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NAME: ____

AUTOMATIC CONTROL ENGINEERING

QUIZ #2

NO NOTES OR TEXTS ALLOWED

NON PROGRAMMABLE CALCULATORS ALLOWED

NO OTHER ELECTRONIC DEVICE ALLOWED

ASK NO QUESTIONS

The equations for a certain system are:

PLANT

$$X \, dW/dt + Y W + Z U = P + D$$

$$U = dR/dt \qquad W = dU/dt$$

DRIVE

$$M P = N Q$$

CONTROLLER

$$Q = K E$$

$$E = C - R$$

where R is the actual state, C is the command state, E is the state error, P is the drive signal, D is a disturbance, Q is the control signal, K is the gain and X Y Z M N are system parameters:

M=8 N=2   X=1 Y=4 Z=4

Determine the proportional gain $\mathbf{K}_P$ and period $\mathbf{T}_P$ for borderline stable operation.

Manipulation of the plant equation gives

$$X\ d^3R/dt^3 + Y\ d^2R/dt^2 + Z\ dR/dt = P + D$$

$$P = X\ d^3R/dt^3 + Y\ d^2R/dt^2 + Z\ dR/dt - D$$

Substitution into the drive equation gives

$$M\ [\ X\ d^3R/dt^3 + Y\ d^2R/dt^2 + Z\ dR/dt - D\ ]$$
$$= N\ K\ [C - R]$$

During borderline stable operation

$$R = R_o + \Delta R\ \mathrm{Sin}[\boldsymbol{\omega} t] \qquad C = C_o \qquad D = D_o$$

Substitution into the RCD equation gives

$$-\ \boldsymbol{\omega}^3\ 8\ \Delta R\ \mathrm{Cos}[\boldsymbol{\omega} t] - \boldsymbol{\omega}^2\ 8\ 4\ \Delta R\ \mathrm{Sin}[\boldsymbol{\omega} t]$$
$$+\ \boldsymbol{\omega}\ 8\ 4\ \Delta R\ \mathrm{Cos}[\boldsymbol{\omega} t]$$
$$-\ 8\ D_o = 2\ \mathbf{K}\ (C_o - R_o - \Delta R\ \mathrm{Sin}[\boldsymbol{\omega} t])$$

This is of the form

$$i\ \mathrm{Sin}[\boldsymbol{\omega} t] + j\ \mathrm{Cos}[\boldsymbol{\omega} t] + k = 0$$

It gives $\boldsymbol{\omega}=2$ and $\mathbf{K}=64$ and $R_o=C_o+4D_o/\mathbf{K}$.

Use Describing function theory to determine the amplitude and period of the limit cycle generated when the system is controlled by an ideal relay controller with a lower saturation limit of -24V DC and an upper limit of +24V DC. Is the system practically stable with this limit cycle?

The describing function for an ideal relay is

$$DF = [4Q_\circ]/[\pi E_\circ]$$

At a limit cycle **DF** = **K.** Manipulation gives

$$\mathbf{E}_\circ = [4Q_\circ]/[\pi \mathbf{K}]$$
$$= [4*24]/[\pi*64]= 0.48$$

The limit cycle period is the borderline period

$$T_\circ = [2*\pi]/\boldsymbol{\omega} = 3.14$$

The system is not identified. So it is impossible to judge whether it is practically stable or not.

Develop a simulation template for getting the response of the system step by step in time.

Manipulation gives the rate equations

$$dR/dt = U$$

$$dU/dt = W$$

$$dW/dt = [P + D - Y W - Z U] / X$$

The time stepping template is

$$R_{NEW} = R_{OLD} + \Delta t\ U_{OLD}$$

$$U_{NEW} = U_{OLD} + \Delta t\ W_{OLD}$$

$$W_{NEW} = W_{OLD} + \Delta t\ [P_{OLD} + D_{OLD} - Y\ W_{OLD} - Z\ U_{OLD}] / X$$

The control signal is

$$Q_{OLD} = K_P\ E_{OLD} + K_I \sum E_{OLD}\Delta t + K_D\ \Delta E_{OLD}/\Delta t$$

$$E_{OLD} = C_{OLD} - R_{OLD} \qquad P_{OLD} = N/M\ Q_{OLD}$$

At end of step make OLD equal NEW for next step.

Determine the system GH function. Determine the overall system characteristic equation.

The plant transfer function is

$$R/[P+D] = 1 / [X S^3 + Y S^2 + Z S]$$

The drive transfer function is

$$P/Q = N / M$$

The controller transfer function is

$$Q/E = K \qquad E = C - R$$

The GH function is

$$K [N/M] / [X S^3 + Y S^2 + Z S]$$
$$= K [2/8] / [[S] [S^2 + 4 S + 4]]$$

The characteristic equation is

$$4 [S] [S^2 + 4 S + 4] + K = 0$$

Use Routh Hurwitz criteria to check the borderline proportional gain of the system. Use the criteria to check the stability when the proportional gain is half the borderline gain.

