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ENGINEERING 6951

AUTOMATIC CONTROL ENGINEERING

FINAL EXAM

FALL 2011

MARKS IN SQUARE [] BRACKETS

INSTRUCTIONS

NO NOTES OR TEXTS ALLOWED

NO CALCULATORS ALLOWED

GIVE CONCISE ANSWERS

ASK NO QUESTIONS

SYSTEM DESCRIPTION

Rollers are used to control the thickness of material sheets. A hydraulic actuator forces rollers onto the sheet. The thickness sensor is located downstream of the rollers and this introduces a transport lag. Here we use the Pade approximant to approximate this lag. The governing equations for the system are:

SENSOR   + T/2 dP/dt + P = - T/2 dR/dt + R

THICKNESS ERROR   E = C − P

CONTROL SIGNAL   Q = K E

OIL FLOW TO ACTUATOR   M = A Q

VELOCITY OF ACTUATOR   B V =   M + N

SHEET THICKNESS   X dR/dt = V

where R is the actual thickness of the sheet at the rollers, P is the thickness of the sheet at the sensor, C is the command thickness, T is the time lag, V is the actuator velocity, M is a control flow to the actuator, N is a disturbance flow due to leakage, Q is the control signal, E is the error signal, T K A B X are constants.

T=2   A=1   B=1   X=1

Sketch an overall block diagram for the system. [10]

The transfer functions are:

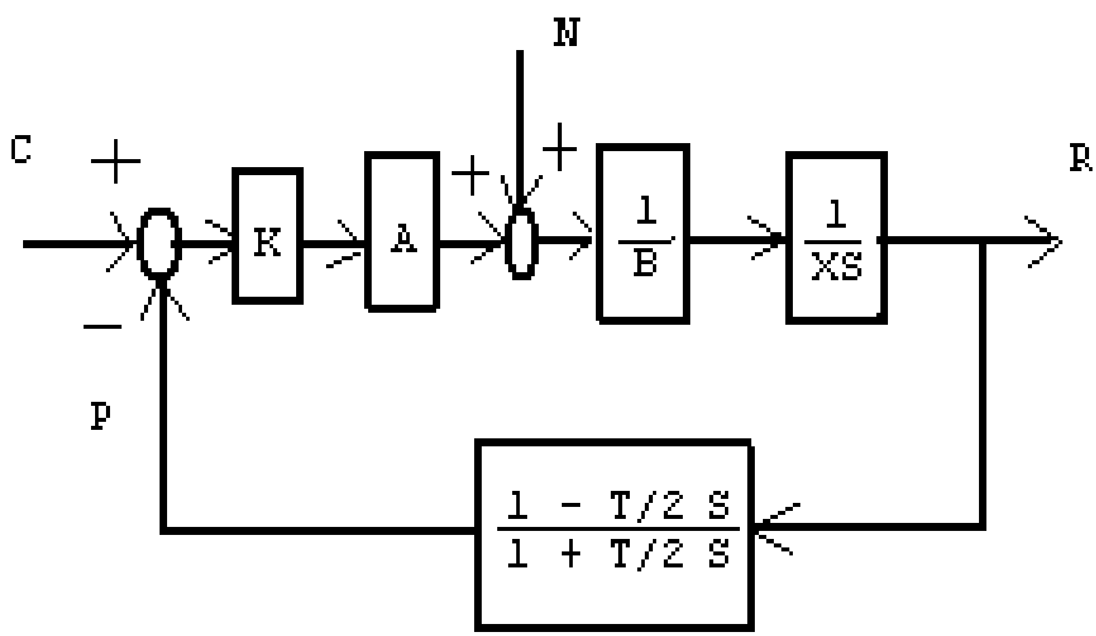
+ T/2 S P + P = - T/2 S R + R

P/R = [1 - T/2 S] / [1 + T/2 S]

E = C − P   Q = K E   M = A Q

B V =   M + N   V/[M + N] = 1/B

X S R = V   R/V = 1/[XS]



Derive equations for the system Ziegler Nichols gains. [Hint: Differentiate sensor equation wrt time.] [10]

