AIRCRAFT ROLL CONTROL

GOVERNING EQUATIONS

$$X\frac{d^2R}{dt^2} + Y\frac{dR}{dt} + ZR = P(t)$$

$$M\frac{dP}{dt} = Q$$

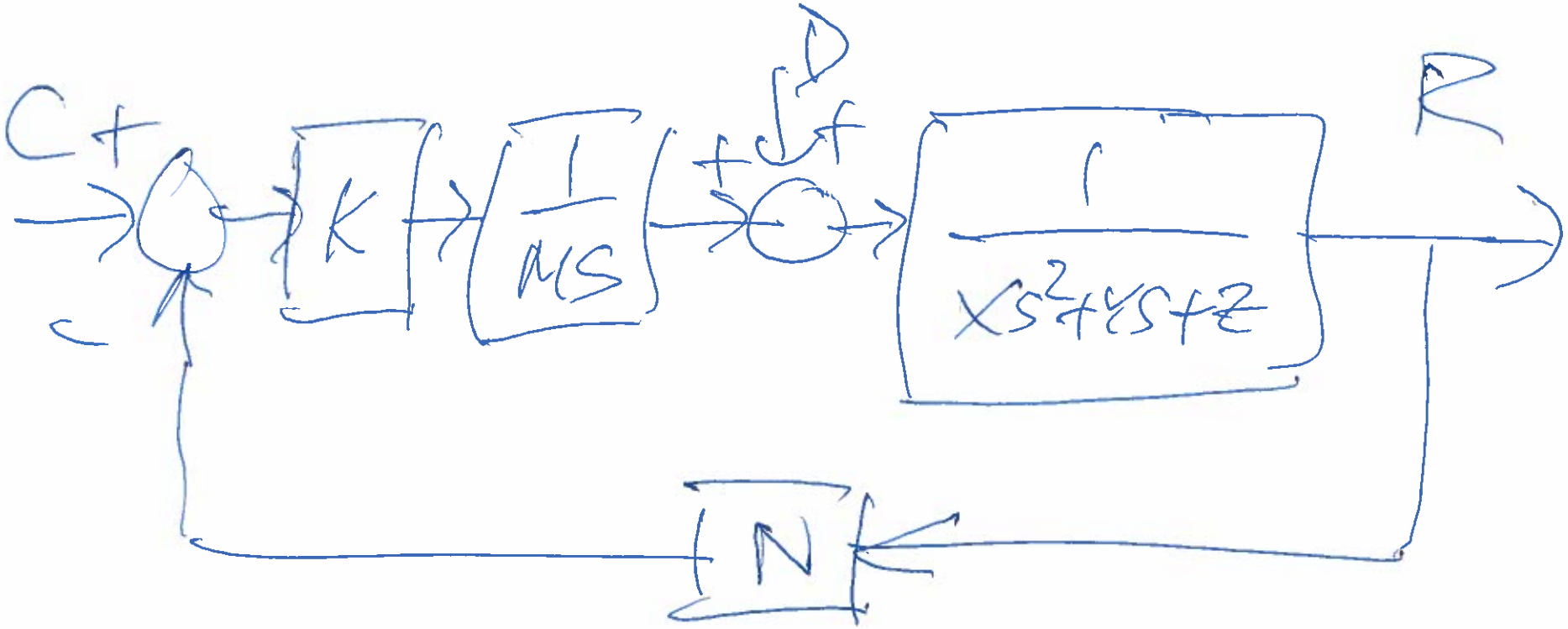
$$Q = KE$$

$$E = C - W$$

$$W = NR$$

CONTROLS EXAMPLE

OVERALL BLOCK DIAGRAM



M=(N=(X=(Y=(Z=(

GH FUNCTION

$$\frac{KN}{MS(XS^2+YS+Z)}$$

BORDERLINE STATE

$K = 1 \quad \omega = 1$

NYQUIST PLOT

$$K = K/2 = 0.5$$

$$G(s) = \frac{0.5}{s(s^2 + s + 1)}$$

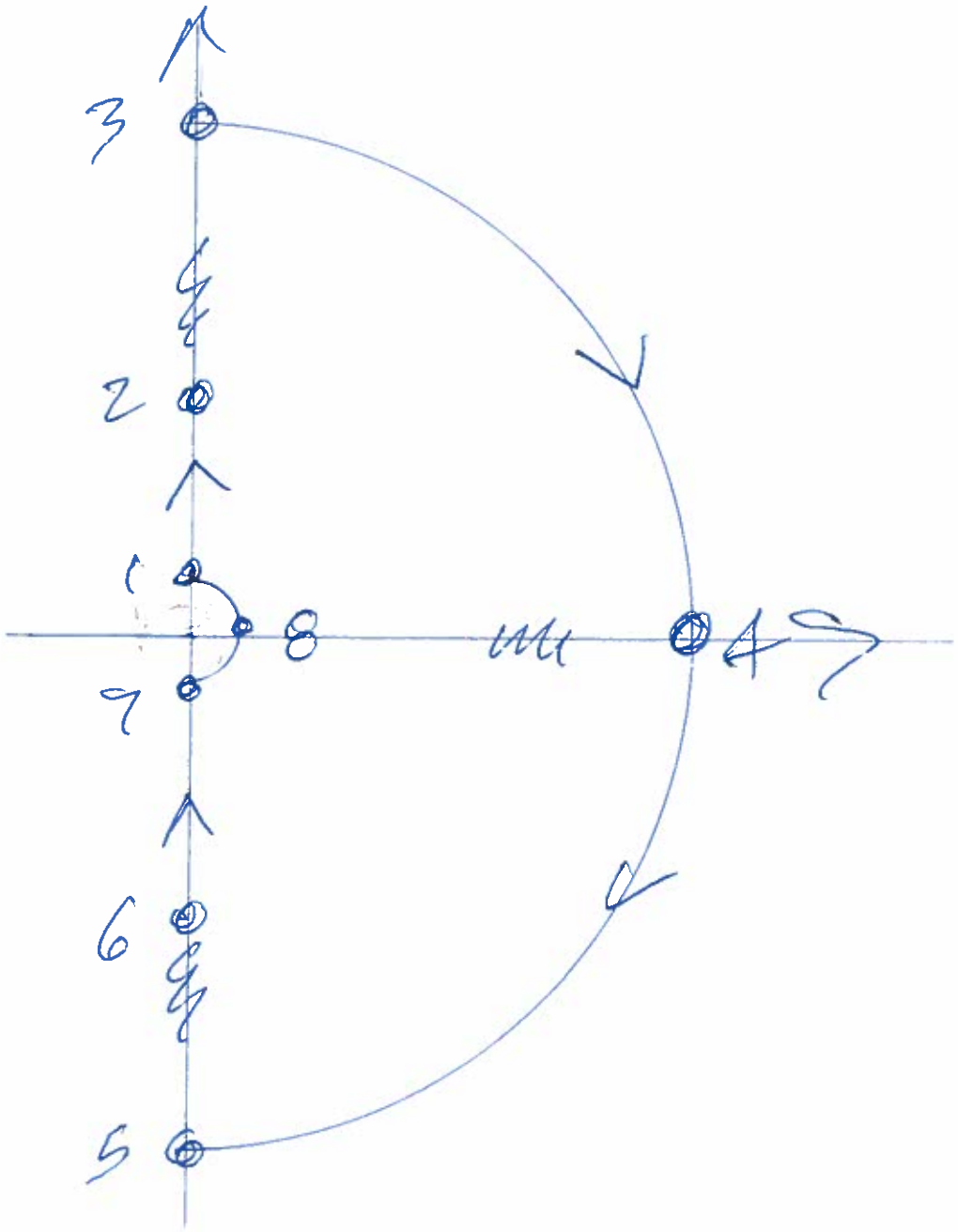
GH POLES

$$\frac{0.5}{s(s+0.5+0.86j)(s+0.5-0.86j)}$$

$s=0 \quad s=-0.5-0.86j \quad s=-0.5+0.86j$

NO UNSTABLE POLES

NYQUIST CONTOUR



IMAGINARY AXIS

$$s = j\omega$$

$$G(s) = \frac{0.5}{s(s^2 + s + 1)}$$

$$= \frac{0.5}{j\omega(-\omega^2 + j\omega + 1)}$$

End Points

As $\omega \Rightarrow 0$ $\quad (-\omega^2 + j2\omega + 1) \Rightarrow 1$

$$GH = \frac{0.5}{j\omega(1)} = -\infty j$$

As $\omega \Rightarrow \infty$ $\quad (-\omega^2 + j\omega + 1) \Rightarrow -\omega^2$

