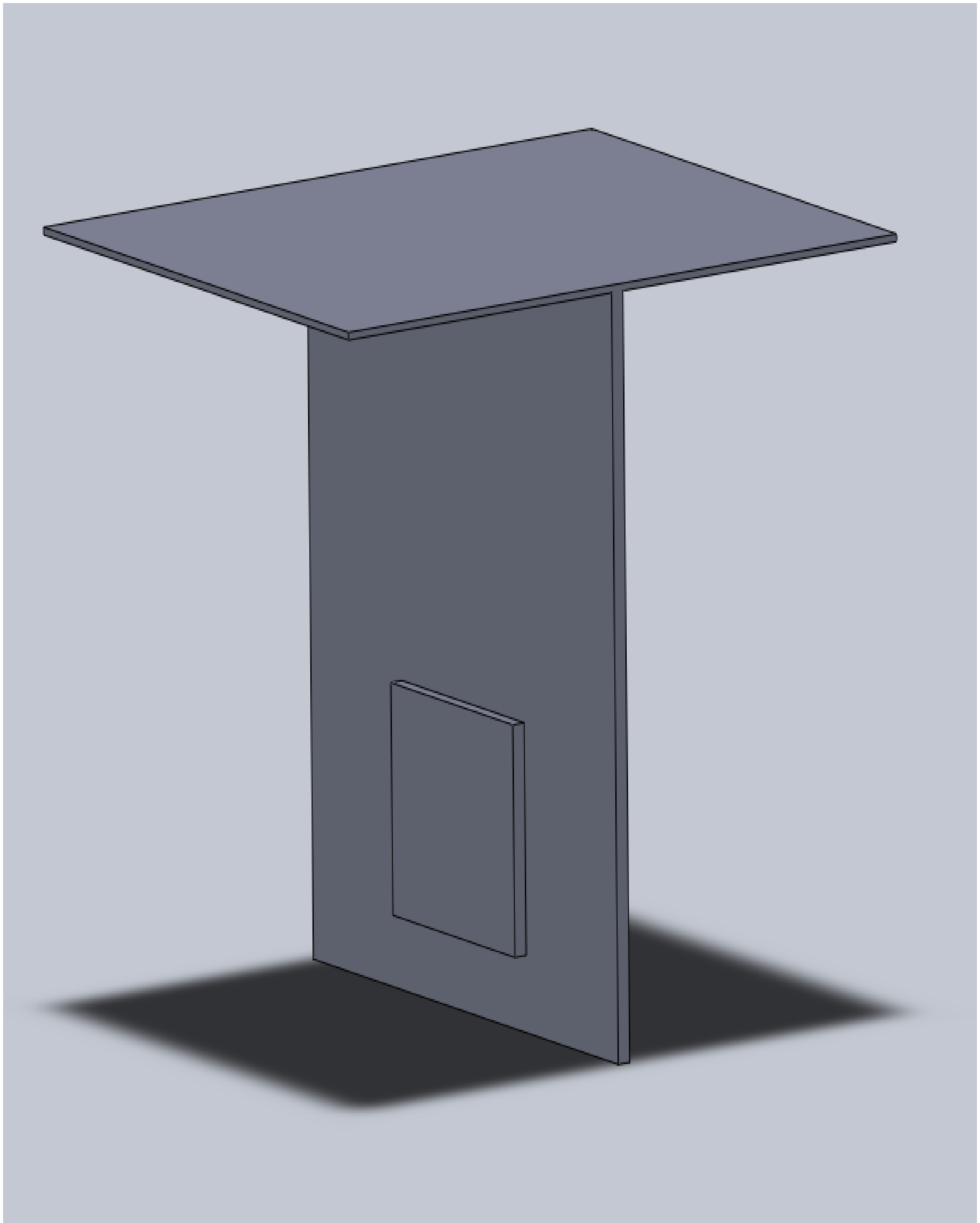
The area the pressure acts over is:

$$W\ dr$$

The total force on the gate is:

$$\int_{-G}^{+G} W\ \rho g\ (H + r)\ dr$$





Evaluation of the integral gives

$$\rho g\ H\ 2G\ W$$

VERTICAL CIRCULAR FLAT GATE

For a horizontal slice of the gate the pressure is

$$\rho g\ (H + r)$$

The area the pressure acts over is:

