FLUIDS IN MOTION

BERNOULLI

EQUATION

BERNOULLI EQUATION

When there is no shaft work and friction is insignificant, conservation of energy for a stream tube shows that h_{OUT} is equal to h_{IN}, which implies that h is constant:

$$C^2/2g + P/\rho g + z = K$$

This equation is known as the Bernoulli Equation. It can also be derived from conservation of momentum. For a short stream tube, a force balance gives:

$$\rho \, DC/Dt = \rho \, (\partial C/\partial t + C\partial C/\partial s) = - \partial P/\partial s - \rho g \, \partial z/\partial s$$

For steady flow this becomes

$$\rho \, CdC/ds = \rho \, d[C^2/2]/ds = - dP/ds - \rho g \, dz/ds$$

Integration of this gives the Bernoulli equation:

$$C^2/2 + P/\rho + gz = \kappa$$

This equation shows that, when pressure goes down in a flow, speed goes up and visa versa. From an energy perspective, flow work causes the speed changes. From a momentum perspective, it is due to pressure forces.

ILLUSTRATION : PITOT TUBE

A sketch of a pitot tube is given on the next page. An application of Bernoulli from a point U far upstream to the stagnation point D on the tube gives

$$(C^2/2 \quad + \quad P/\rho \quad + \quad gz)_U \quad = \quad (C^2/2 \quad + \quad P/\rho \quad + \quad gz)_D$$

