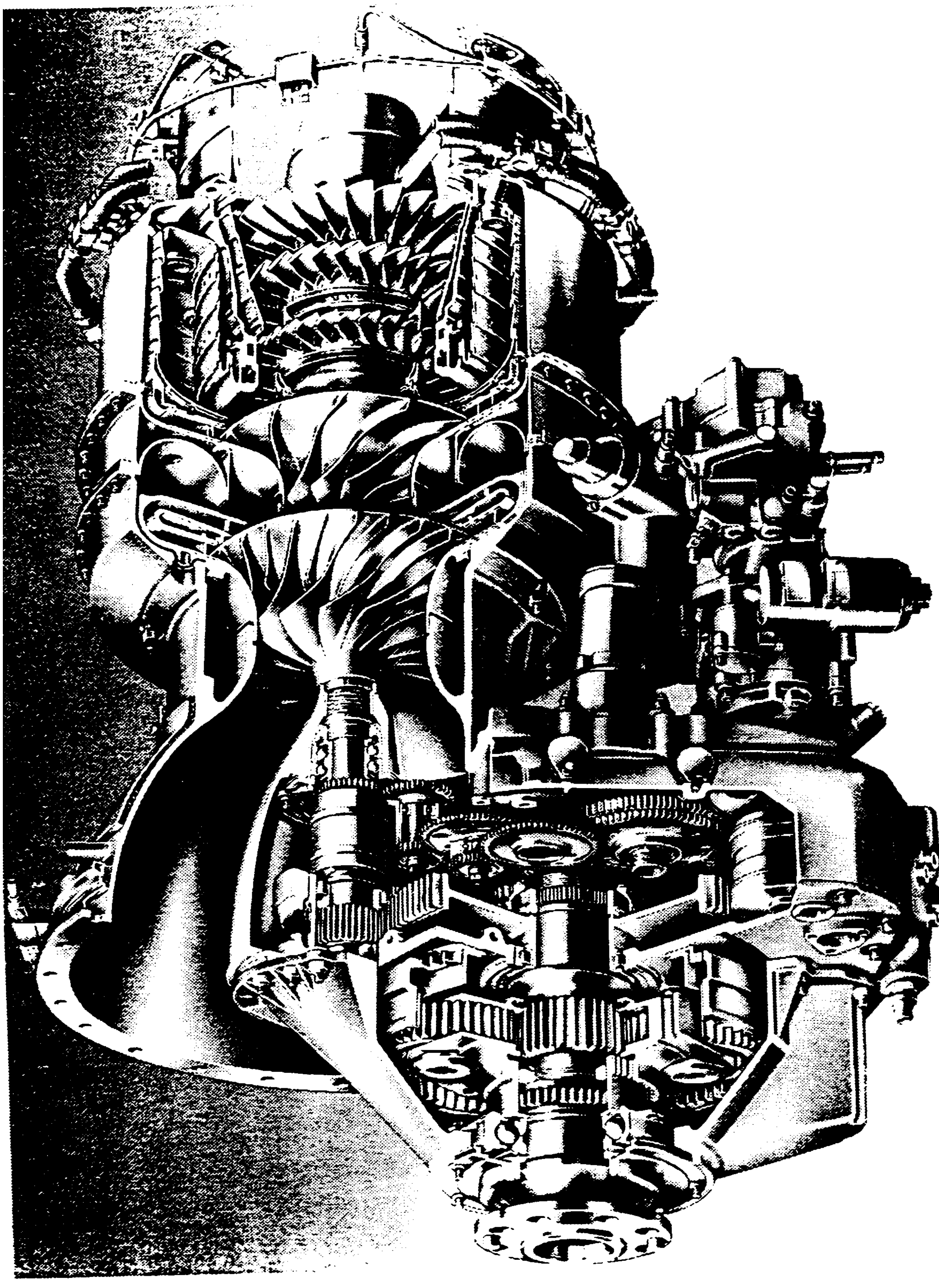
FLUID POWER MACHINES

There are two types of fluid power machines: positive displacement machines and turbomachines. Examples of positive displacement machines include: piston pumps, lobe pumps, screw pumps, gear pumps and vane pumps. They are sometimes referred to as static machines because they do not rely on kinetic energy to operate: they can move at very low speeds. Turbomachines on the other hand are dynamic machines because kinetic energy is critical to how they operate. Most turbomachines are made up of rows of rotor blades separated by stator blades. Stator blades are supposed to direct flow smoothly onto the rotor blades or into a diffuser. There are two types of turbomachines: axial machines and radial machines. The figure on the back of this page gives examples of both. The two stage compressor at the front is an example of a radial machine, while the turbine at the rear is an example of an axial machine. In an axial machine, blades are basically small wings. In a radial machine, blades create tube like passageways. Generally many rows or stages are needed when the pressure across the device is high. Fans and blowers usually move high volumes of low pressure gas: they often have only one set of rotor blades and sometimes no stator blades. Pumps are used to move low or high volumes of low or high pressure liquids. Compressors are designed to compress gases and usually have many rows of rotor and stator blades. Generally highest pressures are generated by positive displacement machines followed by radial machines and then axial machines, while highest flows occur in reverse order starting with axial machines. When asked to design a new turbomachine, manufacturers usually look at the specifications of the machines they already produce to see if any bracket those of the new machine. Such information together with approximate theories is then used to build the new machine. However, the flow within turbomachines is extremely complex. So, once something is built, it usually takes several iterations before the final design emerges. Recently CFD or Computational Fluid Dynamics software packages have been developed for turbomachines. These help but have limitations such as treatment of turbulence.



