EXPERIMENTAL METHODS

QUIZ #1

SYSTEM QUESTION

The equations governing the proportional control of the forward speed of a certain ship are:

Plant $X \, dR/dt + Y R = P + D$

Drive $J \, dP/dt + I P = Q$

Sensor $A \, dW/dt + B W = R$

Controller $Q = G (C-W)$

where R is the actual speed of the ship in meters per second, W is the speed in volts as measured by a sensor, C is the command speed in volts, P is the propulsion force, D is a disturbance force, Q is the control signal, J I X Y A B are constants and G is the controller gain.

$J=1 \quad I=1 \qquad X=1 \quad Y=1 \qquad A=1 \quad B=1$

Derive equations for the borderline gain and the borderline period of the system in terms of system parameters. Calculate the borderline gain and period of the system. Use them to determine the Ziegler Nichols Gains of the system.

The sensor into the plant and the controller into the drive gives:

$$X\ (A\ d^2W/dt^2\ +\ B\ dW/dt)$$
$$+\ Y\ (A\ dW/dt\ +\ B\ W)\ =\ P\ +\ D$$

$$J\ dP/dt\ +\ I\ P\ =\ G\ (C-W)$$

The plant into the drive gives

$$J\ X\ (A\ d^3W/dt^3\ +\ B\ d^2W/dt^2)$$
$$J\ Y\ (A\ d^2W/dt^2\ +\ B\ dW/dt)$$
$$I\ X\ (A\ d^2W/dt^2\ +\ B\ dW/dt)$$
$$+\ I\ Y\ (A\ dW/dt\ +\ B\ W)$$
$$-\ J\ dD/dt\ -\ I\ D\ =\ G\ (C-W)$$

Manipulation gives

$$JXA\ d^3W/dt^3\ +\ (JXB\ +\ JYA\ +\ IXA)\ d^2W/dt^2$$
$$+\ (JYB\ +\ IXB\ +\ IYA)\ dW/dt\ -\ J\ dD/dt$$
$$=\ G\ C\ -\ (G+IYB)\ W\ +\ I\ D$$

During borderline stable operation

$$W = W_o + \Delta W \, \mathrm{Sin}(\boldsymbol{\omega} t)$$

Substitution into the W equation gives

$$- JXA \, \boldsymbol{\omega}^3 \, \Delta W \, \mathrm{Cos}(\omega t) - (JXB+JYA+IXA) \, \boldsymbol{\omega}^2 \, \Delta W \, \mathrm{Sin}(\omega t)$$
$$+ (JYB+IXB+IYA) \, \boldsymbol{\omega} \, \Delta W \, \mathrm{Cos}(\omega t)$$
$$= \mathbf{G} \, C_o - (\mathbf{G}+IYB) \, W_o - (\mathbf{G}+IYB) \, \Delta W \, \mathrm{Sin}(\omega t) + I \, D_o$$

This equation is of the form

$$i \, \mathrm{Sin}[\omega t] + j \, \mathrm{Cos}[\omega t] + k = 0$$

$$i = - (JXB+JYA+IXA) \, \boldsymbol{\omega}^2 + (\mathbf{G}+IYB)$$
$$j = - JXA \, \boldsymbol{\omega}^3 + (JYB+IXB+IYA) \, \boldsymbol{\omega}$$
$$k = - \mathbf{G} \, C_o + (\mathbf{G}+IYB) \, W_o - I \, D_o$$

Mathematics requires that i=0 j=0 k=0. The j equation gives **ω**. The i equation gives **G**. The k equation gives W_o.

$$\boldsymbol{\omega}^2 = (JYB+IXB+IYA) \, / \, (JXA)$$
$$\mathbf{G} = (JXB+JYA+IXA) \, (JYB+IXB+IYA) \, / \, (JXA) - IYB$$
$$W_o = (\mathbf{G} \, C_o + I \, D_o) \, / \, (\mathbf{G}+IYB)$$

The borderline period and gain are:

$$\boldsymbol{\omega}^2 = 3 \qquad \boldsymbol{\omega}=\sqrt{3} \qquad \mathbf{T}=[2\pi] / \boldsymbol{\omega} \qquad \mathbf{G} = 8$$

Develop a simulation template for PID control of the system. Use this to get the ZN response to a step in command 3 steps in time. Let the time step be 0.5 and let the step height be 5. Let the lower saturation limit on the control signal be 0.0 and the upper limit be +10.0. Let D be 0.

