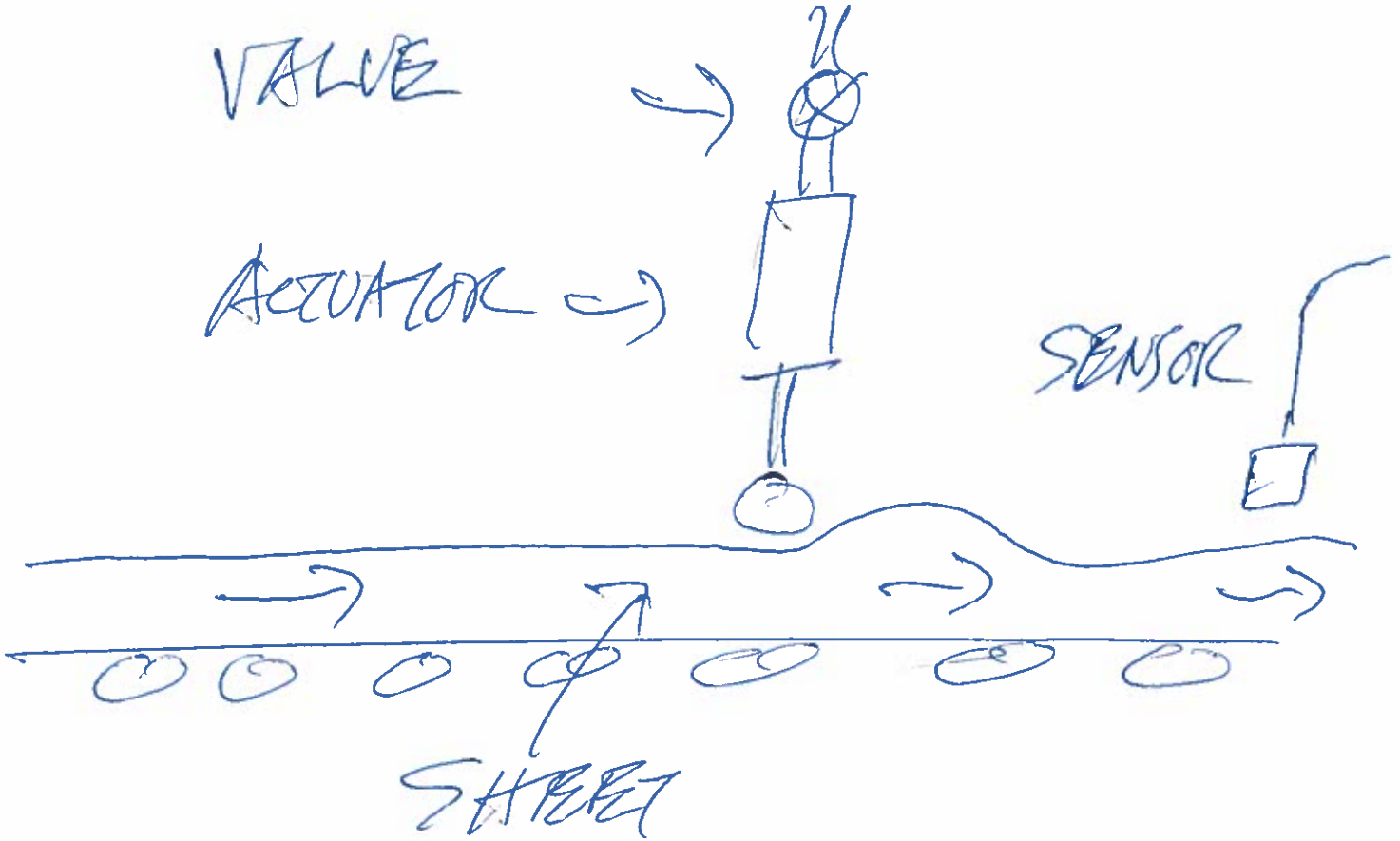
STEEL SHEET THICKNESS



CONTROLS EXAMPLE

GOVERNING EQUATIONS

ACTUATOR $A \frac{dR}{dt} = M + N$

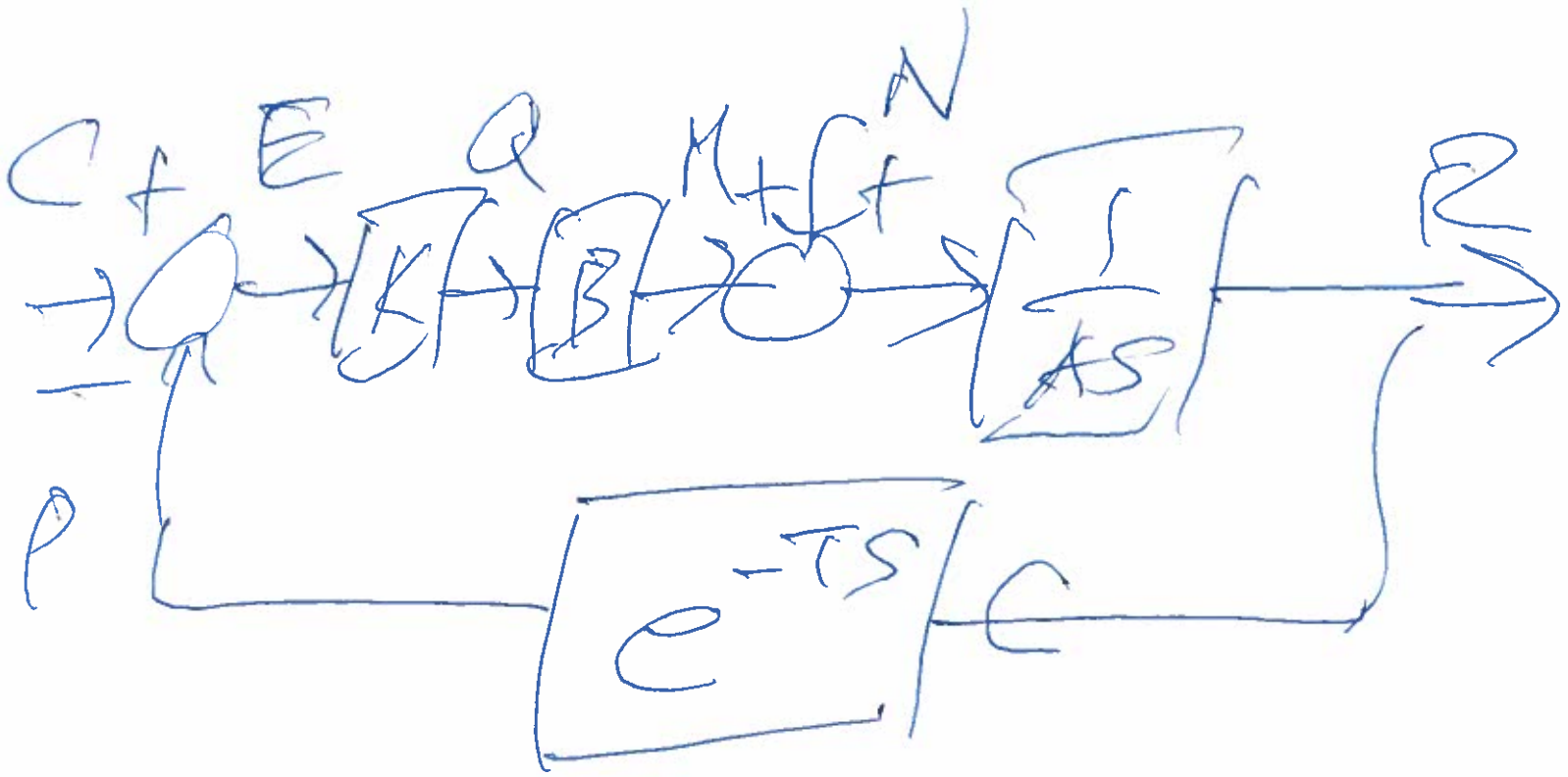
VALVE $M = \beta Q$

CONTROLLER $Q = KE$

ERROR $E = C - P$

SENSOR $P(t) = R(t - T)$

Overall Block Diagram



PADÉ APPROXIMANT

$$e^{-Ts} \cong \frac{1 - \frac{T}{2}s}{1 + \frac{T}{2}s}$$

FIRST ORDER

PADÉ APPROXIMANT

$$e^{-Ts} \cong \frac{1 - \frac{T}{2}s + \frac{T^2}{12}s^2}{1 + \frac{T}{2}s + \frac{T^2}{12}s^2}$$