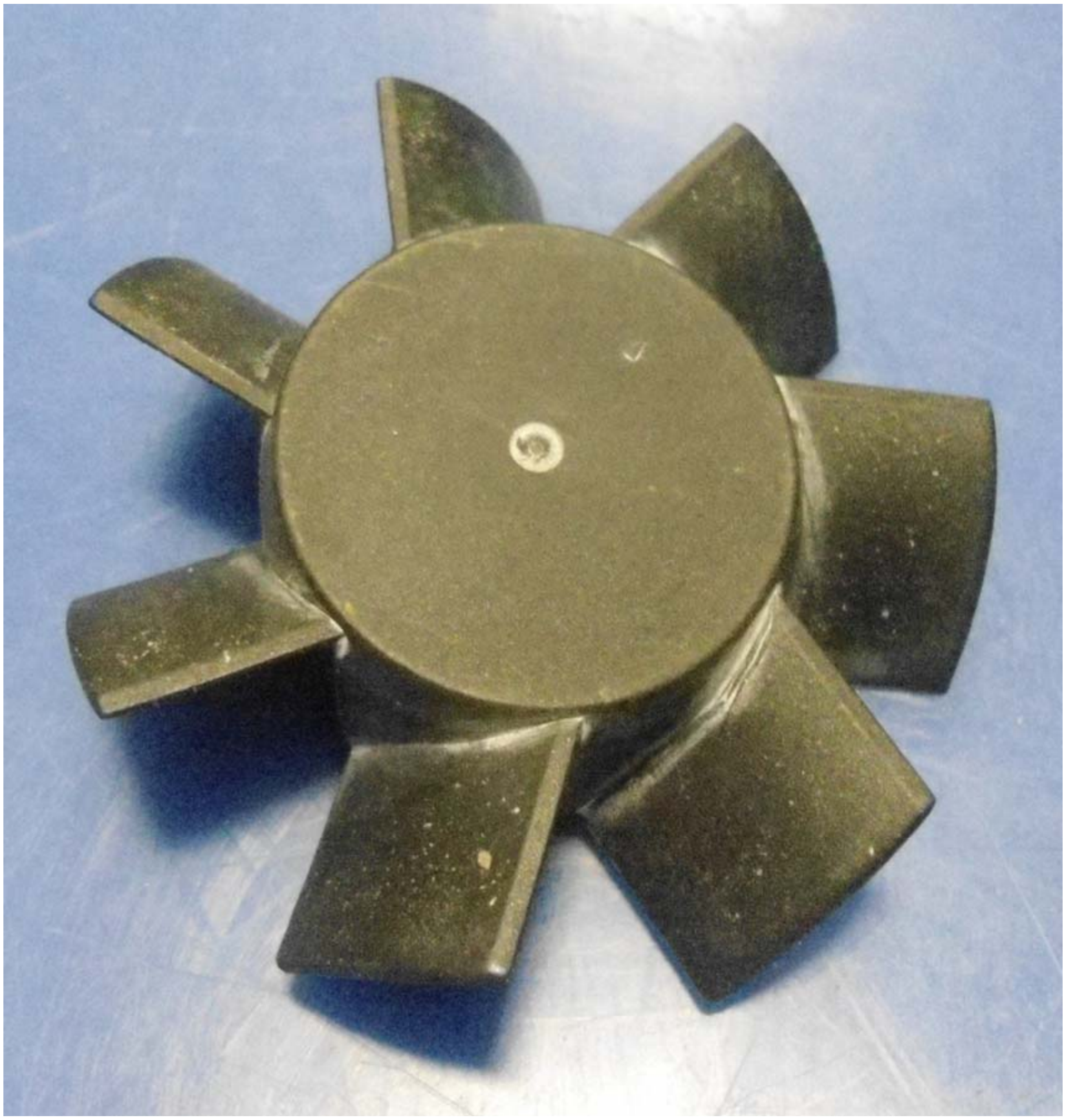
TURBOMACHINE POWER

Swirl is the only component of fluid velocity that has a moment arm around the axis of rotation or shaft of a turbomachine. Because of this, it is the only one that can contribute to shaft power. The shaft power equation is:

$$\mathbf{P} = \Delta\ [T\ \omega] = \Delta\ [\rho Q\ V_T\ R\ \omega]$$

The swirl or tangential component of fluid velocity is V_T. The symbol Δ indicates we are looking at changes from inlet to outlet. The tangential momentum at an inlet or an outlet is $\rho Q\ V_T$. Multiplying momentum by moment arm R gives the torque T. Multiplying torque by the speed ω gives the power $\mathbf{P}$. The power equation is good for pumps and turbines. Power is absorbed at an inlet and expelled at an outlet. If the outlet power is greater than the inlet power, then the machine is a pump. If the outlet power is less than the inlet power, then the machine is a turbine. Geometry can be used to connect V_T to the flow rate Q and the rotor speed ω.

Theoretical analysis of turbomachines makes use of a number of velocities. These are: the tangential flow velocity V_T; the normal flow velocity V_N; the blade or bucket velocity V_B; the relative velocity V_R; the jet velocity V_J.



ELECTRICAL ANALOGY

Electrical power **P** is V I where V is volts and I is current. By analogy, fluid power **P** is P Q where P is pressure and Q is volumetric flow rate. Note that power is force F times speed C. In a flow, force F is pressure P times area A. So power is P times A times C. Now volumetric flow rate Q is C times A. So power becomes P times Q. One can write pressure P in terms of head H as: $P=\rho gH$. Power becomes: $\mathbf{P}=\rho gHQ$. Voltage drop along a wire is $\Delta V=RI$ where R is the resistance of the wire. By analogy, the pressure drop along a pipe due to losses is $\Delta P=RQ^2$ where R is the resistance of the pipe.

TURBOMACHINE SCALING LAWS

Scaling laws allow us to predict prototype behavior from model data. Generally the model and prototype must look the same. This is known as geometric similitude. The flow patterns at both scales must also look the same. This is known as kinematic or motion similitude. Finally, certain force ratios in the flow must be the same at both scales. This is known as kinetic or dynamic similitude. Sometimes getting all force ratios the same is impossible and one must use engineering judgement to resolve the issue.