Differentiation gives

$$+ T/2 \ d^2P/dt^2 + dP/dt = - T/2 \ d^2R/dt^2 + dR/dt$$

$$X \ dR/dt = V \qquad X \ d^2R/dt^2 = dV/dt$$

$$V = [A \ K \ (C-P) + N] / B$$

$$dV/dt = [A \ K \ (dC/dt-dP/dt) + dN/dt] / B$$

Substitution into the sensor equation gives

$$+ T/2 \ d^2P/dt^2 + dP/dt =$$

$$- T/2 \ [A \ K \ (dC/dt-dP/dt) + dN/dt] / [B \ X]$$

$$+ [A \ K \ (C-P) + N] / [B \ X]$$

Assume that the system is borderline with P:

$$P = P_o + \Delta P \ Sin[\omega t]$$

Assume that the inputs are constants:

$$C = C_o \qquad N = N_o$$

Substitution into the sensor equation gives

$$- T/2\ \omega^2\ \Delta P\ \mathrm{Sin}[\omega t]\ + \omega\ \Delta P\ \mathrm{Cos}[\omega t]\ =$$

$$+ T/2\ [A\ \mathbf{K}]\ \omega\ \Delta P\ \mathrm{Cos}[\omega t]\ /\ [B\ X]$$

$$+ [[A\ \mathbf{K}]\ [C_o - P_o] + N_o]\ /\ [B\ X]\ - [A\ \mathbf{K}]\ \Delta P\ \mathrm{Sin}[\omega t]\ /\ [B\ X]$$

This is of the form

$$i\ \mathrm{Sin}[\omega t]\ + \ j\ \mathrm{Cos}[\omega t]\ + \ k\ = \ 0$$

Mathematics requires that

$$i = 0 \qquad j = 0 \qquad k = 0$$

$$\mathbf{K} = [T/2]/A \qquad \omega^2 = [A\mathbf{K}]/[T/2] \qquad P_o = C_o + N_o/[A\mathbf{K}]$$

Plugging in numbers gives

$$\mathbf{K} = 1 \qquad \omega = 1 \qquad P_o = C_o + N_o$$

Develop equations that would allow the behavior of the system to be predicted step by step in time. For this, use an exact representation of the transport lag. [10]

The exact representation of the transport lag is

$$P(t) = R(t-T)$$

When this is used, the only ODE is

$$X \, dR/dt = V$$

An application of time stepping to this gives

$$R_{NEW} = R_{OLD} + \Delta t \; [V_{OLD}/X]$$

The algebraic equations are

$$E_{OLD} = C_{OLD} - P_{OLD} \qquad Q_{OLD} = K \; E_{OLD}$$

$$M_{OLD} = A \; Q_{OLD} \qquad V_{OLD} = [M_{OLD} + N_{OLD}]/B$$

Determine the characteristic equation for the system. [10]

The GH function is:

$$\frac{[A * K] * [1 - T/2 S]}{B * [X S] * [1 + T/2 S]}$$

$$\frac{K * [1 - S]}{S * [1 + S]}$$

This is of the form

$$GH = N/D$$

The characteristic equation is:

$$N + D = 0$$

$$K * [1 - S] + S * [1 + S] = 0$$

$$S^2 + [1 - K] S + K = 0$$

Use the Routh Hurwitz criteria to determine the borderline gain of the system. [5] Is the system stable when K is half the borderline gain? [5]

For stable operation of a system, all coefficients in its characteristic equation must be positive. In addition, certain tests functions must be positive.

A quadratic characteristic equation has the form

$$a S^2 + b S + c = 0$$

It has no test functions. For stable operation, each of its coefficients must be positive: $a>0$ $b>0$ $c>0$.

For thickness control, this implies

$$K > 0 \qquad [1 - K] > 0$$

This gives the borderline gain **K**=1. When K is half the borderline gain **K,** [1-K] is 1/2 which is greater than zero so the system is stable with this gain.

Sketch the Nyquist plot when K is half the borderline gain. [10] What are the system stability margins? [5] Explain the significance of GH equal to minus one. [2.5] What function would model the transport lag exactly? [2.5]

The GH function is

$$\frac{[\ A * K\ ] * [\ 1 - T/2\ S\ ]}{B * [\ X\ S\ ] * [\ 1 + T/2\ S\ ]}$$

$$\frac{K * [\ 1 - S\ ]}{S * [\ 1 + S\ ]}$$

Along the imaginary axis in the S plane S=jω

$$\frac{K * [\ 1 - \omega j\ ]}{\omega j * [\ 1 + \omega j\ ]}$$

$$\frac{K * (\ 1 - \omega j\ ) * (\ 1 - \omega j\ )}{\omega j * (\ 1 + \omega j\ ) * (\ 1 - \omega j\ )}$$

$$\frac{K * (\ 1 - 2\omega j - \omega^2\ )}{\omega j * (\ 1 + \omega^2\ )}$$

As ω approaches zero, the GH function reduces to

$$\frac{K * (1)}{\omega j * (1)}$$

which tends to minus infinity j.