The characteristic equation is

$$4\ [S]\ [\ S^2 + 4\ S + 4\ ] + K = 0$$

$$4\ S^3 + 16\ S^2 + 16\ S + K = 0$$

This is of the form

$$a\ S^3 + b\ S^2 + c\ S + d = 0$$

For stable operation all coefficients be positive and the test function x=bc-ad must also be positive. For borderline operation x is zero. Substitution into the test function gives

$$16 * 16 - 4 * \mathbf{K} = 0 \qquad \mathbf{K} = 64$$

Manually sketch the GH plot for the case where the system has a proportional controller and the gain is half the borderline gain. Interpret the plot. Write an m code to construct the plot.

The GH function is

$$GH = K / [\ [S]\ [\ S^2 + 4\ S + 4\ ]\ ]$$

$$= [64/2]\ [2/8] / [\ [S]\ [\ S^2 + 4\ S + 4\ ]\ ]$$

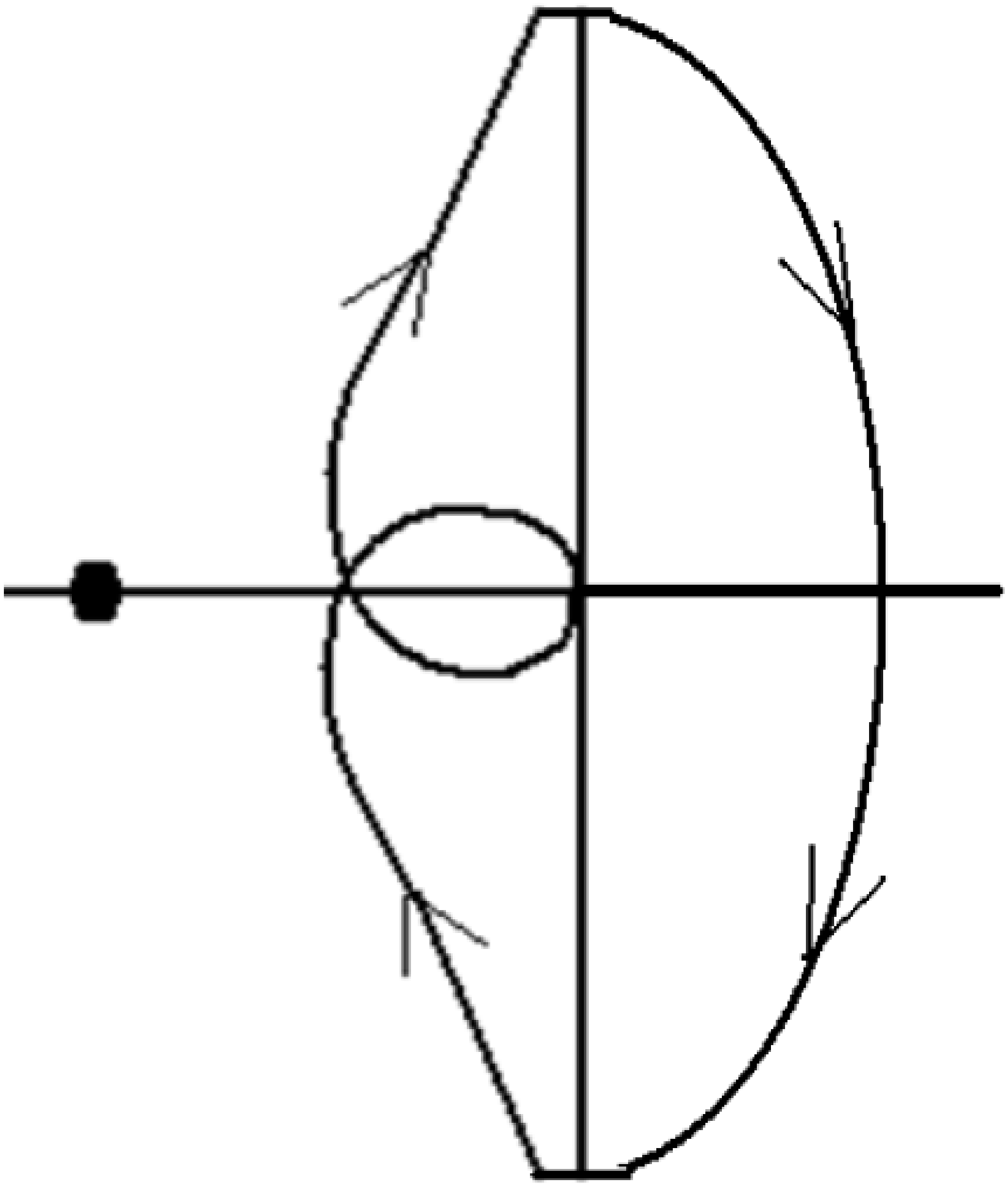
Because of the pole at the origin in the S plane, the GH plot will be closed by an infinite radius clockwise semicircle. Along the imaginary axis in the S plane S=jω. Substitution into GH gives