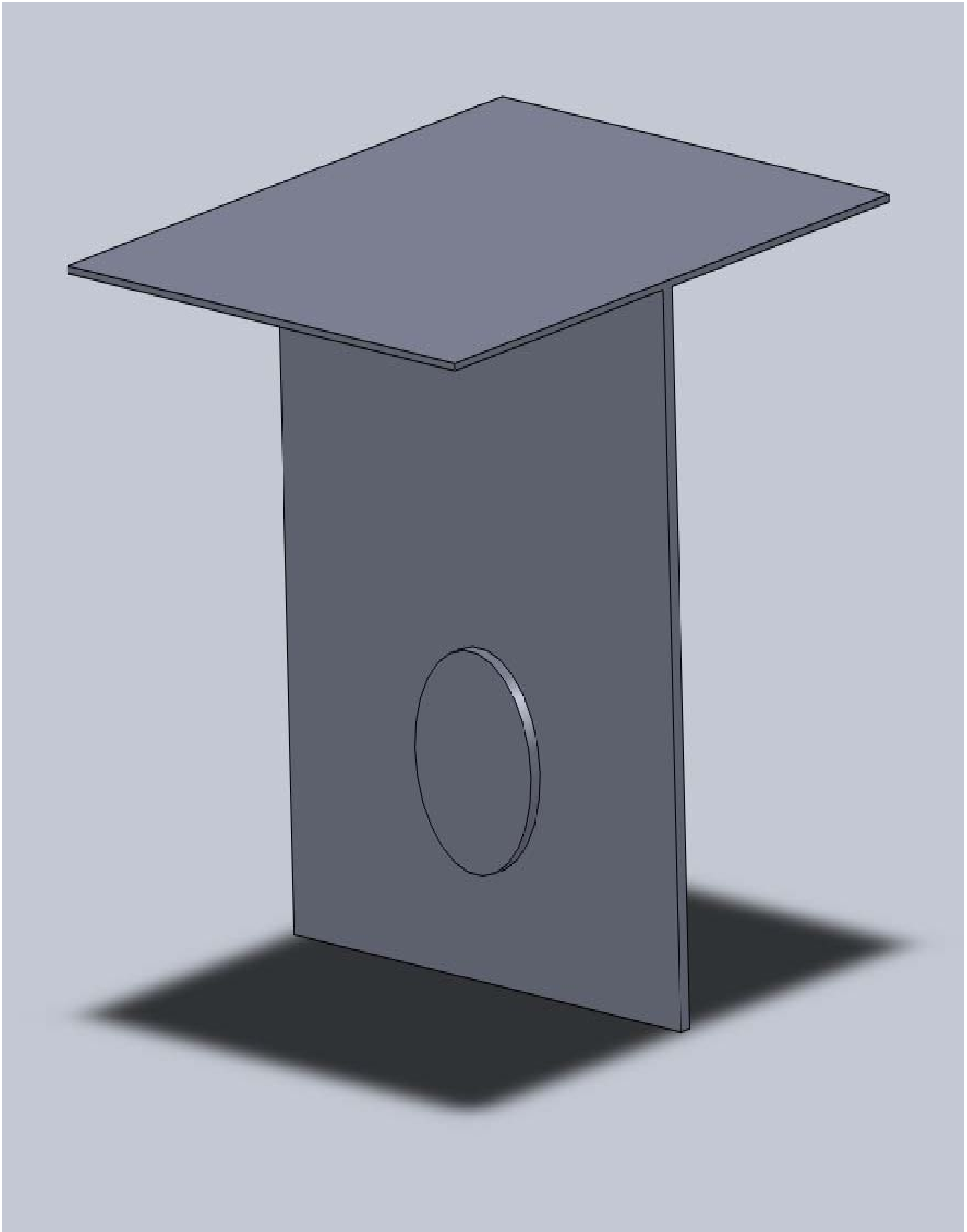
$$2\ \sqrt{[G^2-r^2]}\ \ dr$$

The total force on the gate is:

$$\int_{-G}^{+G} \rho g\ (H + r)\ 2\ \sqrt{[G^2-r^2]}\ \ dr$$

Evaluation of the r integral gives

$$\rho g\ H\ \pi G^2$$



HEMISPHERICAL SIDE GATE

For a horizontal slice of the gate the pressure is

$$\rho g \ (H - G \cos\theta)$$

Angle θ is measured from top to bottom (like latitude on earth). The area the pressure acts over is:

$$G \ d\theta \quad G \sin\theta \quad \pi$$

The total vertical force on the gate is:

$$\int_0^{\pi} [\ \rho g \ (H - G \cos\theta) \quad G \quad G \sin\theta \quad \pi \ [-\cos\theta] \ ] \ d\theta$$