SCALING LAWS FOR TURBINES

For turbines, we are interested mainly in the power of the device as a function of its rotational speed. The simplest way to develop a nondimensional power is to divide power $\mathbf{P}$ by something which has the units of power. The power in a flow is equal to its dynamic pressure P times its volumetric flow rate Q:

$$P\ Q$$

So, we can define a power coefficient C_P:

$$C_P = \mathbf{P} / [P\ Q]$$

To develop a nondimensional version of the rotational speed of the turbine, we can divide the tip speed of the blades $R\omega$ by the flow speed U, which is usually equal to a jet speed V_J. So, we can define a speed coefficient C_S:

$$C_S = R\omega / V_J$$

SCALING LAWS FOR PUMPS

For a pump, it is customary to let N be the rotor RPM and D be the rotor diameter. All flow speeds U scale as ND and all areas A scale as D^2. Pressures are set by the dynamic pressure $\rho U^2/2$. Ignoring constants, one can define a reference pressure $[\rho N^2 D^2]$ and a reference flow $[ND^3]$. Since fluid power is just pressure times flow, one can also define a reference power $[\rho N^3 D^5]$. Dividing dimensional quantities by reference quantities gives the scaling laws:

Pressure Coefficient $C_P = P / [\rho N^2 D^2]$

Flow Coefficient $C_Q = Q / [ND^3]$

Power Coefficient $C_{\mathbf{P}} = \mathbf{P} / [\rho N^3 D^5]$

On the pressure versus flow characteristic of a pump, there is a best efficiency point (BEP) or best operating point (BOP). For geometrically similar pumps that have the same operating point on the C_P versus C_Q curve, the coefficients show that if D is doubled, P increases 4 fold, Q increases 8 fold and $\mathbf{P}$ increases 32 fold, whereas if N is doubled, P increases 4 fold, Q doubles and $\mathbf{P}$ increases 8 fold.

PELTON WHEEL TURBINE THEORY

The power output of the turbine is: $\mathbf{P} = T\ \omega$ where T is the torque on the rotor and ω is the rotational speed of the rotor. The torque is:

$$T = \Delta\ (\rho Q\ V_T\ R)$$

The tangential flow velocities at inlet and outlet are:

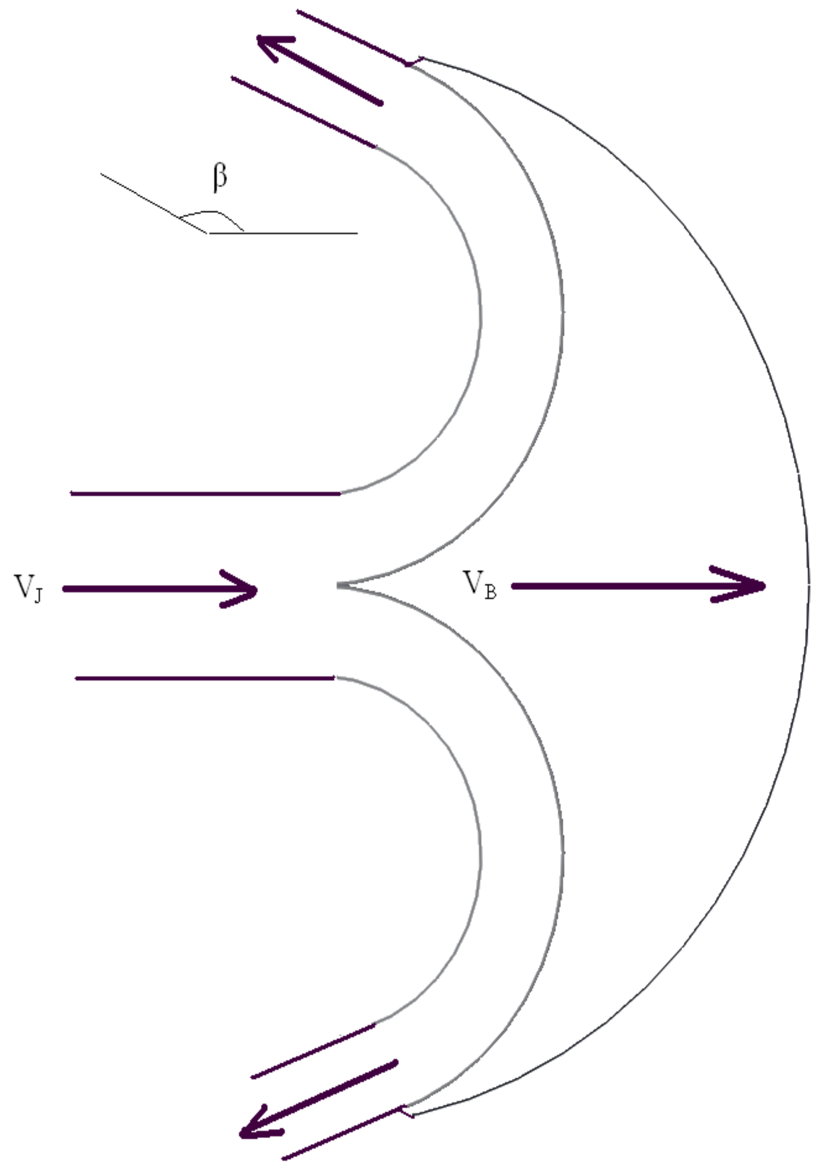
$$V_{IN} = V_J \qquad V_{OUT} = (V_J - V_B)\ K\ \text{Cos}\beta + V_B$$

