

NAME :

AUTOMATIC CONTROL ENGINEERING

ENGINEERING 6951

FINAL EXAMINATION

FALL 2014

INSTRUCTIONS

NO NOTES OR TEXTS ALLOWED

NO CALCULATORS ALLOWED

NO ELECTRONIC DEVICES ALLOWED

ASK NO QUESTIONS

THIS EXAM HAS 12 QUESTIONS

ANSWER ANY 10 QUESTIONS

[MARKS FOR BEST 10 OUT OF 12]

EACH QUESTION IS WORTH 10%.

The equations governing the back and forth motion for an hydraulic actuator are:

PLANT

$$M \, dR/dt = P + D$$

DRIVE

$$P = N \, Q$$

CONTROLLER

$$Q = K \, E$$

$$E = C - R$$

where R is the actual position of the actuator rod in cm, C is the command position, E is the position error, P is the control valve flow, D is a leak flow, Q is the control signal, K is the controller gain and M and N are system parameters:

$$M=4 \quad N=2$$

1. Determine the proportional gain and period for borderline stable operation of the system. Does this system have Ziegler Nichols gains? [10]

Substitution into the plant equation gives

$$M \frac{dR}{dt} = N K (C-R) + D$$

During borderline stable operation

$$R = R_o + \Delta R \sin[\omega t] \quad C = C_o \quad D = D_o$$

Substitution into the plant equation gives

$$\begin{aligned} M \Delta R \omega \cos[\omega t] + N K \Delta R \sin[\omega t] \\ = \\ N K (C_o - R_o) + D_o \end{aligned}$$

This is of the form

$$i \sin[\omega t] + j \cos[\omega t] + k = 0$$

Mathematics requires that

$$i = 0 \quad j = 0 \quad k = 0$$

This gives $\omega=0$ $T=\infty$ $K=0$ $R_o=C_o+D_o/[NK]$.

2. Develop a simulation template for getting the response of the system step by step in time. Use it, with a time step of 1 second, to move 4 steps in time, when there is a step in command with height 10cm and the proportional gain K is +1. [10]

The plant equation can be written as

$$M \Delta R / \Delta t = M [R_{\text{NEW}} - R_{\text{OLD}}] / \Delta t = P_{\text{OLD}} + D_{\text{OLD}}$$

Manipulation gives the template

$$R_{\text{NEW}} = R_{\text{OLD}} + \Delta t (P_{\text{OLD}} + D_{\text{OLD}}) / M$$