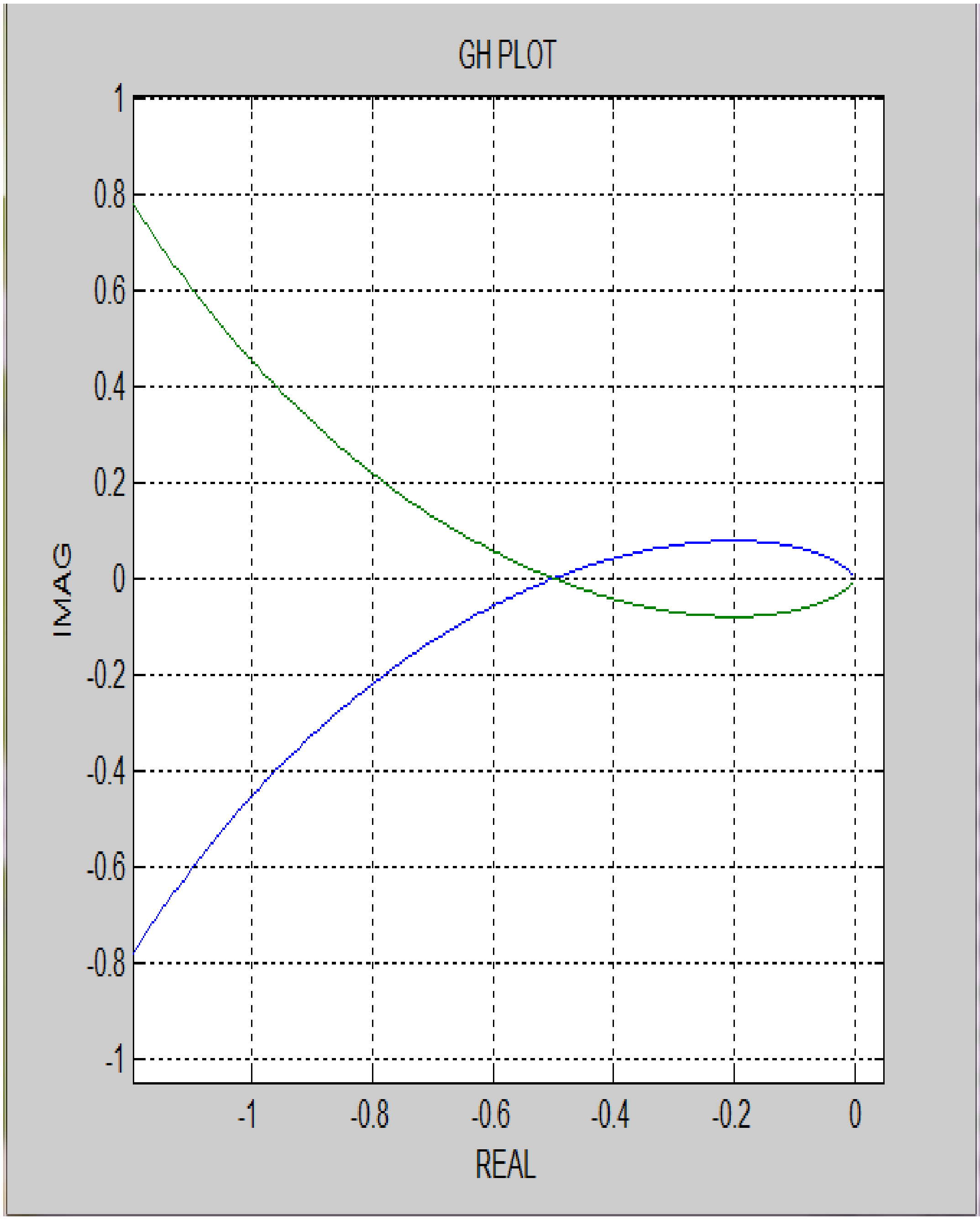
$$GH = 8 / [\ [\omega j]\ [\ -\ \omega^2 + 4\omega\ j + 4\ ]\ ]$$

If the real parts inside the square brackets on the right sum to zero, the remaining imaginary terms in the GH function produce a real number. This gives

$$\omega^2 = 4 \qquad GH = -0.5$$

As ω tends to zero, GH tends to minus ∞j. As ω tends to infinity, GH tends to plus 0j. The GH plot is sketched on the next page.




GH PLOT
IMAG
REAL
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0

Use the Root Locus concept to check the borderline gain and period of the system.

The GH function is

$$GH = \mathbf{K}\ [2/8] \ /\ [\ [S]\ [\ S^2 + 4\ S + 4\ ]\ ]$$
$$= \mathbf{K}/4 \ /\ [\ S\ (S+2)\ (S+2)\ ]$$

Nyquist suggests $\omega=2$. Substitution of $S=j\omega$ with $\omega=2$ into the GH function gives

$$GH = \mathbf{K}/4 \ /\ [\ [j\omega]\ [j\omega+2]\ [j\omega+2]\ ]$$

$$= \mathbf{K}/4 \ /\ [\ [j2]\ [j2+2]\ [j2+2]\ ]$$

This gives angles which add up to 180°:

$$90^{\circ} \quad 45^{\circ} \quad 45^{\circ}$$

It gives magnitudes which imply **K** is 64:

$$4 \quad 2 \quad \sqrt{8} \quad \sqrt{8}$$