The governing equations can be rewritten as

$$dR/dt = (P + D - Y\ R) / X$$

$$dP/dt = (Q - I\ P) / J$$

$$dW/dt = (R - B\ W) / A$$

$$Q = G_P\ E + G_I \int E d\tau + G_D\ dE/dt$$

$$E = (C-W)$$

Application of time stepping gives

$$R_{NEW} = R_{OLD} + \Delta t * (P_{OLD} + D_{OLD} - Y\ R_{OLD}) / X$$

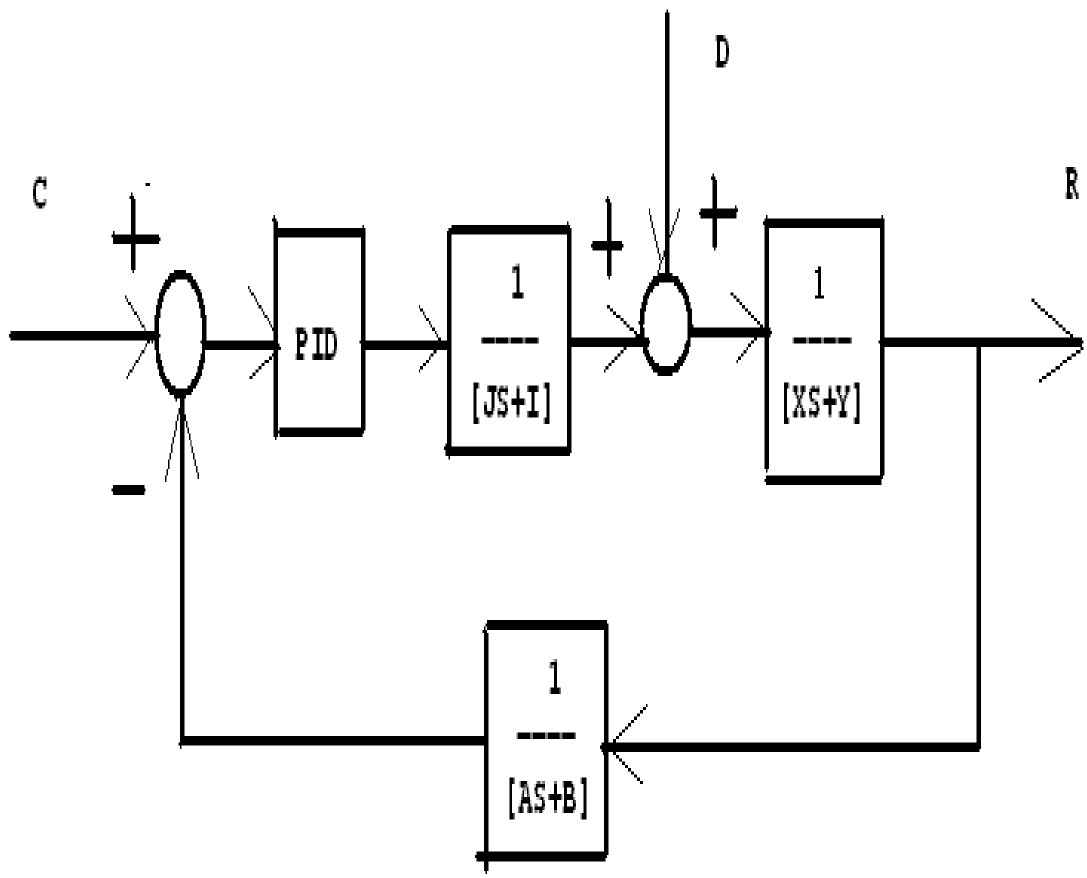
$$P_{NEW} = P_{OLD} + \Delta t * (Q_{OLD} - I\ P_{OLD}) / J$$