The drive and controller equations become

$$P_{\text{OLD}} = N Q_{\text{OLD}}$$

$$Q_{\text{OLD}} = K E_{\text{OLD}}$$

$$E_{\text{OLD}} = C_{\text{OLD}} - R_{\text{OLD}}$$

STEP #1

$$R_{\text{OLD}} = 0 \quad C_{\text{OLD}} = 10 \quad E_{\text{OLD}} = 10$$

$$Q_{\text{OLD}} = 1[10] = 10 \quad P_{\text{OLD}} = 2 [10] = 20$$

$$R_{\text{NEW}} = 0 + 1 [20 + 0] / 4 = 5$$

3. Write a short m code based on the simulation template. Include statements in the code to mimic control of the system by a computer. Describe the influence of loop rate on performance. [10]

```
% hydraulic actuator

clear all

n=2.0; m=4.0;
rold=0.0; pold=0.0;
dold=0.0; command=10.0;
is=10; id=20; delt=0.01;
gain=1.0; nit=1000; mit=0;

for it=1:nit

mit=mit+1;

% sensor
if(mit==is) ...
sensor=rold; ...
end;

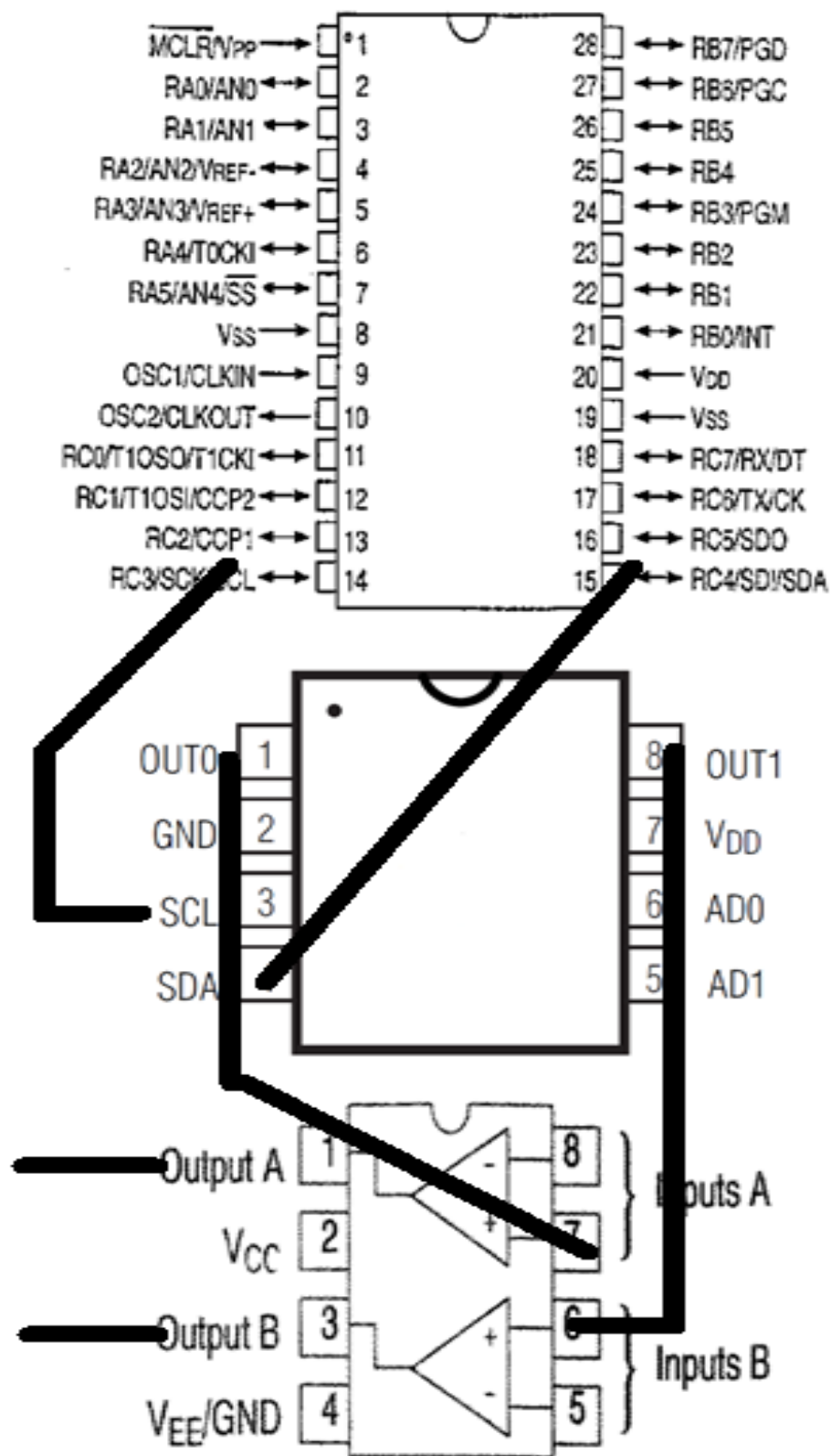
% drive
if(mit==id) ...
error=command-sensor; ...
control=gain*error; ...
pold=n*control; mit=0; ...
end;

% time step
rnew=rold+delt*(pold+dold)/m;
rold=rnew; r(it)=rnew;
time=it*delt; t(it)=time;

end

% response
plot(t,r)
xlabel('time')
ylabel('position')
title('actuator')
```