The speed at D is zero. The z is the same for both points. Manipulation gives

$$C_U = \sqrt{\ } \, [2(P_D - P_U)/\rho]$$

$$Q = \Sigma \; C_k \; A_k$$

$$A_k = \pi \; D_k \; \Delta D$$

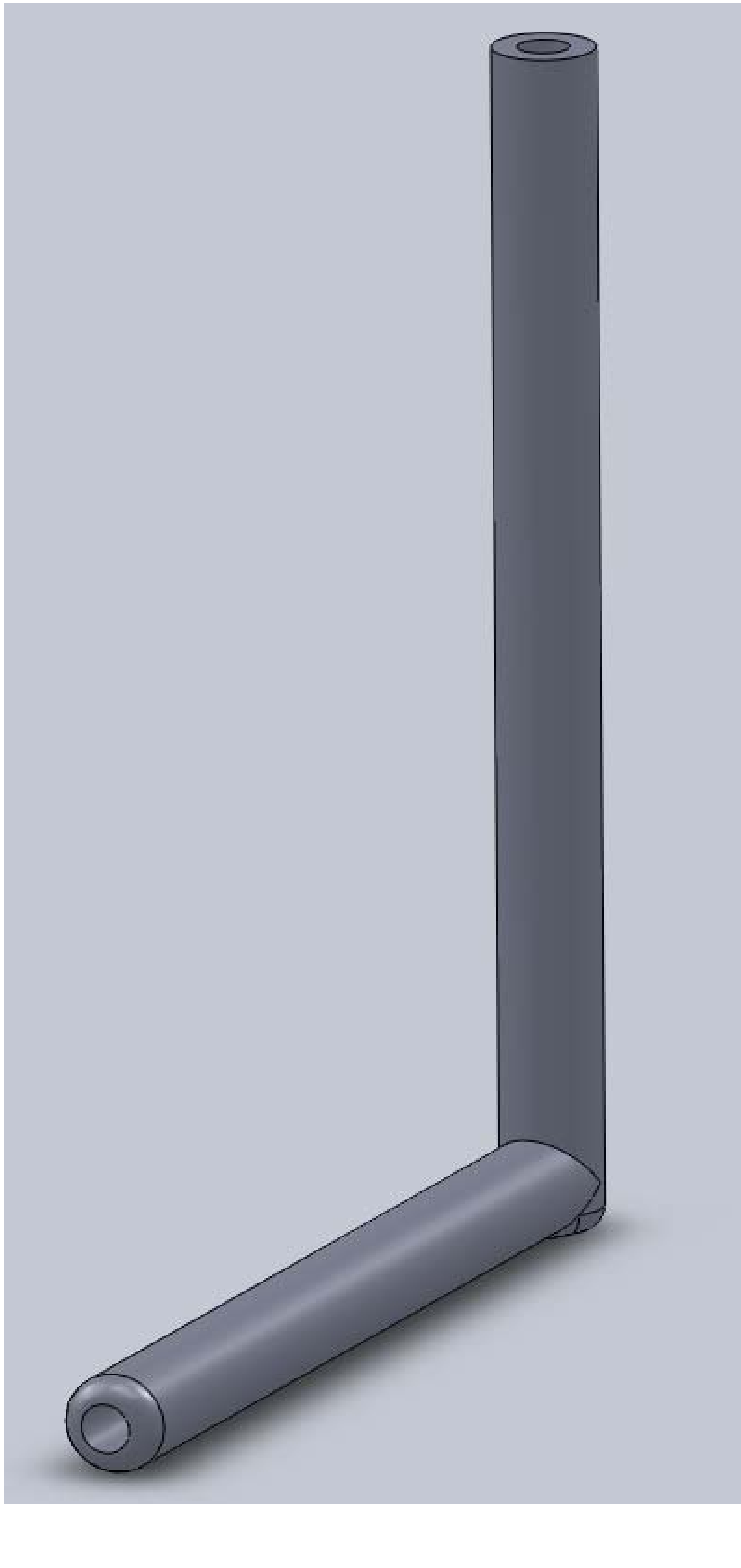

ILLUSTRATION : CONICAL INLET

A sketch of a conical inlet is given on the next page. An application of Bernoulli from a point U far outside the inlet to a point D just inside the inlet gives

$$(C^2/2 + P/\rho + gz)_U = (C^2/2 + P/\rho + gz)_D$$

The speed at U is zero. The z is the same for both points. Manipulation gives

$$C_D = \sqrt{[2(P_U - P_D)/\rho]}$$

$$Q = K\ C_D\ A_D \qquad A = \pi D^2/4$$

The constant K accounts for losses.