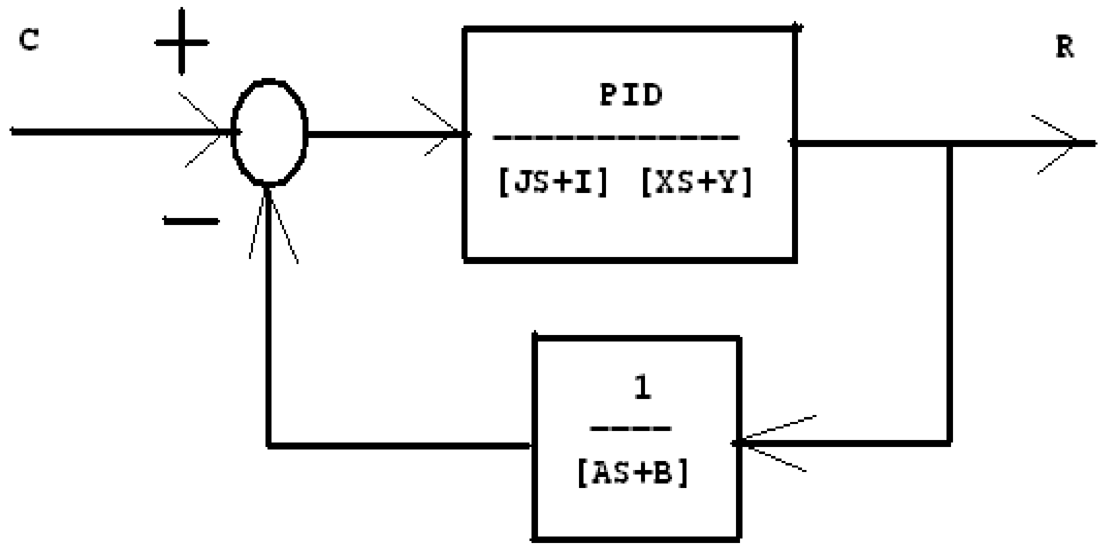
$$W_{NEW} = W_{OLD} + \Delta t * (R_{OLD} - B\ W_{OLD}) / A$$

$$Q_{OLD} = G_P\ E_{OLD} + G_I\ \Sigma\ E_{OLD}\ \Delta t + G_D\ \Delta E_{OLD}/\Delta t$$

$$E_{OLD} = C_{OLD} - W_{OLD}$$

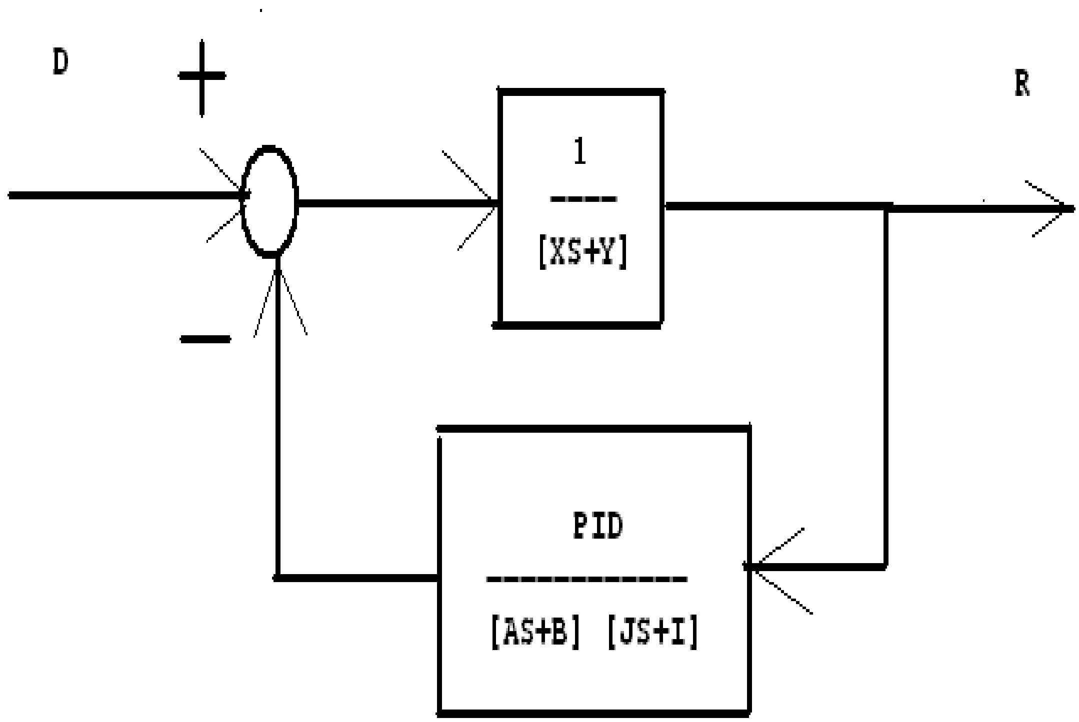
Sketch the overall block diagram for the system. Reduce this down to the standard form for each input. Derive the transfer function for each input.





