

## FLUID MECHANICS I

### QUIZ #1

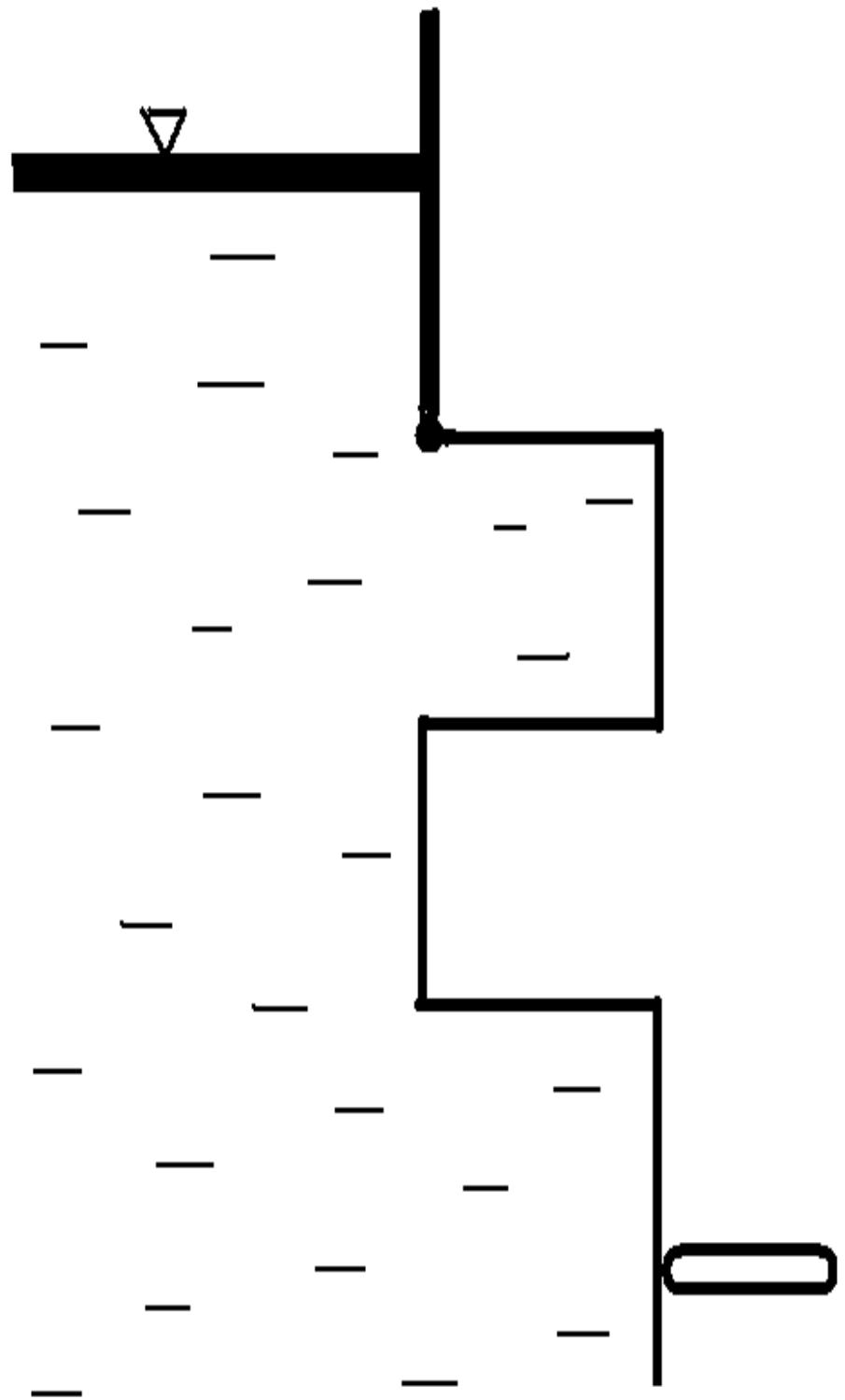
Consider the box shaped 2D gate shown in the sketch. Determine the vertical and horizontal loads on the gate using the pressure weight method. Determine the moment of the loads about the pivot. For this let all flat surfaces be 1m long.