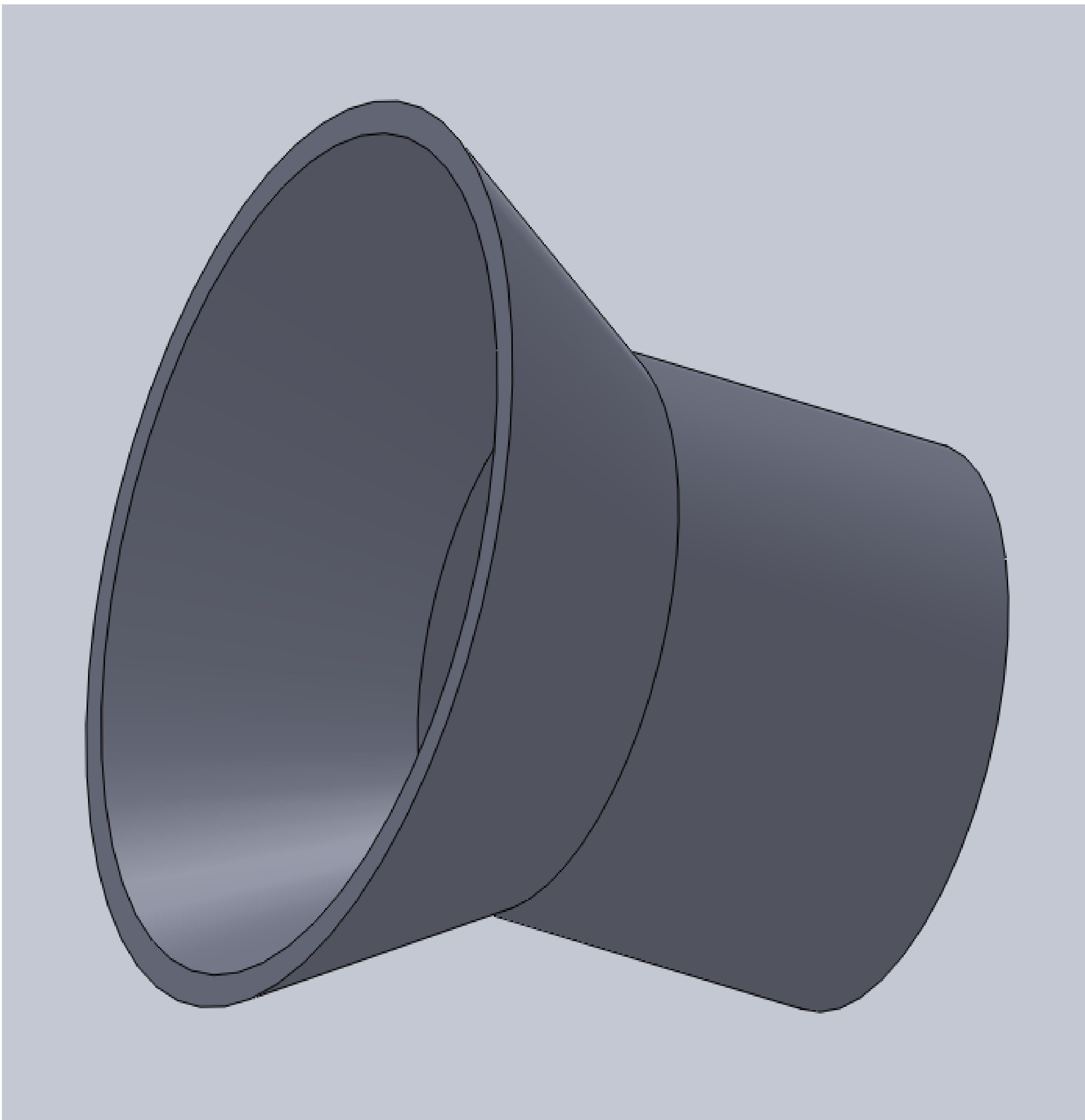

ILLUSTRATION : CONSTRICTIONS

Examples of constrictions include the venturi contraction and the orifice plate. An application of Bernoulli from a point U upstream to a point D at the constriction gives

$$(C^2/2 + P/\rho + gz)_U = (C^2/2 + P/\rho + gz)_D$$

Conservation of Mass gives

$$[\rho CA]_U = [\rho CA]_D$$

Mass into Bernoulli gives

$$C_D = \sqrt{[2(P_U-P_D)/\rho]}/\sqrt{[1-(A_D/A_U)^2]}$$

Further manipulation gives

$$C_D = \sqrt{[2(P_U-P_D)/\rho]}/\sqrt{[1-(D_D/D_U)^4]}$$

$$Q = K\ C_D\ A_D \qquad A = \pi D^2/4$$

The constant K accounts for losses.