SECOND ORDER

GH FUNCTIONS

EXACT $\dfrac{K_B e^{-TS}}{AS}$

PADE $\dfrac{K_B}{AS}\left(\dfrac{1-\frac{T}{2}S}{1+\frac{T}{2}S}\right)$

CHARACTERISTIC EQUATION

$$KB\left(1-\frac{I}{2}S\right)+AS\left(1+\frac{I}{2}S\right)=0$$

$$A\frac{I}{2}S^2+\left(A-KB\frac{I}{2}\right)S+KB=0$$

MASS DASHPOT SPRING

ZERO DISAPOT

$$A - KB\frac{T}{2} = 0$$

$$K = \frac{2A}{BT}$$

$$A = 1 \quad B = 1 \quad T = 2$$

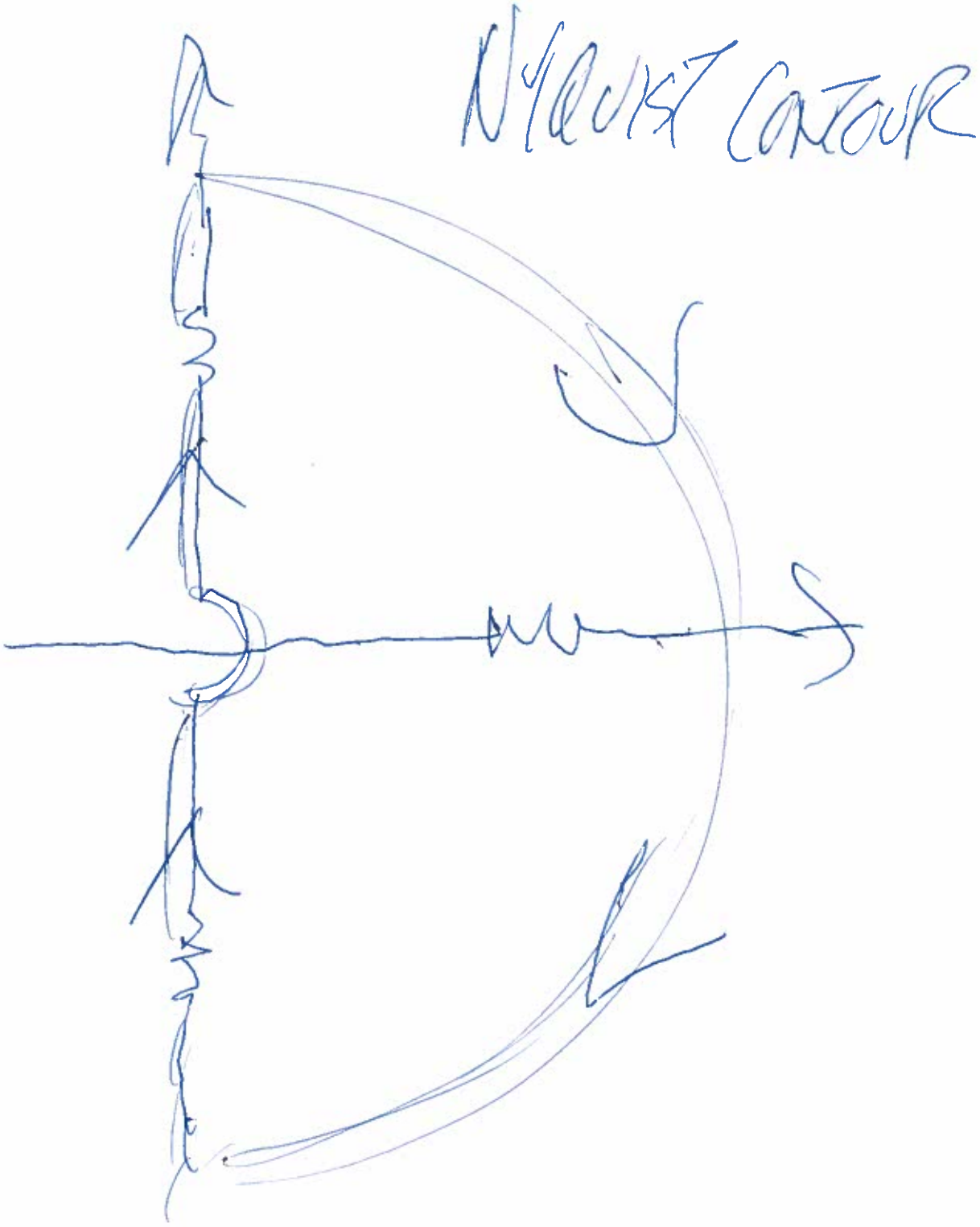
$$K = \frac{2*1}{1*2} = 1$$

GH Plot $K = \frac{R}{2} = 0.5$

$$GH = \frac{KB\left(1 - \frac{I}{2}s\right)}{As\left(1 + \frac{I}{2}s\right)}$$

$$= \frac{0.5 * (* (1 - s)}{1 * s * (1 + s)}$$

Poles $\Rightarrow s = 0 \quad s = -1 \Rightarrow N_p = 0$


NYQUIST CONTOUR

IMAGINARY PART

$$\frac{0.5 * 1 * (1 - j\omega)}{1 * j\omega (1 + j\omega)}$$

$$= \frac{0.5 * 1 * (1 - j\omega)(1 - j\omega)}{1 * j\omega (1 + j\omega)(1 - j\omega)}$$