Evaluation of the integral gives

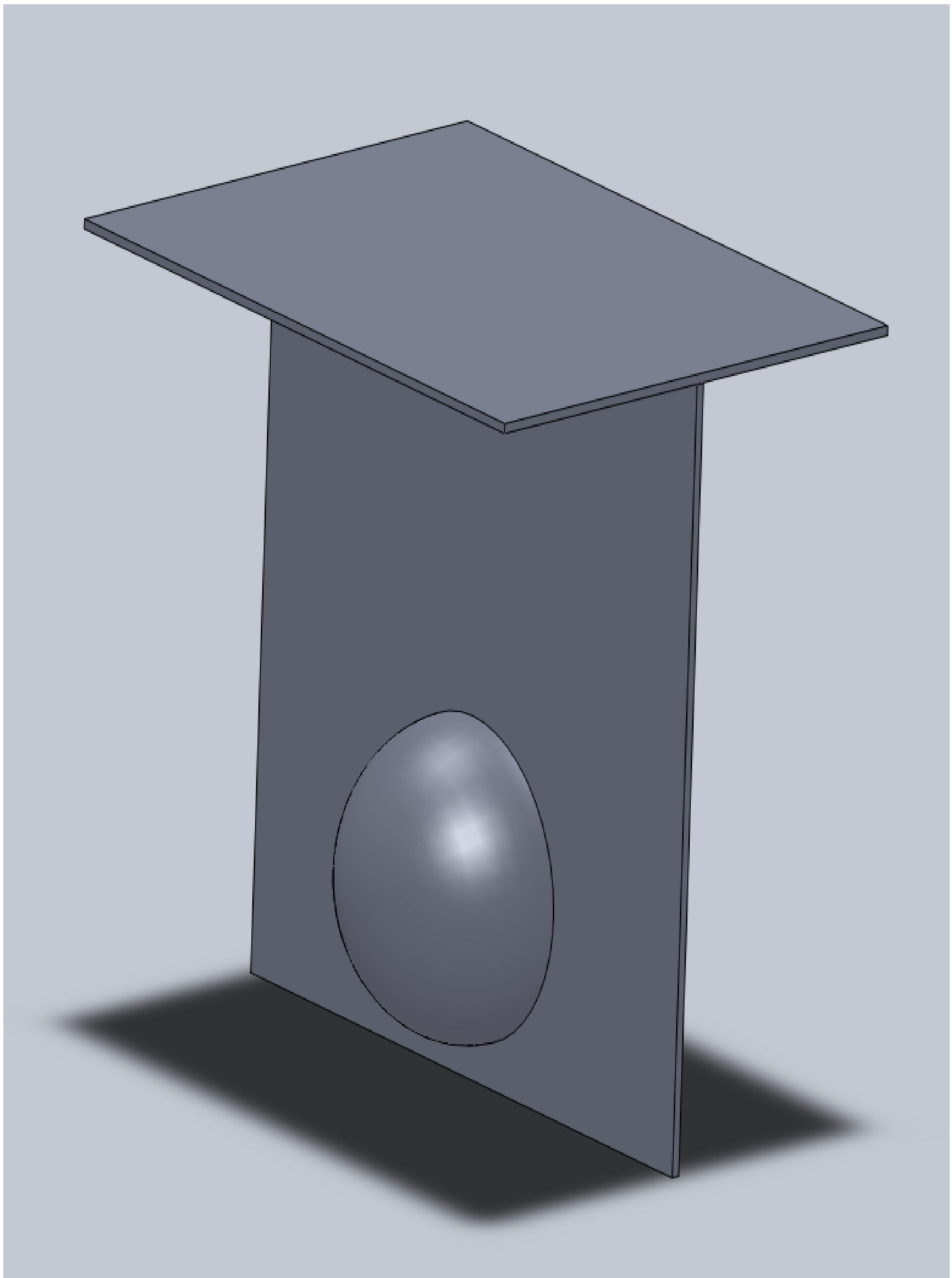
$$\rho g \ [4/3 \ \pi \ G^3] \ / \ 2$$

The horizontal force on the gate is:

$$\int_{-\pi/2}^{+\pi/2} \int_0^{\pi} [\ \rho g \ (H - G \cos\theta) \ G \quad G \sin\theta \quad [+\sin\theta] \ d\theta \ ] \ \cos\sigma \ d\sigma$$

Angle σ is measured around the slice (like longitude on earth). Evaluation of the integral gives

$$\rho g \ H \ \pi \ G^2$$



HEMISPHERICAL WATER TANK

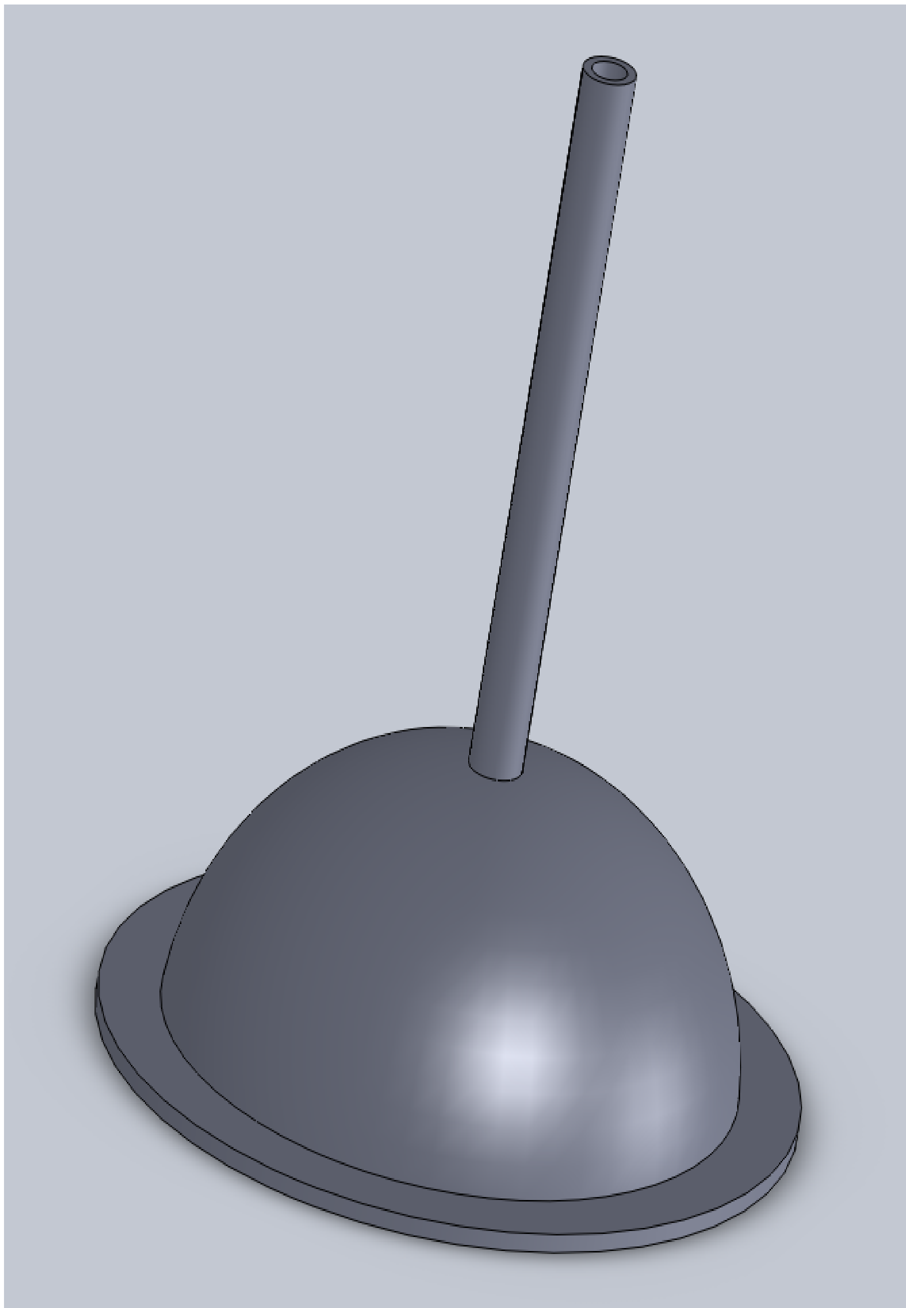
A certain hemispherical water tank sits on a concrete foundation. The tank diameter is 5m. At the top of the tank, there is a small diameter vertical fill tube that is open at the bottom to the tank and open at the top to the atmosphere. The water level in the tube is 5m above the top of the tank. Using the Pressure Weight Method, calculate the vertical force in wall at the base of the tank needed to counteract hydrostatic pressure load. Check your answer using the Panel Method and also using Analytical Integration. The tank wall is 1cm thick and is made out of steel. Calculate the force on the concrete foundation.