As ω approaches infinity the GH function reduces to

$$\frac{K * (-\omega^2)}{\omega j * (+\omega^2)} \qquad \frac{-K}{\omega j}$$

which tends to plus zero j.

A real axis cross over occurs when ω^2 is equal to one. In this case, the GH function becomes

$$\frac{0.5 * (- 2\omega j)}{\omega j * (1 + \omega^2)} \qquad \frac{0.5 * (- 2j)}{j * (2)}$$

which is equal to minus 0.5.

The GH plot is shown on the next page. Inspection of the plot shows that the net clockwise rotations is zero. Inspection of the GH function shows that the number of unstable poles is zero. The number of unstable zeros is:

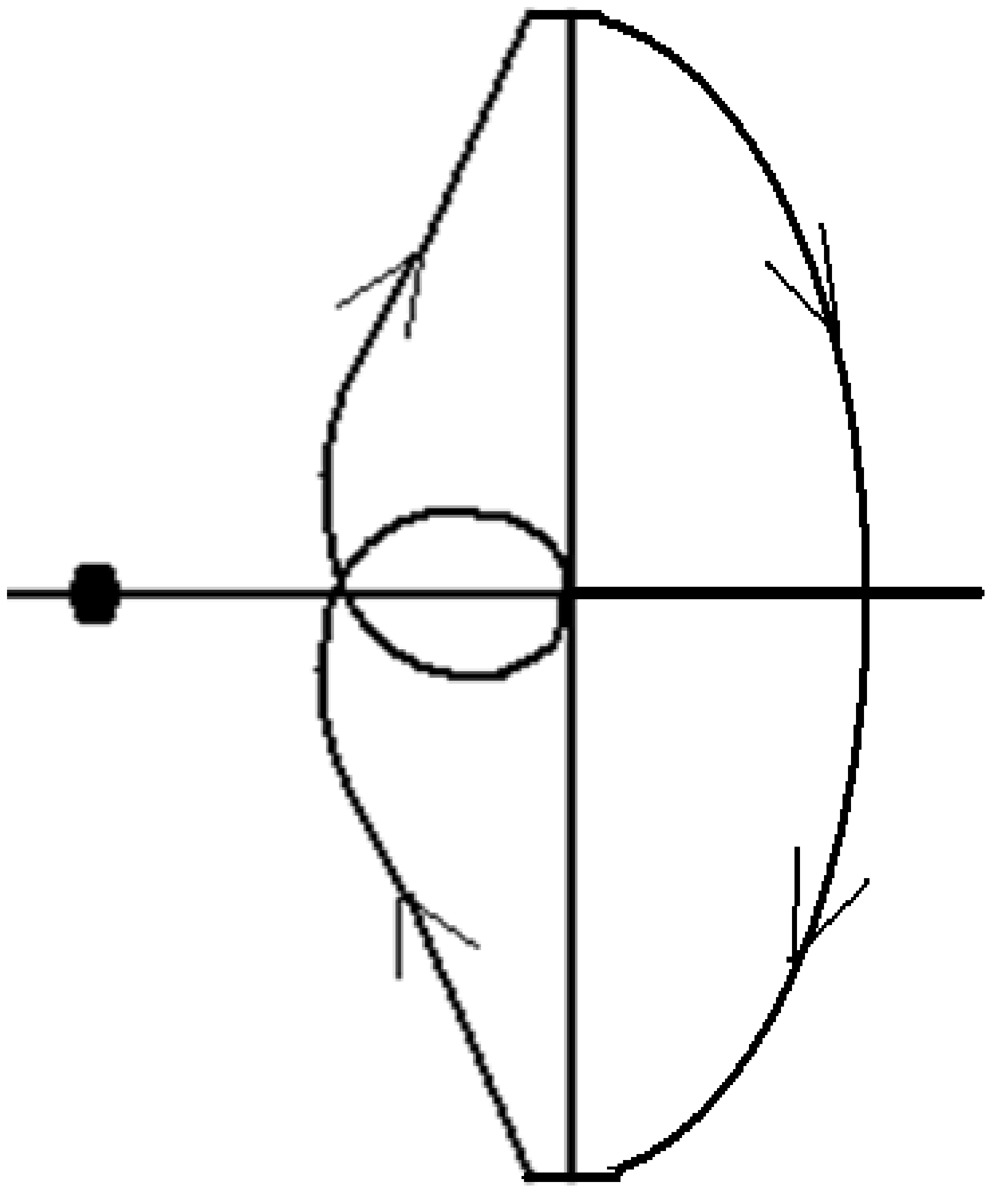
$$N = N_Z - N_p \qquad N_Z = N + N_p$$

This gives N_Z equal to zero. So the system is stable.