4. Sketch a PIC circuit for the system. [10]



5. Write a short PIC code for the system. [10]

```
// hydraulic actuator //
// header files //
#include <16f876.h>
#fuses HS,NOWDT
#fuses NOPROTECT,NOLVP
#fuses NOBROWNOUT,NOPUT
#device ADC=10 // 10 BIT
#use delay(clock=20000000)
#use i2c(master,sda=PIN_C4,scl=PIN_C3,slow)
#org 0x1F00,0x1FFF{}

void one(int bits);
void two(int bits);

// declare variables //
float target,data,error;
float control,gain,dump;
int power;

void main()
{
    // setup ports //
    setup_adc_ports(ALL_ANALOG);
    setup_adc(ADC_CLOCK_INTERNAL);
    setup_timer_2(T2_DIV_BY_16,254,1);
    set_adc_channel(1); delay_us(21);
    delay_ms(7000);

    gain=1.0; target=0.6*1024; dump=1024.0;

    while(true)
    {
        // sensor //
        data=read_adc(); delay_ms(5);

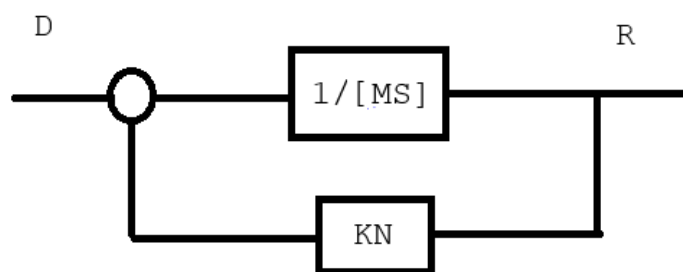
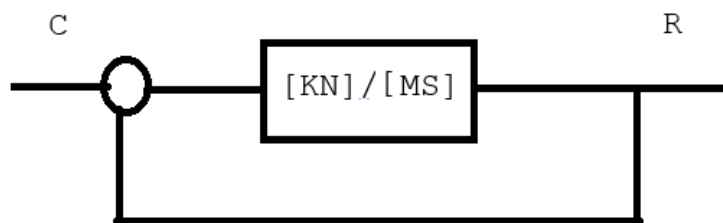
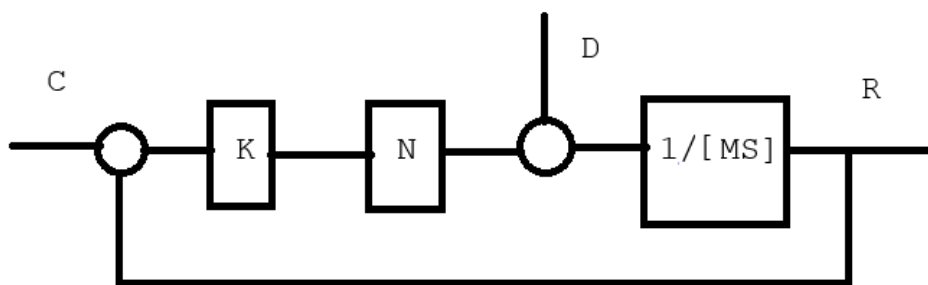
        // control //
        error=target-data;
        control=gain*error;
        if(control>dump)
            control=dump; end;
        power=control*254.0/1024.0;

        // drive //
        if(error>0.0)
            one(power); two(0); end;
        if(error<0.0)
            one(0); two(power); end;

    }
}

void one(int bits) {
    i2c_start();
    i2c_write(0x5e);
    i2c_write(0);
    i2c_write(bits);
    i2c_stop();}
void two(int bits) {
    i2c_start();
    i2c_write(0x5e);
    i2c_write(1);
    i2c_write(bits);
    i2c_stop();}
```

6. Sketch the overall block diagram for the system. Reduce it down to the standard form with command as input. Reduce it down to the standard form with disturbance as input. Determine the transfer function connecting the response to the command. Determine the transfer function connecting the response to the disturbance. [10]



$$R/C = G/[1+GH] = [K N] / [M S + K N]$$

$$R/D = G/[1+GH] = 1 / [M S + K N]$$

7. Use Partial Fraction Expansion and Inverse Laplace Transformation to get the response of the system when there is a step in command with height 10cm and the proportional gain K is +1. [10]

$$\begin{aligned} R/C &= G/[1+GH] = [K \ N] / [M \ S + K \ N] \\ &= 1 * 2 / [4 * S + 1 * 2] \\ &= 0.5 / [S + 0.5] \end{aligned}$$

$$\begin{aligned} R &= 0.5 / [S + 0.5] \ C \\ &= 0.5 / [S + 0.5] \ 10/S \end{aligned}$$

$$R = 10/S - 10/[S+0.5]$$

$$R = 10 - 10 e^{-0.5t}$$

8. Determine the system GH function. Determine the system characteristic equation. Use Routh Hurwitz criteria to check the borderline proportional gain of the system. Use the criteria to check the stability of the system when the proportional gain K is +1 and when the gain K is -1. [10]

The GH function is

$$GH = [K \ N] / [M \ S]$$

The characteristic equation is

$$M \ S + K \ N = 0$$

For stability all coefficients must be positive. A zero coefficient means the system is borderline.

$$\mathbf{K} = 0$$

Positive coefficient when K positive so stable.

Negative coefficient when K negative so unstable.

9. Manually sketch the GH plot for the case where the proportional gain K is +1. Manually sketch the GH plot for the case where the proportional gain K is -1. Interpret the plots. [10]