where β is the bucket outlet angle and K is a loss factor. The blade and jet velocities are:

$$V_B = R\ \omega \qquad V_J = k\ \sqrt{[2P/\rho]}$$

So power becomes:

$$\mathbf{P} = \rho Q\ (V_J - V_B)\ (1 - K\ \text{Cos}\beta)\ V_B$$


β
V_J
V_B

FRANCIS TURBINE THEORY

The power output of the turbine is: $\mathbf{P} = T\ \omega$ where T is the torque on the rotor and ω is the rotational speed of the rotor. The torque is:

$$T = \Delta\ (\rho Q\ V_T\ R)$$

The tangential flow velocities at inlet and outlet are:

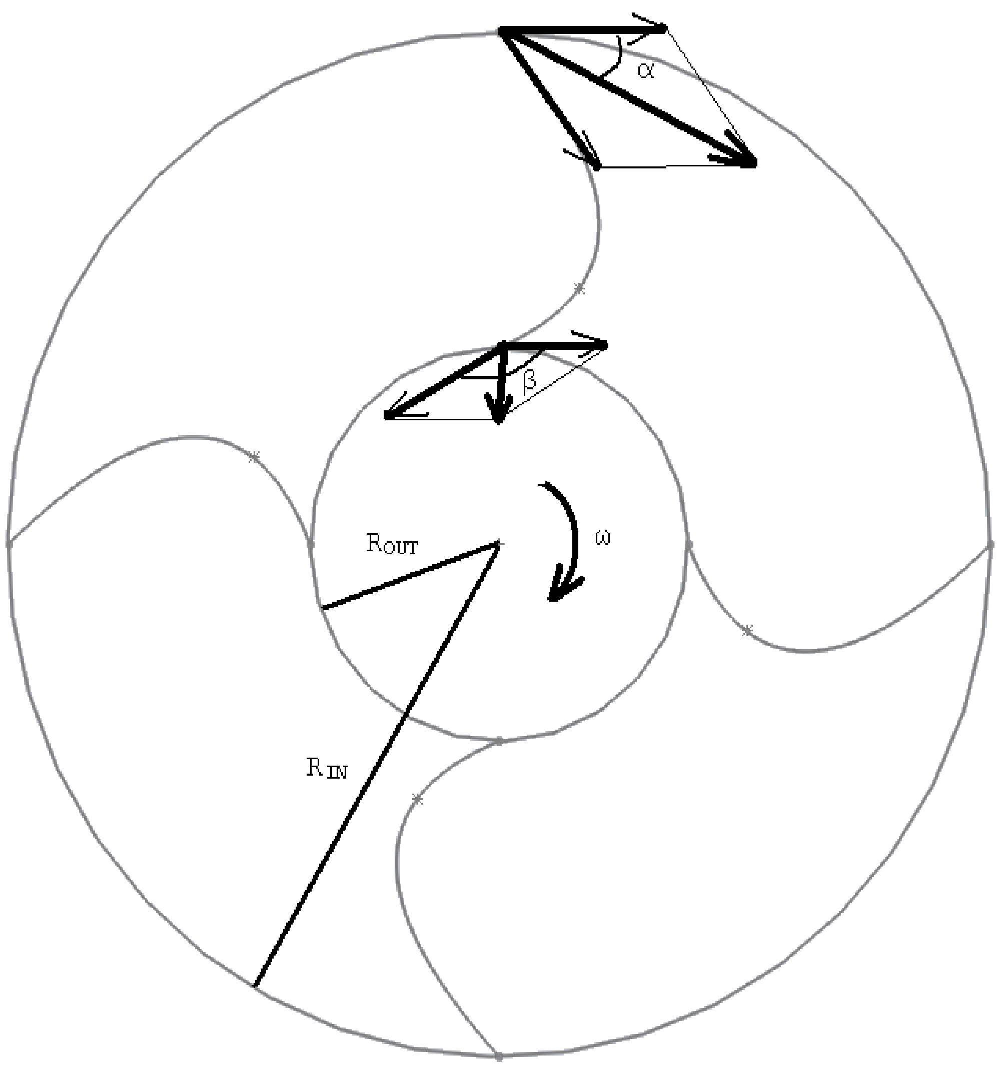
$$V_{IN} = V_N\ \mathrm{Cot}[\alpha] \qquad V_{OUT} = V_B + V_N\ \mathrm{Cot}[\beta]$$