The gain margin is one over the magnitude of GH where it crosses the negative real axis. Here it is 2. The phase margin is the angle to where the GH plot crosses a unit circle centered on the origin. One could get this by plotting more points on the GH plot.

A GH plot is basically a polar open loop frequency response plot. When GH is equal to minus one, a command sine wave produces a response which has the same magnitude as the command but is 180° out of phase. If the command was suddenly removed and the loop was suddenly closed, the negative of the response would take the place of the command and keep the system oscillating. If the gain was bigger than **K**, the command would produce a response bigger than itself. When this takes over, it would produce growing or unstable oscillations. If the gain was smaller than **K**, the command would produce a response smaller than itself. When this takes over, it would produce decaying or stable oscillations.

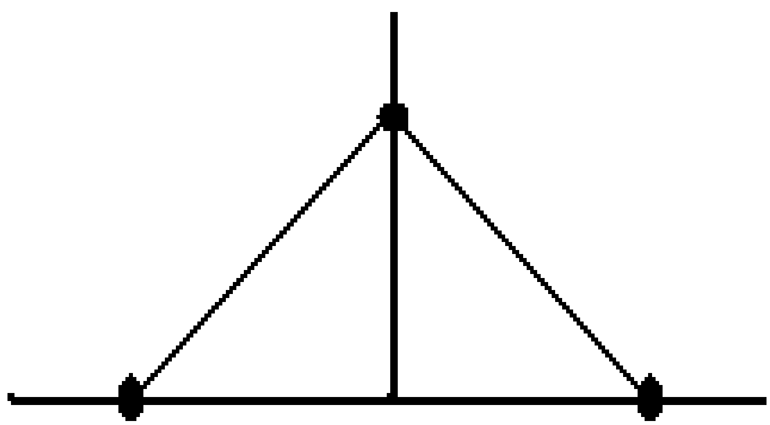
The function e^{-TS} models transport lags exactly.



Use Root Locus concept to check the borderline gain. [10]

The GH function is:

$$\frac{K * [1 - S]}{S * [1 + S]}$$



Nyquist suggests ω=1. In this case the angles are:

-45 -90 -45 = -180

The magnitudes are:

[**K** * √2] / [1 * √2] = 1 **K** = 1

Determine the amplitude and the period of the limit cycle generated when the system is controlled by an ideal relay controller with DF=1/E_o. [5] Is the limit cycle stable? [2.5] Is the system practically stable? [2.5]. For this problem, assume that R is in millimeters. [BONUS: For relay with deadband: What is the critical deadband?]

At a limit cycle the DF is equal to the borderline gain:

$$\mathbf{DF} = \mathbf{K} = 1/\mathbf{E_o}$$

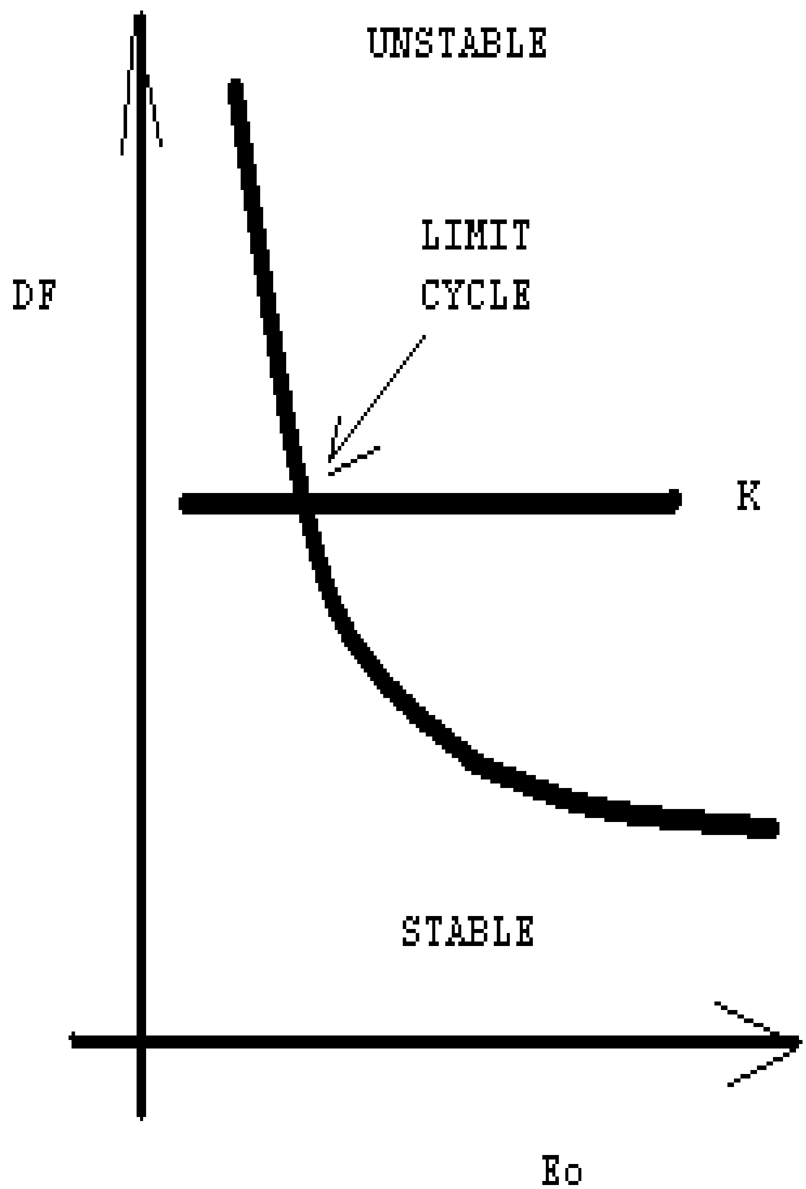
$$\mathbf{E_o} = 1/\mathbf{K} = 1$$

The limit cycle frequency is $\boldsymbol{\omega}$=1. So the period is

$$\mathbf{T_o} = 2\pi/\boldsymbol{\omega} = 2\pi$$

This is a system which is stable when K is below the borderline gain **K**. So the limit cycle is stable. If the sheet being rolled was around 1mm thick, the system would be practically unstable. If the sheet was 10cm thick, it might be practically stable.

Relay with deadband controller: DF theory shows that the critical deadband is half the limit cycle amplitude.


UNSTABLE
LIMIT
CYCLE
DF
K
STABLE
Eo

Write a short m code that would predict the behavior of the system step by step in time. For this, use an exact representation of the transport lag. [5] Add statements to the code that would mimic loop rate phenomena. [5]

```
A=1;B=1;X=1;K=0.5;
ROLD=0.0; POLD=0.0;
COLD=1.0; NOLD=0.0;
NIT=10000; MIT=50; PIT=10;
EOLD=COLD-POLD; JIT=0;
QOLD=K*EOLD; DELT=0.01;
for IT=1:NIT
JIT=JIT+1;
if(JIT==1)
if(IT>MIT)
 POLD=R(IT-MIT);
 EOLD=COLD-POLD;
end;end;
if(JIT==PIT)
QOLD=K*EOLD;
JIT=0;end;
MOLD=A*QOLD;
VOLD=[MOLD+NOLD]/B;
RNEW=ROLD+DELT*[VOLD/X];
ROLD=RNEW;R(IT)=RNEW;
T(IT)=IT*DELT;
end
plot(T,R)
```

BONUS [5]

Briefly explain EITHER the basis for the Nyquist procedure OR the basis for the Describing Function procedure.

NYQUIST PROCEDURE: The 1+GH function is made up of S-Z and S-P factors, where Z denote a zero while P denote a pole. When S moves clockwise around a contour which surrounds the entire right half or unstable half of the S plane, unstable zero factors cause clockwise rotations of 1+GH whereas unstable pole factors cause counterclockwise rotations. Only unstable zero or pole factors cause such rotations. Stable zero or pole factors only cause the 1+GH function to swing back and forth or nod up and down. The net clockwise rotations N is equal to the number of unstable zeros N_Z minus the number of unstable poles N_P. One gets N by inspection of the 1+GH plot and N_P from inspection of the 1+GH function. This allows one to calculate $N_Z = N + N_P$.

DESCRIBING FUNCTION PROCEDURE: Many systems with nonlinear controllers behave like a borderline stable system with a borderline proportional gain. The controller seems to be able to adjust its gain to make the system borderline stable. The describing function DF for a nonlinear controller approximates this adjustable gain.