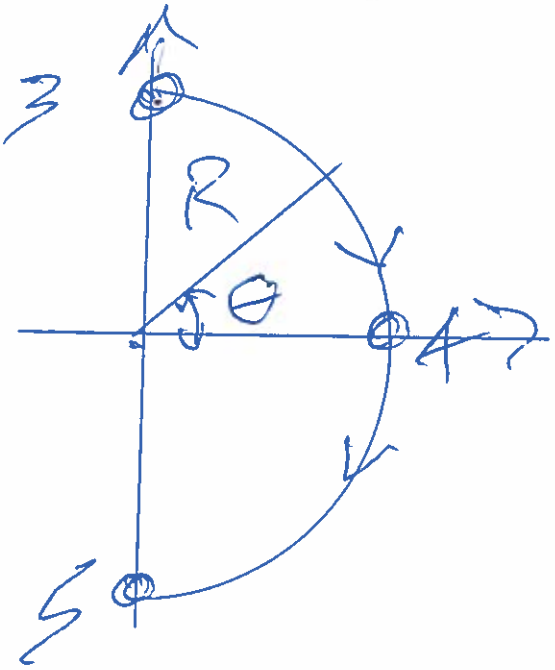
$$GH = \frac{0.5}{j\omega(-\omega^2)} = +0\, j$$

REAL AXIS CROSSOVER

$$-\omega^2 + 1 = 0 \qquad \omega^2 = 1 \qquad \omega = 1$$

$$GH = \frac{0.5}{j\omega(j\omega)} = \frac{0.5}{-\omega^2} = -0.5$$

INFINITE RADIUS SEMICIRCLE



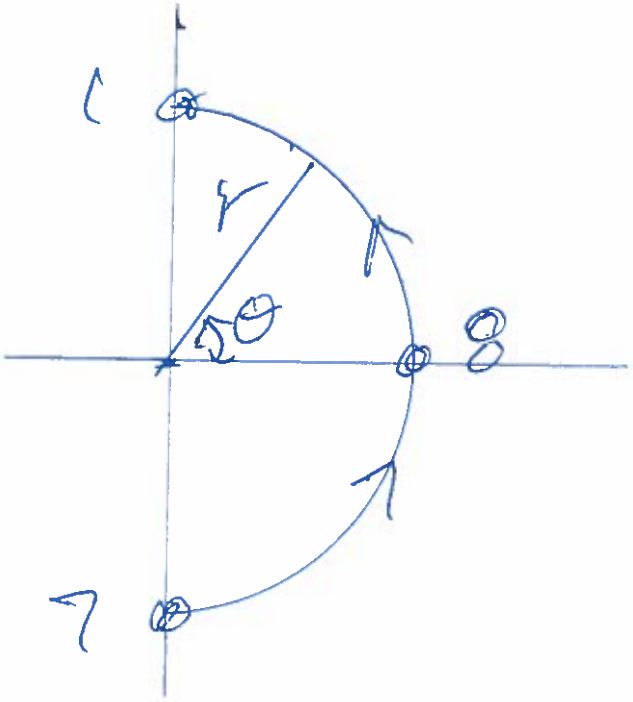
$$GH \Rightarrow \frac{0.5}{s(s^2)} \Rightarrow \frac{0.5}{R\angle\theta\ R^2\angle 2\theta} \Rightarrow 0\angle -3\theta$$