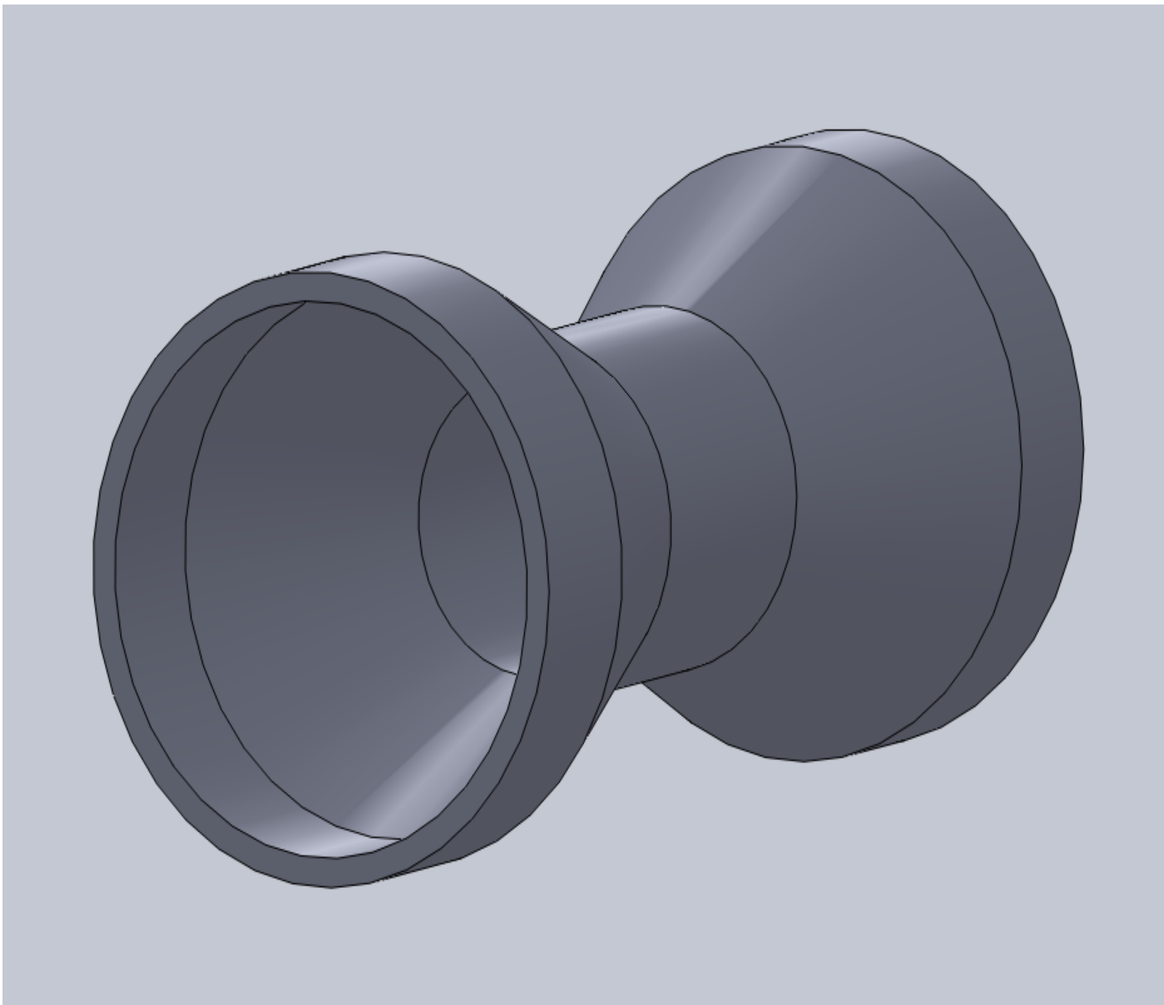

ILLUSTRATION : IDEAL PIPE FLOW

The sketch on the next page shows a large reservoir connected to a simple turbine by a pipe. An application of Bernoulli from a point U on the reservoir surface to a point D just outside the pipe exit gives

$$(C^2/2 \quad + \quad P/\rho \quad + \quad gz)_U \quad = \quad (C^2/2 \quad + \quad P/\rho \quad + \quad gz)_D$$

Because of the size of the reservoir, the speed at U is zero. Let the z at U relative to the outlet be H. The gage pressure at both points is zero. Manipulation gives

$$C_D = \sqrt{[2gH]}$$

Recall from mechanics that if a body was to free fall from a height S the following equations apply

$$V = g\ t \qquad S = g\ t^2\ /2$$

Manipulation gives

$$t = \sqrt{[2S/g]} \qquad V = \sqrt{[2gS]}$$

So it appears the water has free fallen from a height H.