Pressure Weight Method: Imagine the tank wall is cut just where it joins the bottom plate. A free body diagram shows that the force balance on tank and water above the cut gives: wall force plus pressure force minus water weight minus wall weight must total to zero. The forces are:

$$\text{pressure force} : \rho g \, H \, \pi \, G^2$$

$$\text{water weight} : \quad \rho g \, [4/3 \, \pi \, G^3] \, / \, 2$$

$$\text{wall weight} : \sigma g \, [4\pi \, G^2] \, t \, / \, 2$$



Analytical Integration Method: For a horizontal slice of the tank the pressure is:

$$\rho g \ (H - G \cos\theta)$$

Angle θ is measured from top to bottom (like latitude on earth). The area the pressure acts over is:

$$G \, d\theta \quad G \sin\theta \quad 2\pi$$

The vertical direction is $+\cos\theta$. The vertical force is:

$$\int_{0}^{+\pi/2} \rho g \ (H - G \cos\theta) \quad G \quad G \sin\theta \quad 2\pi \cos\theta \quad d\theta$$

Evaluation of the integral gives:

$$\rho g \ H \ \pi \ G^2 \ - \ \rho g \ [4/3 \ \pi \ G^3] \ / \ 2$$

Panel Method: The panel method replaces the integral with a sum. The tank is broken into flat panels. The pressure is evaluated at the centroid of each panel. The area of each panel is the length of the panel times 2π times the radius out to the centroid. The panel normal is $+\cos\theta$.

Force on Concrete Foundation: This is just the weight of the steel in the tank walls plus the weight of the water.

```
%
%   HEMISPHERICAL WATER TANK
%
    PANELS=20;PI=3.14159;
    RADIUS=2.5;DEPTH=5.0;
    LENGTH=DEPTH+RADIUS;
    CHANGE=[PI/2]/PANELS;
    FLAT=2.0*RADIUS*sin(CHANGE/2.0);
    OUT=sqrt(RADIUS^2-(FLAT/2)^2);
    GRAVITY=9.81;DENSITY=1000.0;
    WEIGHT=GRAVITY*DENSITY;
%
%   INTEGRATION METHOD
    ONE=+WEIGHT*LENGTH*PI*RADIUS^2;
    TWO=-WEIGHT*PI*RADIUS^3*2/3;
    INTEGRAL=ONE+TWO
%
%   PANEL METHOD
    LIFT=0.0;
    ANGLE=CHANGE/2.0;
    for STEPS=1:PANELS
    HEIGHT=LENGTH-OUT*cos(ANGLE);
    AREA=FLAT*OUT*sin(ANGLE)*2*PI;
    PRESSURE=WEIGHT*HEIGHT;
    NORMAL=+cos(ANGLE);
    BIT=PRESSURE*AREA*NORMAL;
    LIFT=LIFT+BIT;
    ANGLE=ANGLE+CHANGE;
    end
    PANEL=LIFT
%
%   PRESSURE WEIGHT METHOD
    LOAD=+WEIGHT*LENGTH*PI*RADIUS^2;
    WATER=+WEIGHT*PI*RADIUS^3*2/3;
    BOX=LOAD-WATER
```