where α is the inlet guide vane angle and β is the blade outlet angle. The blade and normal velocities are:

$$V_B = R\ \omega \qquad V_N = Q\ /\ [\pi\ 2R\ h]$$

where h is the depth of the rotor. So power becomes:

$$\mathbf{P} = \rho Q\ (V_{IN}\ R_{IN} - V_{OUT}\ R_{OUT})\ \omega$$
$$= \rho Q\ (\ [V_T V_B]_{IN} - [V_T V_B]_{OUT}\ )$$



α
β
ω
ROUT
RIN

KAPLAN TURBINE THEORY

The power output of the turbine is: $\mathbf{P} = T\ \omega$ where T is the torque on the rotor and ω is the rotational speed of the rotor. The torque is:

$$T = \Delta\ (\rho Q\ V_T\ R)$$

The tangential flow velocities at inlet and outlet are:

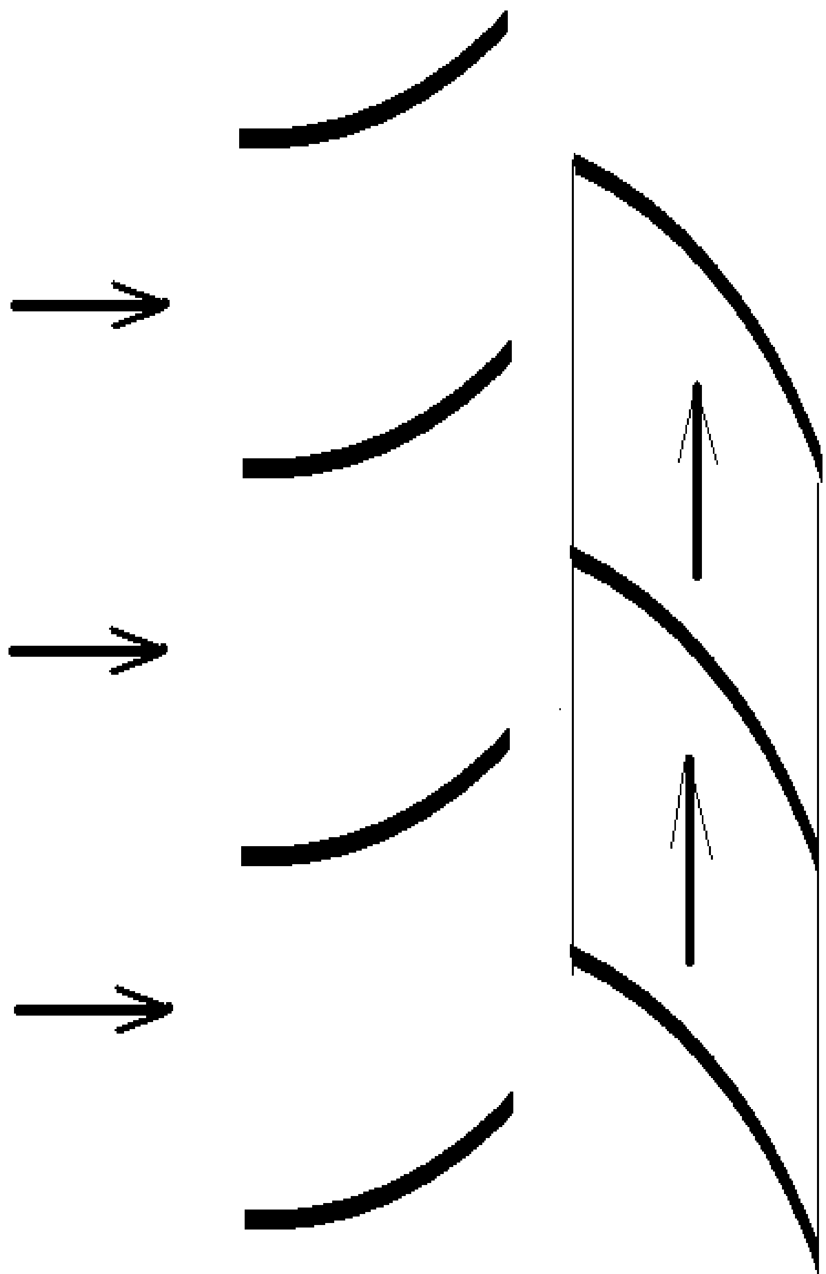
$$V_{IN} = V_N\ \mathrm{Cot}[\alpha] \qquad V_{OUT} = V_B + V_N\ \mathrm{Cot}[\beta]$$