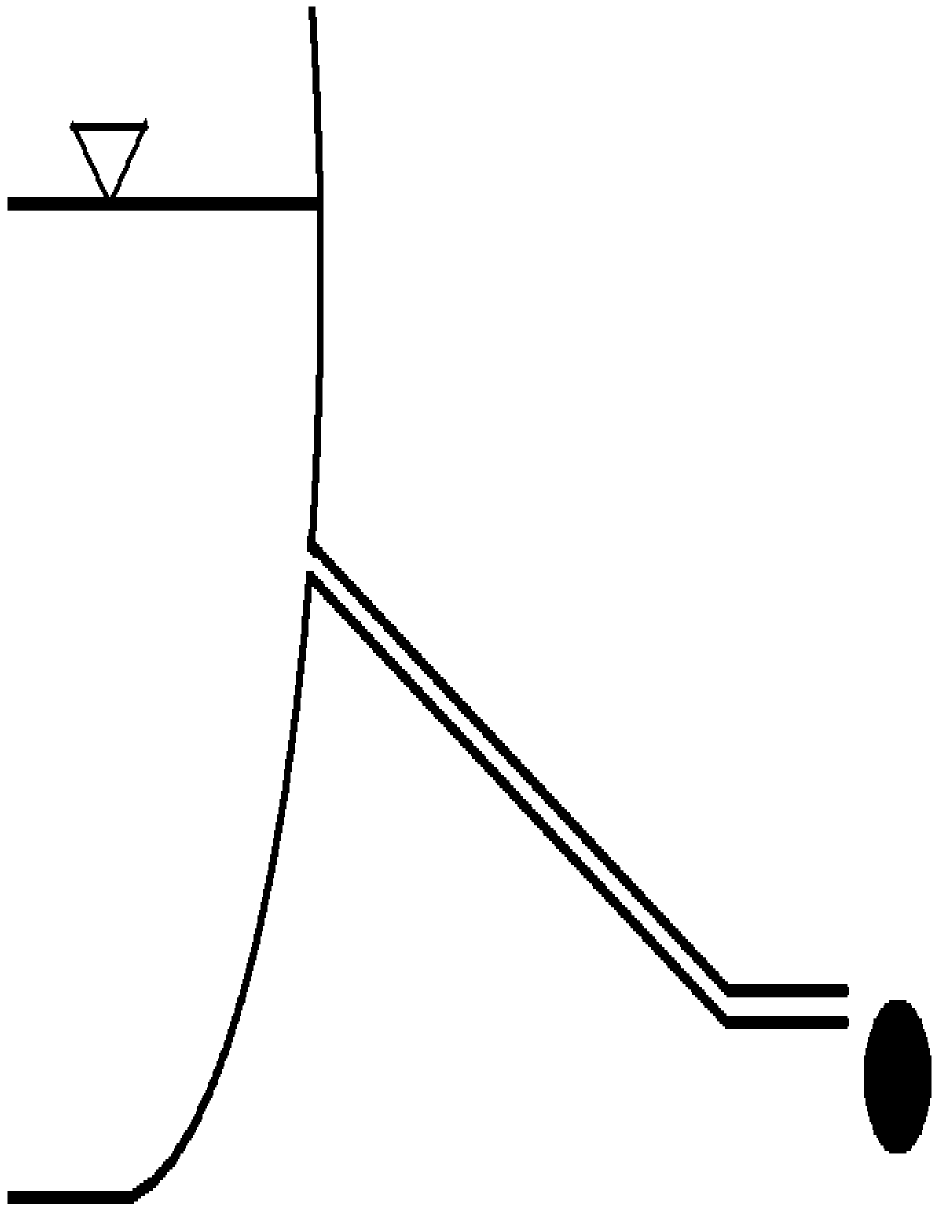

ILLUSTRATION : FLOW POWER

The force associated with pressure is

$$F = P\ A$$

The force moves at speed C and gives the power

$$\mathbf{P} = F\ C$$

The volumetric flow is

$$Q = A\ C$$

So the flow power is

$$\mathbf{P} = P\ Q$$

For a jet the dynamic pressure is

$$\rho\ C^2/2$$

The jet power is

$$\mathbf{P} = \rho\ C^3/2\ A$$

ILLUSTRATION : REAL PIPE FLOW

Conservation of Energy for real pipe flow gives

$$h_{IN} = h_{OUT} + h_L$$

where h is the flow head

$$h = C^2/2g + P/\rho g + z$$

The head loss is

$$h_L = (fL/D + \Sigma K)\ C^2/2g$$

For a pipe connecting a reservoir

$$h_{IN} = H \qquad h_{OUT} = C^2/2g$$

Assume that f=0.01, L=1000m, D=0.1m and $\Sigma K=0$. In this case Conservation of Energy becomes

$$H = C^2/2g + 100\ C^2/2g$$

$$C = \sqrt{[2gH]} / 10$$

Power scales as C^3 so the ideal power case is 1000 times bigger than the real power case.