The GH function is

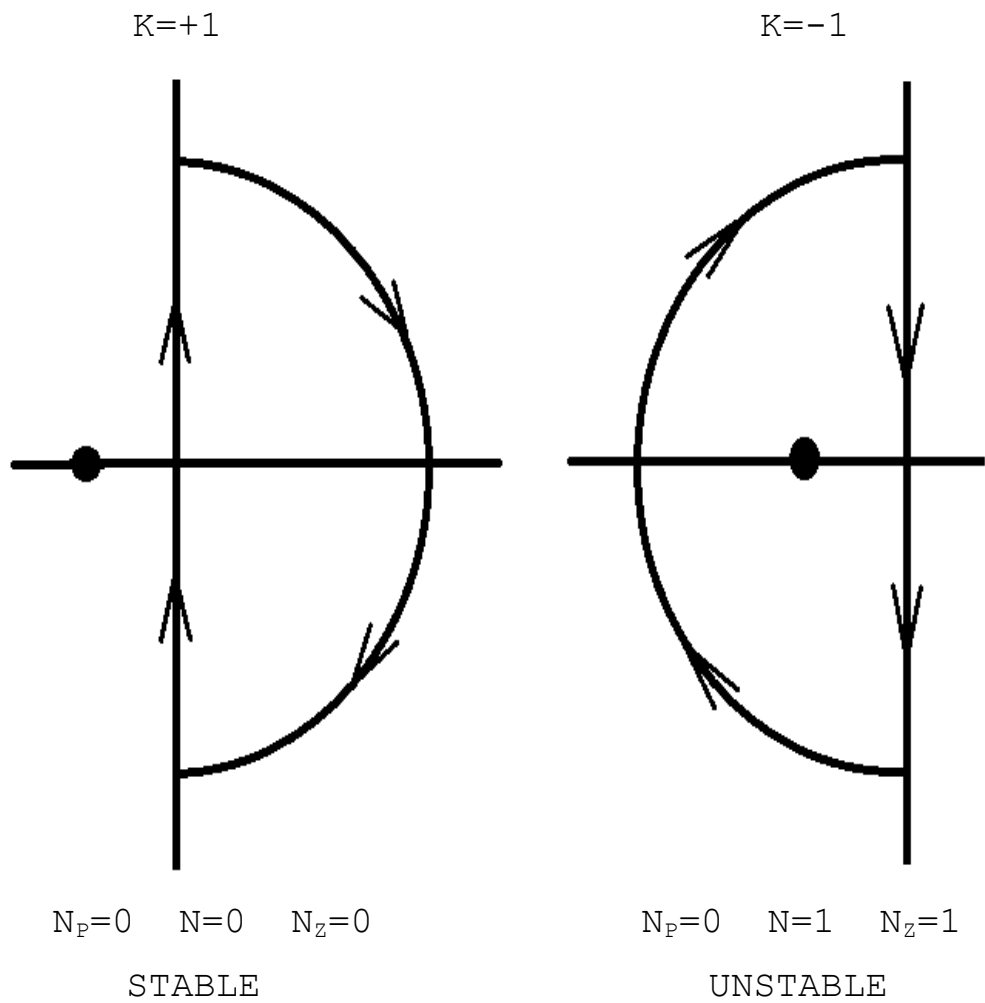
$$GH = [K \ N] / [M \ S]$$

Along imaginary axis this reduces to

$$GH = [K \ N] / [M \ \omega j] = - [K \ N] / [M \ \omega] \ j$$

Along infinitesimal radius semi circle

$$GH= [K \ N] / [M \ r \angle +\theta] = [K \ N] / [M \ r] \ \angle -\theta$$



10. Use the Root Locus concept to check the borderline gain and the borderline period of the system. Where in the S plane is the root of the overall characteristic equation located when the gain K is +1 and when the gain K is -1? [10]

The GH function is

$$GH = [K \ N] / [M \ S]$$

Nyquist suggests $\omega=0$. At $S=0$ this makes denominator zero which suggests **K** must be zero.

The characteristic equation is

$$M \ S + K \ N = 0$$

$$4 \ S + 2 \ K = 0$$

When $K=+1$ then $S=-0.5$. When $K=-1$ then $S=+0.5$.

11. Write an m code to construct the GH plot. Write an m code to construct the Root Locus plot. [10]

```
clear all
nit=100; K=+1.0;
m=4.0; n=2.0;
w=0.1; dw=0.1;
for it=1:nit
s=complex(0.0,w);
num=n*K;
den=m*s;
gh=num/den;
p(it)=real(gh);
q(it)=imag(gh);
w=w+dw;
end
plot(p,+q,p,-q)
title('nyquist')
xlabel('real')
ylabel('imag')

clear all
m=4.0; n=2.0;
gp=[-2.0:0.1:2.0];
for it=1:length(gp)
q=[m n*gp(it)];
p(:,it)=roots(q);
end
plot(real(p),imag(p),'x')
title('root locus')
xlabel('real')
ylabel('imag')
```

12. The controller has saturation limits of -24V DC and $+24\text{V DC}$ and the stroke of the actuator relative to its centered position is -15cm and $+15\text{cm}$. Can this system undergo a stable limit cycle? Is the system practically stable? [10]

The borderline gain is zero. All gains greater than zero are stable. All DFs for nonlinear controllers would be positive and thus stable. So all oscillations must decay to zero. So a limit cycle with a finite amplitude would not exist and the system would be practically stable.

BONUS QUESTION [5]

List the steps for design of a control system.

Understand process.

Select sensors.

Select drives.

Do PID analysis.

Simulate process.

Modify plant.

Build prototype.