where α is the inlet guide vane angle and β is the blade outlet angle. The blade and normal velocities are:

$$V_B = (R_O+R_I)/2\ \omega \qquad V_N = Q\ /\ [\pi\ (R_O\ R_O - R_I\ R_I)]$$

where R_O and R_I are outer radius and inner radius of the blade respectively. So power becomes:

$$\mathbf{P} = \rho Q\ (V_{IN}\ R_{IN} - V_{OUT}\ R_{OUT})\ \omega$$
$$= \rho Q\ (\ [V_T V_B]_{IN} - [V_T V_B]_{OUT}\ )$$



CENTRIFUGAL PUMP THEORY

The power output of the pump is:

$$\mathbf{P} = T\ \omega = \Delta\ (\rho Q\ V_T\ R)\ \omega$$

The tangential flow velocities at inlet and outlet are:

$$V_{IN} = V_N\ \mathrm{Cot}[\alpha] \qquad V_{OUT} = V_B + V_N\ \mathrm{Cot}[\beta]$$

The blade and normal velocities are:

$$V_B = R\ \omega \qquad V_N = Q\ /\ [\pi\ 2R\ h]$$

Power output is also

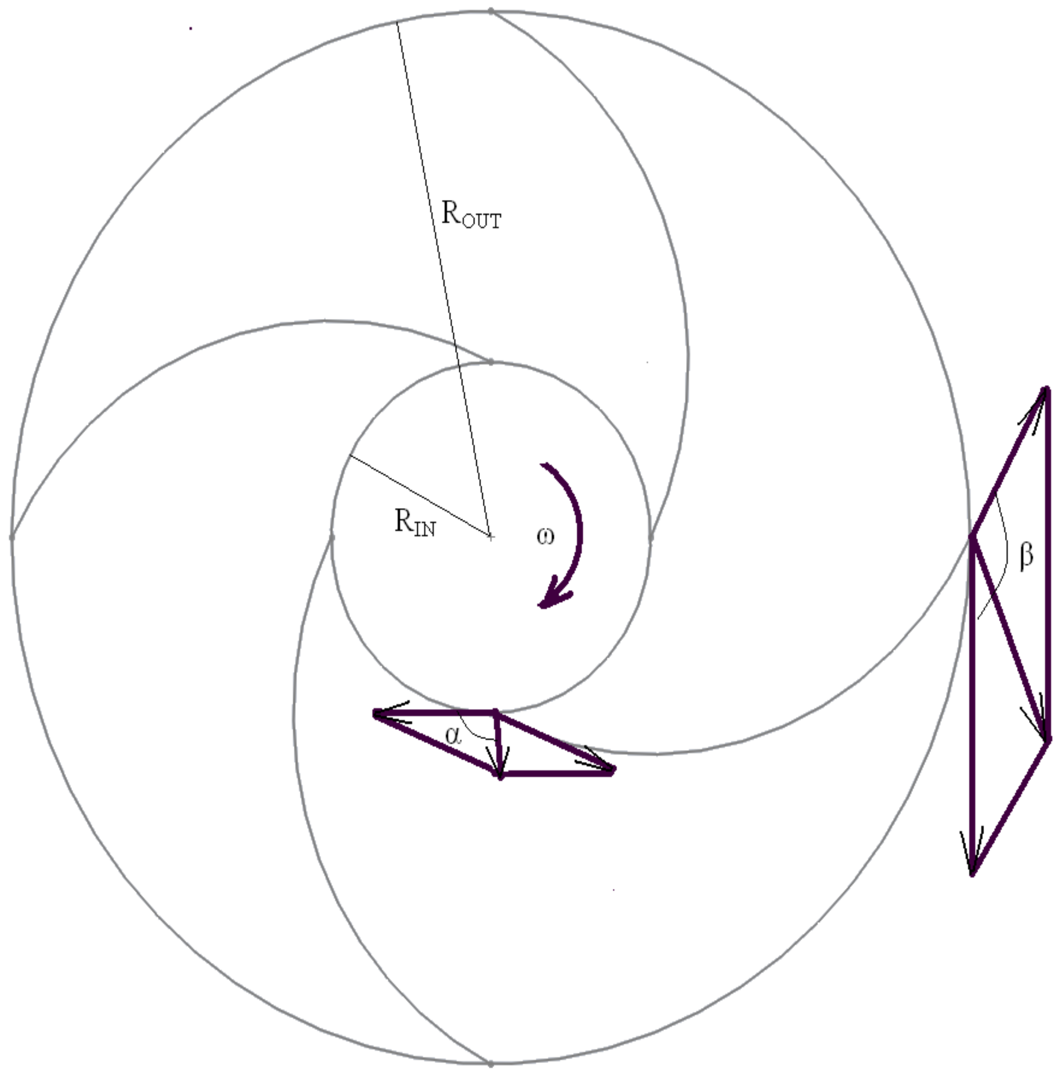