(3) $GH \Rightarrow 0\angle -270 \Rightarrow +0j$

(4) $GH \Rightarrow 0\angle 0 \Rightarrow +0$

(5) $GH \Rightarrow 0\angle +270 \Rightarrow -0j$

INFINITESIMAL RADIUS SEMICIRCLE



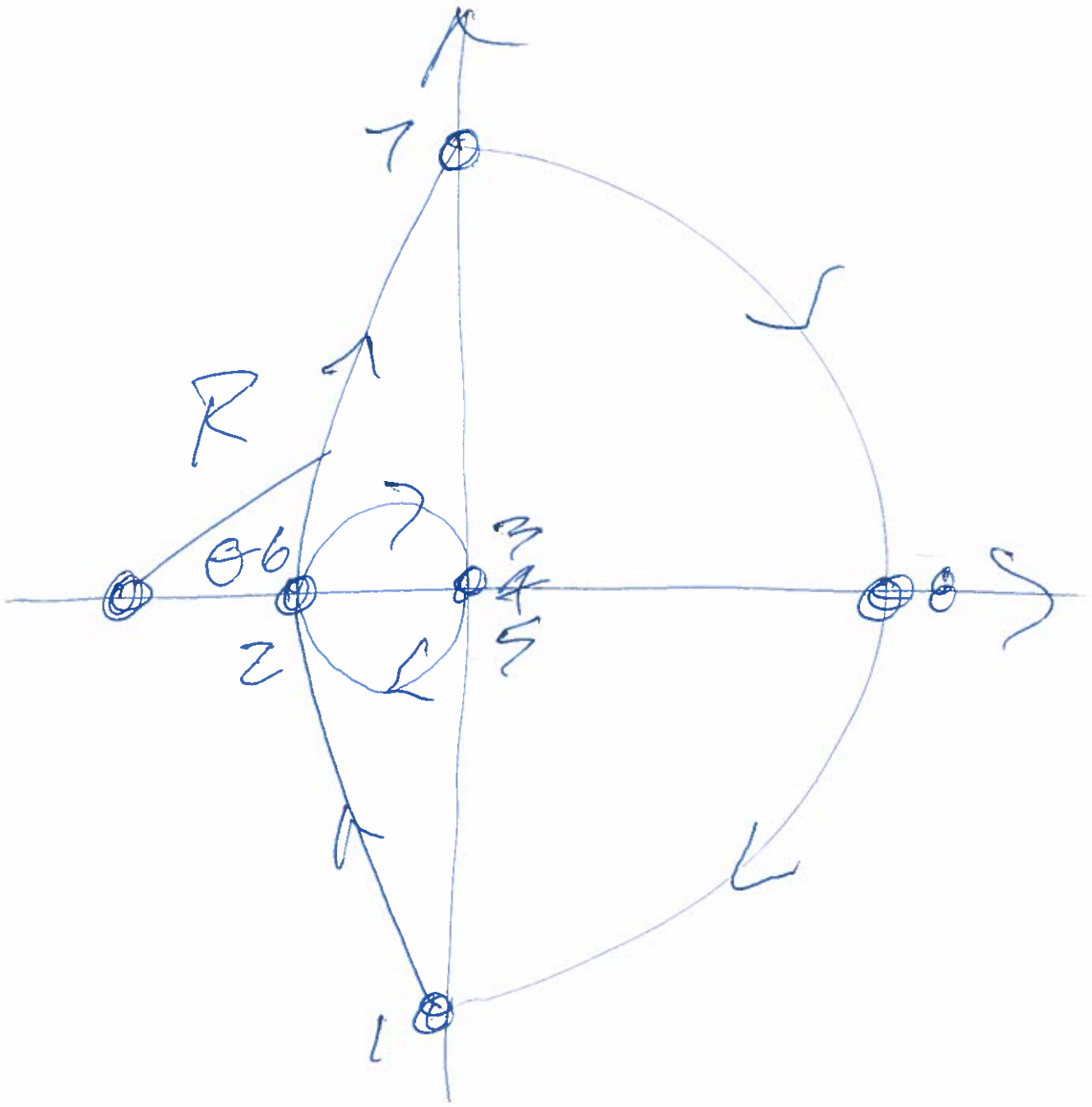
$$GH \Rightarrow \frac{0.5}{s(1)} \Rightarrow \frac{0.5}{r\angle\theta} \Rightarrow \infty\angle -\theta$$

