FLUIDS IN MOTION

BASIC FLUID

PROPERTIES

PRESSURE

Pressure is force intensity. It is force per unit area. Its SI units are N/m^2. It acts normal to any surface exposed to it. In a gas, pressure is due mainly to rebound forces associated with the high speed motion of its molecules. In a liquid, intermolecular forces also contribute.

TEMPERATURE

In both liquids and gases, temperature is basically a measure of the kinetic energy of molecular motion. Its SI units are degrees Celsius and degrees Kelvin.

DENSITY

Density is mass per unit volume. Its SI units are kg/m^3.

SURFACE TENSION

At an interface between a gas and a liquid, there is an imbalance of intermolecular forces which makes it seem like there is a membrane under tension at the interface. Its units are N/m. Water spiders walk on this membrane.

VISCOSITY

Viscosity relates the shear stress acting on a fluid element to its rate of shear strain:

$$\tau = \mu \, dU/dn$$

In a flow, viscosity causes a lateral transfer of momentum. In a gas, viscosity is due mainly due to the high speed random motion of its molecules. In a liquid, it is due mainly to intermolecular forces.

COMPRESSIBILITY

The compressibility of a fluid connects volume or density changes to changes in pressure. The bulk modulus K of a liquid relates changes in pressure to changes in volume:

$$\Delta P = - K \, \Delta V/V$$

Mass is constant for a bit of fluid. This implies

$$\Delta[M] = \Delta[\rho V] = V\Delta\rho + \rho\Delta V = 0$$

$$\Delta V/V = - \Delta\rho/\rho$$

With this the bulk modulus equation becomes

$$\Delta P = + K \Delta\rho/\rho$$

Manipulation gives

$$\Delta P/\Delta\rho = K/\rho$$

Thermodynamics gives for a gas

$$P/\rho = RT \qquad P = N \rho^n$$

Differentiation and manipulation gives

$$\Delta P/\Delta\rho = n/\rho \, N \rho^n = n \, P/\rho = nRT$$

One can show that the wave speed for a fluid is

$$a = \sqrt{\Delta P/\Delta\rho}$$

One can also show that across a wave

$$|\Delta P| = \rho a \, |\Delta U|$$

APPLICATION : DRUM VISCOMETER

A drum viscometer consists of a drum which rotates inside a sleeve. A liquid fills the gap between them. Let the gap be h. The shear stress due to rotation is:

$$\mu\ R\omega/h$$

This acts over the cylindrical area

$$2\pi\ R\ L$$

The force due to this is

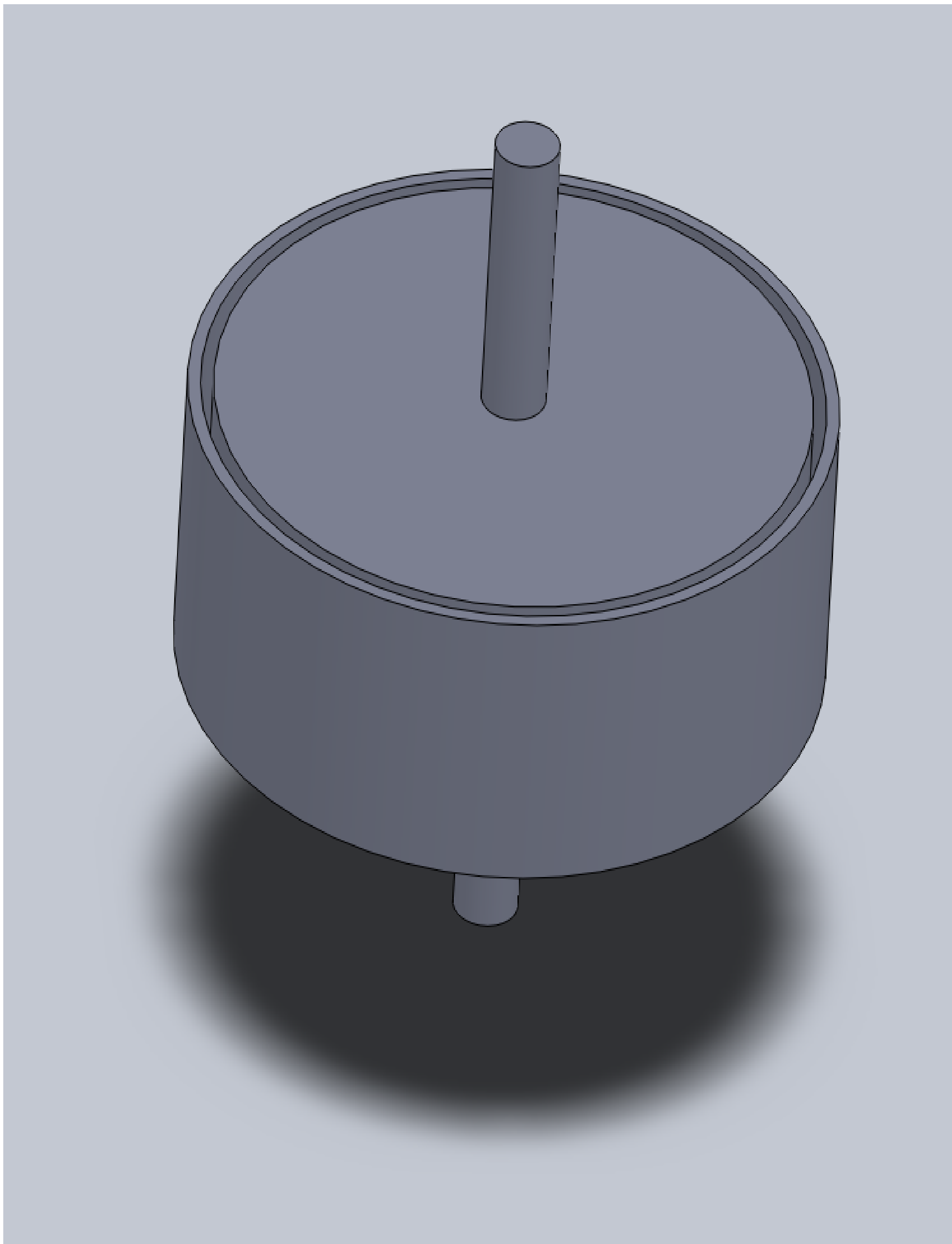
$$\mu\ R\omega/h\ 2\pi\ R\ L$$

The torque needed to rotate the drum is

$$T = R\ \mu\ R\omega/h\ 2\pi\ RL$$

With known geometry and measured torque, one gets

$$\mu = [\ T\ h\ ]\ /\ [\ 2\pi\ R^3\ L\ \omega\ ]$$



APPLICATION: DISK VISCOMETER

A disk viscometer consists of a disk which rotates inside a can. A liquid fills the gap between the disk and the can. Let the gap be h. The shear stress due to rotation is:

$$\mu \; r\omega/h$$

This acts over the annular area

$$2\pi \; r \; dr$$

The force due to this is

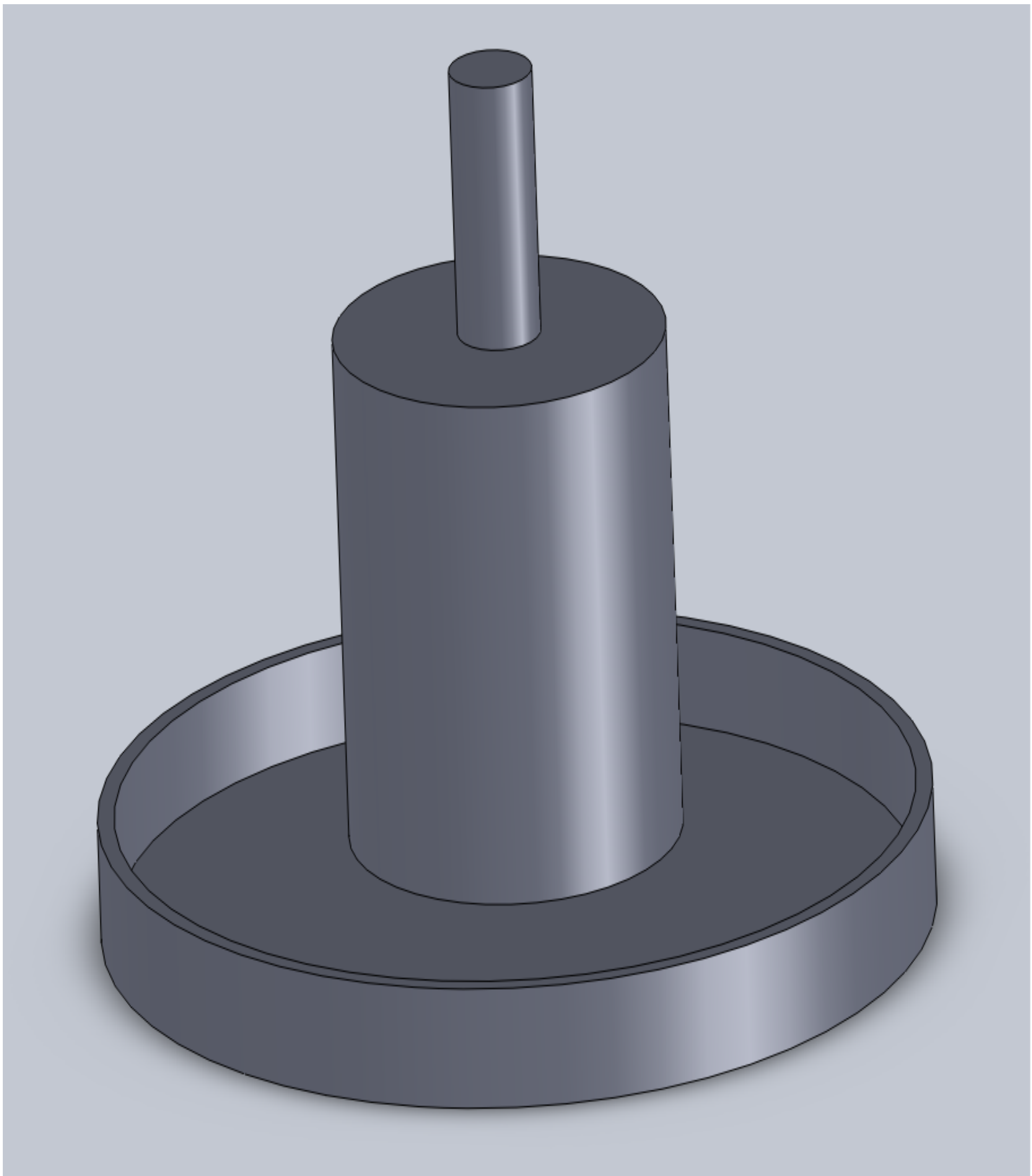
$$\mu \; r\omega/h \; 2\pi \; r \; dr$$

The torque needed to rotate the disk is

$$T = \int r \; \mu \; r\omega/h \; 2\pi \; r \; dr$$

With known geometry and measured torque, one gets

$$\mu = [\; 2 \; T \; h \;] \; / \; [\pi \; R^4 \; \omega]$$



APPLICATION : CAPILLARY VISCOMETER

A small diameter tube is known as capillary. Conservation of Mass considerations give for flow in such a tube

$$\partial U/\partial s = 0$$

while Conservation of Momentum considerations give

$$0 = - r\ \partial P/\partial s + \partial/\partial r\ (r\ \mu\ \partial U/\partial r)$$

Integration of Momentum gives

$$0 = - \partial P/\partial s\ r^2/2 + r\ \mu\ \partial U/\partial r + K$$

where K=0 because r can be zero. Manipulation gives

$$\partial U/\partial r = r/[2\mu]\ \partial P/\partial s$$

Integration of this equation gives

$$U = r^2/[4\mu]\ \partial P/\partial s + C$$

At r equal to R, U is zero. So U becomes

$$U = - [R^2-r^2]/[4\mu]\ \partial P/\partial s$$

Integration gives the volumetric flow rate

$$Q = \int U \, 2\pi r \, dr$$

$$= - \int [R^2 - r^2]/[4\mu] \, \partial P/\partial s \, 2\pi r \, dr$$

$$= - [\pi R^4]/[8\mu] \, \partial P/\partial s$$

For a tube L meters long open at both ends with its outlet H meters below its inlet, this equation becomes

$$Q = [\pi R^4]/[8\mu] \, [\rho g H]/L$$

Manipulation of this equation gives

$$\mu = [\rho g H][\pi R^4] \, / \, [8QL]$$

This is the equation for a tube viscometer.