$$\frac{R}{C} = \frac{G}{1 + GH}$$

$$= \frac{PID\ [AS+B]}{[XS+Y]\ [JS+I]\ [AS+B] + PID}$$



$$\frac{R}{D} = \frac{G}{1 + GH}$$

$$= \frac{[AS+B]\ [JS+I]}{[XS+Y]\ [JS+I]\ [AS+B]\ +\ PID}$$

Use the ROUTH HURWITZ criteria to derive an equation for the borderline G of the system in terms of system parameters. Calculate the borderline G of the system. Is the system stable when G is half the borderline G?

The characteristic equation is

$$[XS+Y]\ [JS+I]\ [AS+B] + G = 0$$

$$XJA\ S^3 + (YJA+IXA+BXJ)\ S^2 + (YJB+YAI+IXB)\ S + YIB + G = 0$$

$$S^3 + 3\ S^2 + 3\ S + 1 + G = 0$$

This is of the form:

$$a\ S^3 + b\ S^2 + c\ S + d = 0$$

For borderline operation Routh Hurwitz gives:

$$b*c - a*d = 0$$

Substitution into this gives

$$3 * 3 = 1 * (1+\mathbf{G})$$

$$\mathbf{G} = 8$$

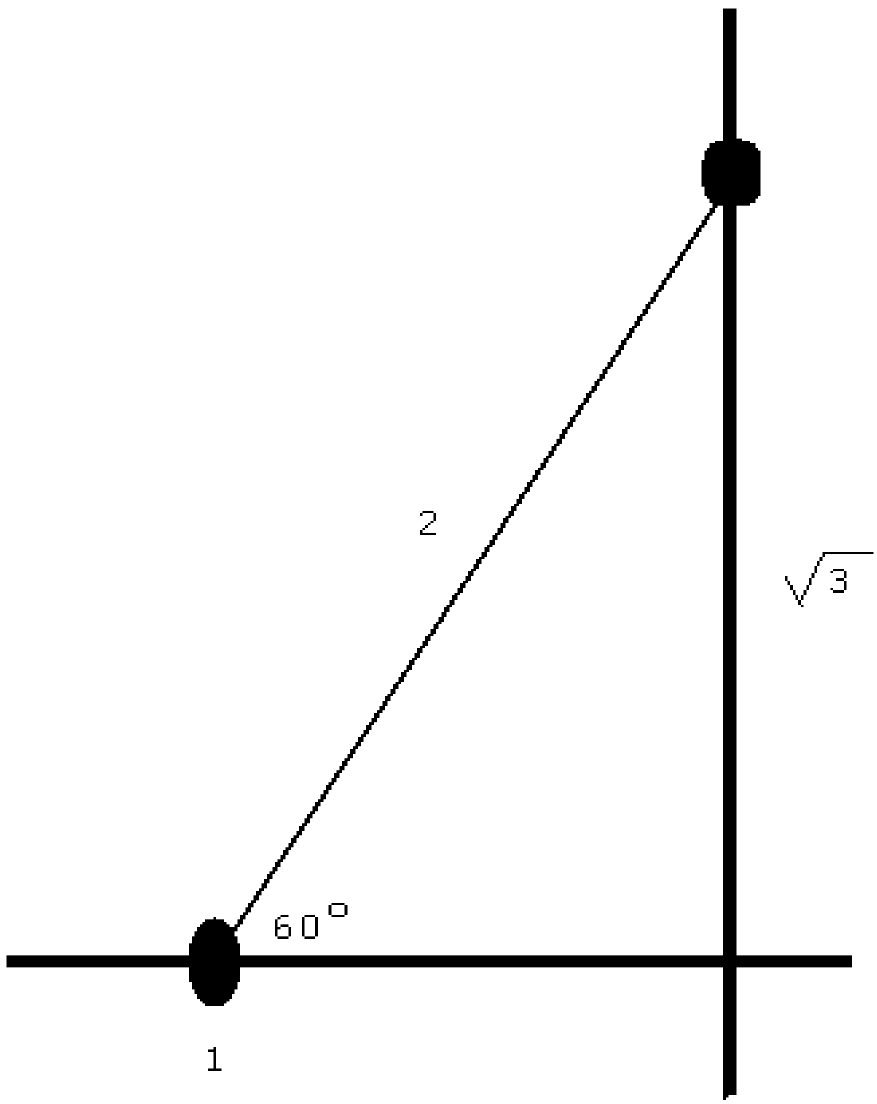
Use the ROOT LOCUS magnitude and angle concepts to check the borderline G of the system.

The GH function is

$$GH = \frac{G}{[XS+Y]\ [JS+I]\ [AS+B]}$$

$$GH = \frac{G}{[S+1]\ [S+1]\ [S+1]}$$

There are 3 identical poles. The angle from each pole to the imaginary axis must add up to 180°. This implies the angle for each is 60°. For the intersection of each pole vector with the imaginary axis, geometry gives $\omega=\sqrt{3}$. The length of each pole vector is 2. The magnitude of GH must be 1. This implies that the borderline gain is **G** = 2*2*2 = 8.



2
$\sqrt{3}$
60°
1

Sketch the NYQUIST plot for the case where the gain G is half the borderline G. Estimate the gain and phase margins from the plot.