① $GH \Rightarrow \infty\angle +90 \Rightarrow +\infty\, j$

⑧ $GH \Rightarrow \infty\angle 0 \Rightarrow +\infty$

① $GH \Rightarrow \infty\angle -90 \Rightarrow -\infty\, j$

NYQUIST PLOT



NET CLOCKWISE ROTATIONS

$$N = N_Z - N_P \geq 0$$

UNSTABLE POLES

$$N_P \geq 0$$

UNSTABLE ZEROS

$$N_Z = N + N_P \geq 0$$

ROOT LOCUS CHECK

$$G(s) = \frac{K}{s(s^2+s+1)}$$

$$= \frac{K}{s(s+0.5+0.86j)(s+0.5-0.86j)}$$

CHARACTERISTIC EQUATION

ZERO ROOT

$1+GH=0 \Rightarrow GH=-1$

IMPLICATIONS

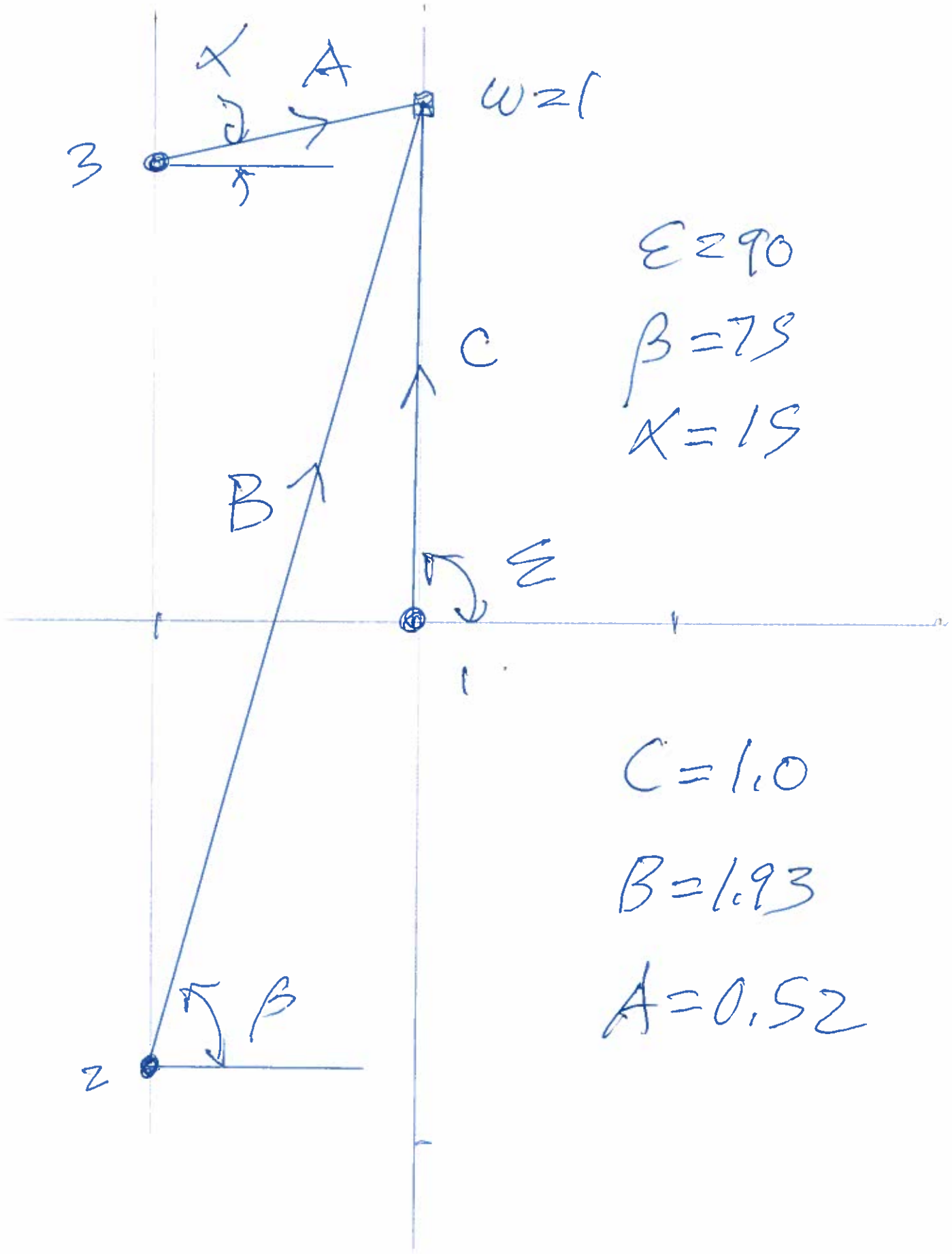
MAGNITUDE 1

PHASE $\pm 180^\circ$

$$K = 1 \qquad \omega = 1$$

$$\frac{K}{s(s+0.5+0.86j)(s+0.5-0.86j)} = \frac{1}{j(\underset{(1)}{(j+0.5+0.86j)}\underset{(2)}{(j+0.5-0.86j)})}$$

(1) (2) (3)



$\varepsilon = 90$

$\beta = 75$

$\alpha = 15$

$C = 1.0$

$B = 1.93$

$A = 0.52$

MAGNITUDE

$$\frac{1}{1.0 \angle 1.93 \angle 0.52} = 1 \quad \checkmark$$

PHASE

$$90 + 75 + 15 = 180 \quad \checkmark$$