For a steady flow the head loss is H. Solving for H gives

$$H = \mu \, [8QL] \, / \, [\rho g \, \pi R^4]$$

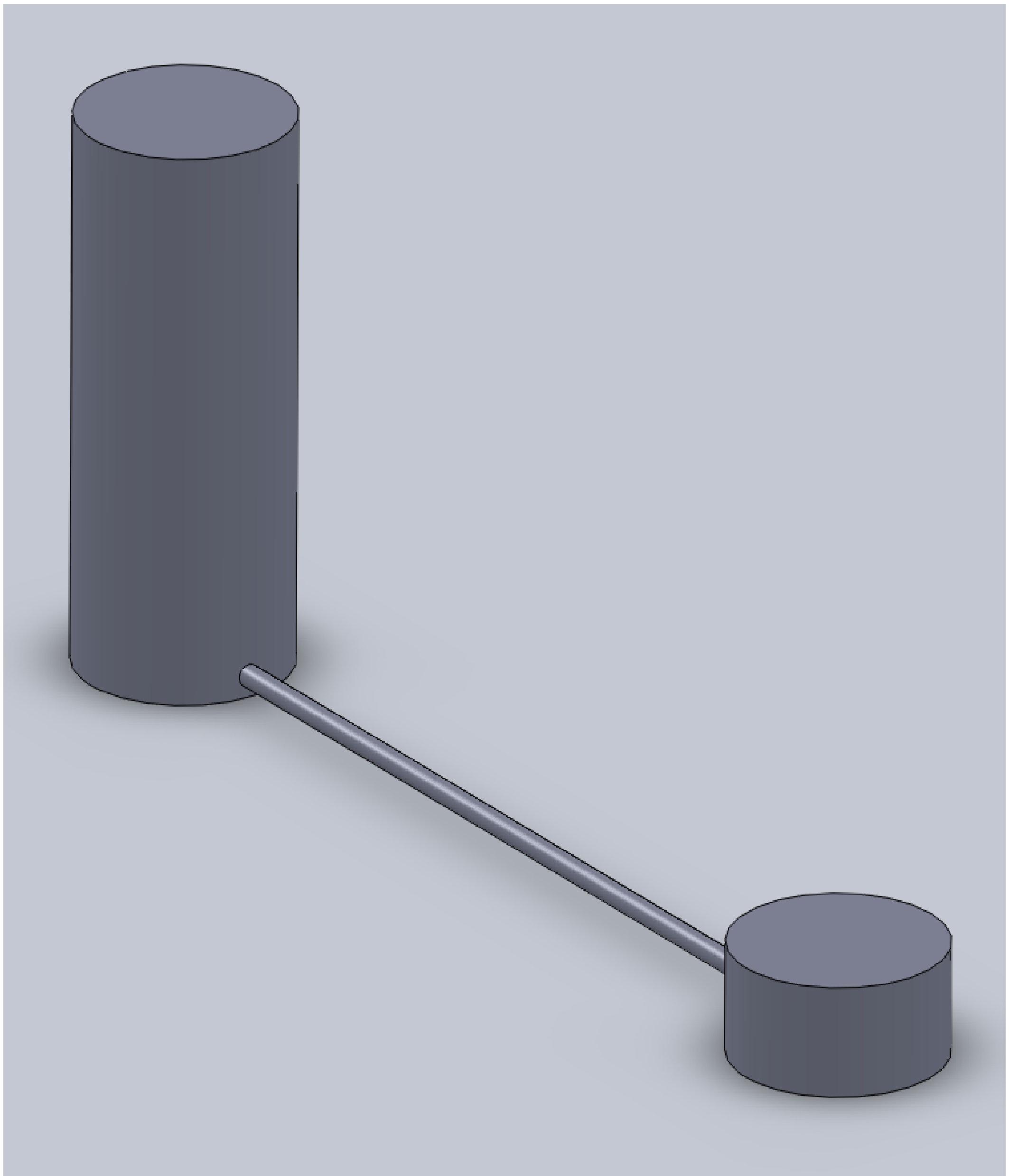
In terms of average flow speed **U** the flow is