$$\mathbf{P} = P\ Q$$

Manipulation gives

$$P = \mathbf{P}\ /\ Q = \Delta\ (\rho\ V_T\ R)\ \omega$$

$$= \rho\ (V_{OUT}\ R_{OUT} - V_{IN}\ R_{IN})\ \omega$$

$$= \rho\ (\ [V_T V_B]_{OUT} - [V_T V_B]_{IN}\ )$$



R_{OUT}
R_{IN}
ω
α
β

PROPELLOR PUMP THEORY

The power output of the pump is:

$$\mathbf{P} = T\ \omega = \Delta\ (\rho Q\ V_T\ R)\ \omega$$

The tangential flow velocities at inlet and outlet are:

$$V_{IN} = V_N\ \mathrm{Cot}[\alpha] \qquad V_{OUT} = V_B + V_N\ \mathrm{Cot}[\beta]$$

The blade and normal velocities are:

$$V_B = (R_O+R_I)/2\ \omega \qquad V_N = Q\ /\ [\pi\ (R_O\ R_O - R_I\ R_I)]$$

Power output is also

$$\mathbf{P} = P\ Q$$

Manipulation gives

$$P = \mathbf{P}\ /\ Q\ = \Delta\ (\rho\ V_T\ R)\ \omega$$

$$= \rho\ (V_{OUT}\ R_{OUT} - V_{IN}\ R_{IN})\ \omega$$

$$= \rho\ (\ [V_T V_B]_{OUT} - [V_T V_B]_{IN}\ )$$