The GH function is

$$GH = \frac{\mathbf{G}/2}{[XS+Y]\ [JS+I]\ [AS+B]}$$

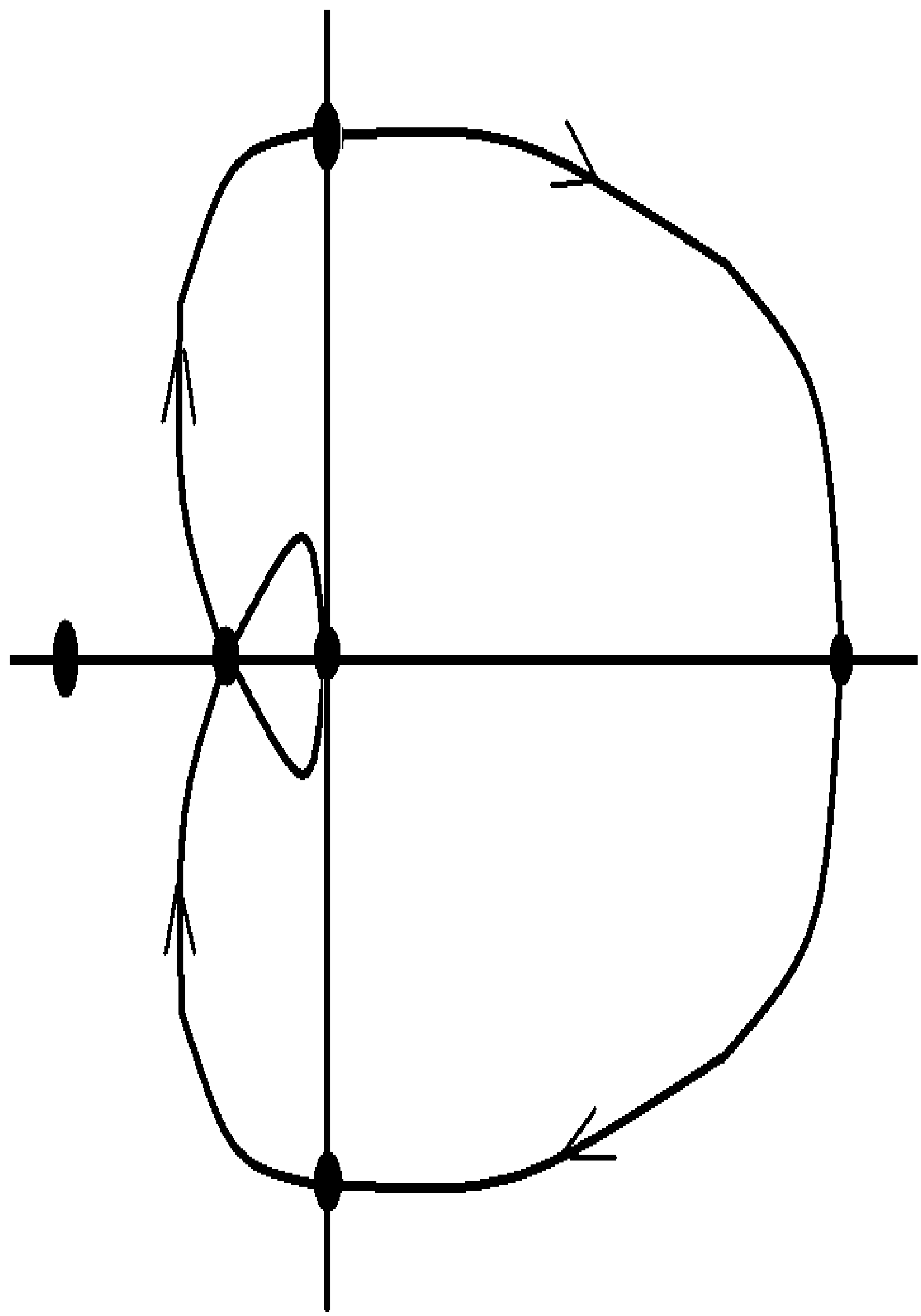
$$GH = \frac{4}{[S+1]\ [S+1]\ [S+1]}$$

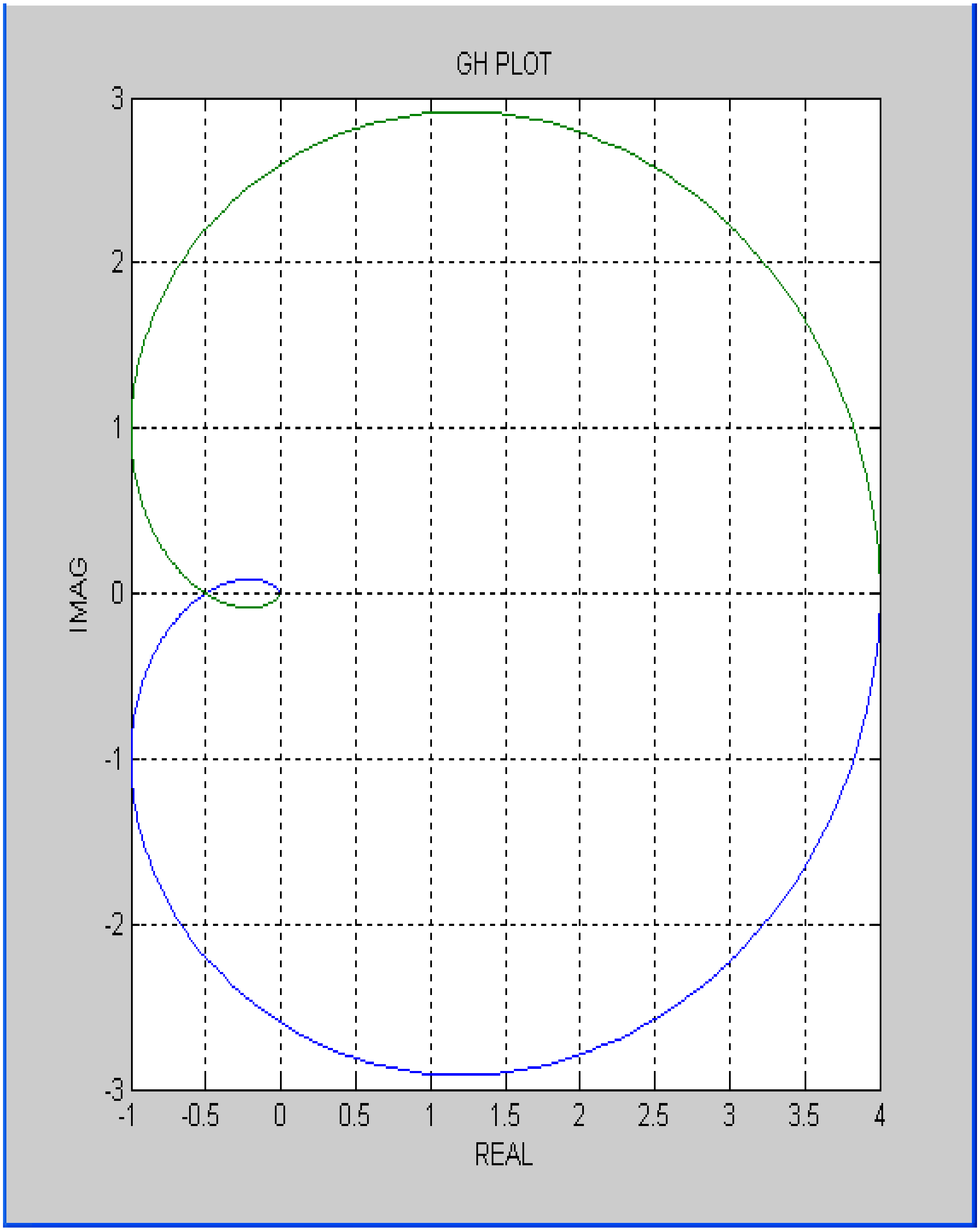
$$GH = \frac{4}{S^3 + 3\ S^2 + 3\ S + 1}$$

Substitution of $S=j\omega$ gives

$$GH = \frac{4}{-\omega^3 j - 3\ \omega^2 + 3\omega j + 1}$$

This shows that as ω tends to zero this becomes +4 while as ω tends to infinity it becomes +0j. There is a real axis crossover at -0.5 when ω is $\sqrt{3}$ and an imaginary axis crossover at -2.59j when ω is $1/\sqrt{3}$.




GH PLOT
IMAG
REAL
3
2
1
0
-1
-2
-3
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4

Determine the amplitude and period of the limit cycle generated when the system is controlled by a 5V ideal relay controller. Is the system practically stable with this limit cycle?

The describing function for an ideal relay controller is:

$$DF = [4Q_o]/[\pi E_o]$$

At a limit cycle, this is equal to the borderline proportional gain **G**. Manipulation gives:

$$\mathbf{G} = \mathbf{DF} = [4Q_o]/[\pi \mathbf{E}_o]$$

The limit cycle amplitude is:

$$\mathbf{E}_o = [4Q_o]/[\pi \mathbf{G}]$$

$$= [4*5]/[\pi*8] = 0.8$$

The limit cycle period is:

$$\mathbf{T}_o = [2\pi]/\boldsymbol{\omega}$$

$$= [2\pi]/\sqrt{3} = 3.6$$