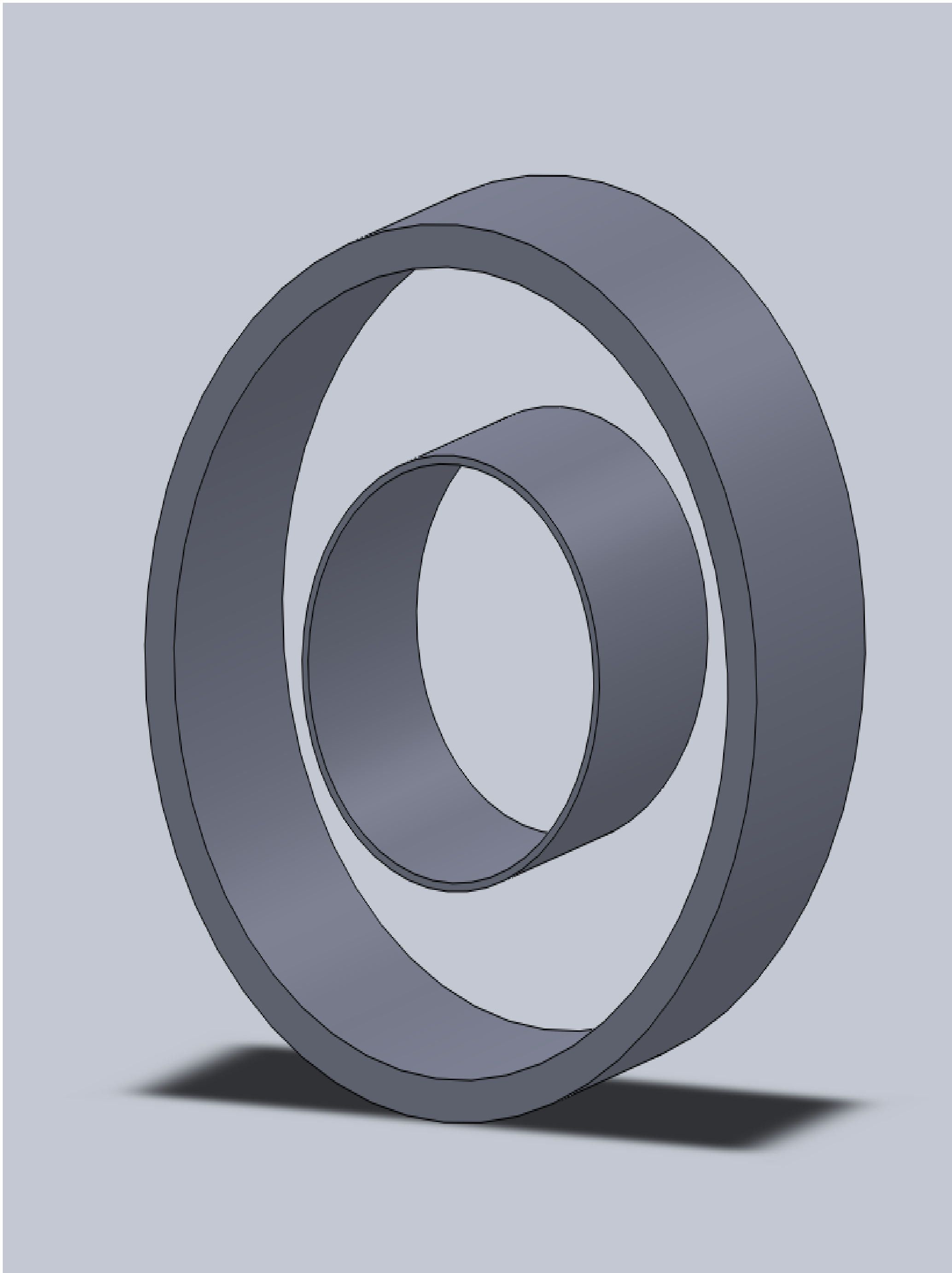
$$Q = \mathbf{U} \, \pi R^2 = \mathbf{U} \, \pi D^2/4$$

With this head becomes

$$H = 64/Re \; L/D \; \mathbf{U}^2/[2g]$$

$$= f \; L/D \; \mathbf{U}^2/[2g]$$





APPLICATIONS: SURFACE TENSION

WATER DROPLET IN AIR

AIR BUBBLE IN WATER

$$P \pi R^2 = 2\pi R \sigma$$

$$P = 2\sigma/R$$

AIR BUBBLE IN AIR

$$P \pi R^2 = 2\ 2\pi R \sigma$$

$$P = 4\sigma/R$$

CAPILLARY TUBE

$$\rho g\ \pi R^2\ h = \sigma\ 2\pi R\ \mathrm{Cos}[\beta]$$

$$h = 2\sigma\ \mathrm{Cos}[\beta]\ /\ [\rho g\ R]$$