A catamaran dugout canoe consists of 2 dugout canoes strapped together. Each dugout is  $2R$  wide. The spacing between them is  $2H$ . They are submerged a distance  $R$  below the water line. Determine the location of the metacenter relative to the water line. For this let  $H=3$   $R=1$   $L=6$ . Note that  $B$  is located a distance  $[4R]/[3\pi]$  below the water line.

Dimensional analysis for a pump gives the following scaling equations for pressure and flow.

$$C_P = P / [\rho N^2 D^2] \quad C_Q = Q / [ND^3]$$

Using these equations determine scaling equations for pump power and head. Discuss the implications of the pump scaling laws with respect to pump size.



For the gate the vertical load is:

$$F_V = V_B - W$$

Geometry shows that:

$$W = 2\rho g 1*1 = 20000N \quad V_B = \rho g 1*4 = 40000N$$

So the vertical load is:

$$F_V = 40000 - 20000 = 20000N$$

The horizontal load on the gate is:

$$F_H = H_R + H_T$$

Geometry shows that:

$$H_R = \rho g 1*3 = 30000N \quad H_T = \rho g 3*3/2 = 45000N$$

So the horizontal load is:

$$F_H = 30000 + 45000 = 75000N$$

The pivot moment is:

$$H_R*1.5 + H_T*2.0 + V_B*0.5 - W*0.5 = 145000 Nm$$

To work this problem using analytical integration, for each bit of the gate, you would set up a coordinate system which runs along the bit. Let the coordinate be  $C$  and let the length of the bit be  $H$ . Let the two ends of the bit be  $A$  and  $B$ . The pressure at the two ends would be  $\rho gh_A$  and  $\rho gh_B$ . The pressure variation along bit would be:

$$P = P_A + (P_B - P_A) / H \cdot C$$

Integration of  $P \, dC$  gives the load on the bit:

$$\begin{aligned} & P_A H + (P_B - P_A) / H \cdot H^2 / 2 \\ & = P_A H + (P_B - P_A) \cdot H / 2 \end{aligned}$$

Note that the 1<sup>st</sup> term in the load is basically the area under the rectangular part of the pressure profile while the 2<sup>nd</sup> term is basically the area under the triangular part of the pressure profile. Integration of  $PC \, dC$  divided by the load would give the location of the load on the bit. Knowing the orientation of the bit, one could break the load into horizontal and vertical components.

For the catamaran dugout canoe the roll K is:

$$K = 2 (H^2 2R + 2R^3/3) L$$

The volume V is:

$$V = 2 \pi R^2/2 L$$

Theory shows that:

$$K \theta = V S = V BM \theta$$

Manipulation gives:

$$BM = K/V$$

$$BM = [2 (H^2 2R + 2R^3/3) L] / [\pi R^2 L]$$

$$BM = 4H^2/\pi R + 4R/3\pi$$

The distance of M above the water line is:

$$MO = BM - BO = 4H^2/\pi R + 4R/3\pi - 4R/3\pi$$

$$= 4H^2/\pi R = [4 3^2] / [\pi 1] = 11.5m$$

The pitch  $K$  of the canoe is:

$$K = 2 (L/2)^3 / 3 (2R + 2R)$$

The volume  $V$  is again:

$$V = 2 \pi R^2 / 2 L$$

Manipulation gives:

$$BM = L^2 / [3\pi R] \quad BO = 4R / 3\pi$$

$$MO = BM - BO = 36/3\pi - 4/3\pi = 32/3\pi = 3.4m$$

For a canoe made up of several pairs of legs:

$$BM = \sum K / \sum V$$

where the  $K$  and the  $V$  for each pair are:

$$K = (4H^2R + 4R^3/3) L$$

$$V = 2 A L$$