$$= \frac{0.5 * (1 - 2j\omega - \omega^2)}{j\omega (1 + \omega^2)}$$

END POINTS

$\omega \to 0 \qquad GH \to -\infty\, j$

$\omega \to \infty \qquad GH \to +0\, j$

CROSS OVER

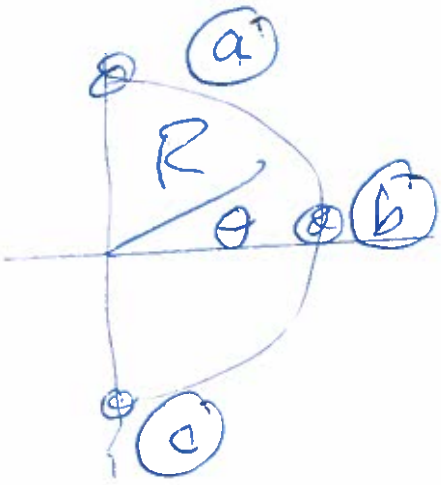
$1 - \omega^2 = 0 \Rightarrow \omega = 1$

$$GH = \frac{0.5(-2j\omega)}{j\omega(1+\omega^2)} = -1.5$$

INFINITE RADIUS SEMICIRCLE

$$S = R\angle\theta$$

$$R \to \infty$$



$$G(s) \Rightarrow \frac{0.5 * 1 * (-s)}{1 * s * (+s)} \Rightarrow -\frac{0.5}{s}$$

$$\Rightarrow \frac{-0.5}{R\angle\theta} \Rightarrow 0^{+}\angle 180^{\circ} - \theta$$

(a) $0^{+}\angle 90 \quad +0^{+}j$

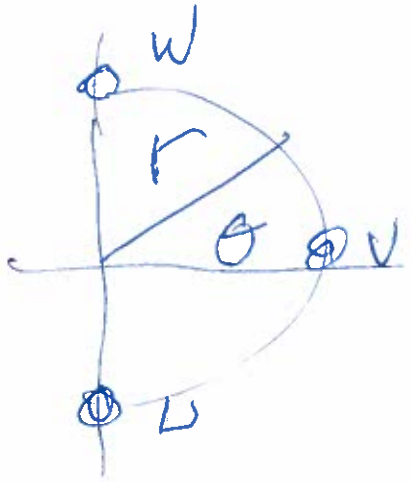
(b) $0^{+}\angle 0 \quad +0^{+}$

(c) $0^{+}\angle 270 \quad -0^{+}j$

INFINITESIMAL RADIUS SEMICIRCLE

$$S = r\angle\theta$$

$$r \to 0$$



$$G(s) \Rightarrow \frac{0.5 * 1 * (1)}{1 * s * (1)} \Rightarrow \frac{0.5}{s}$$

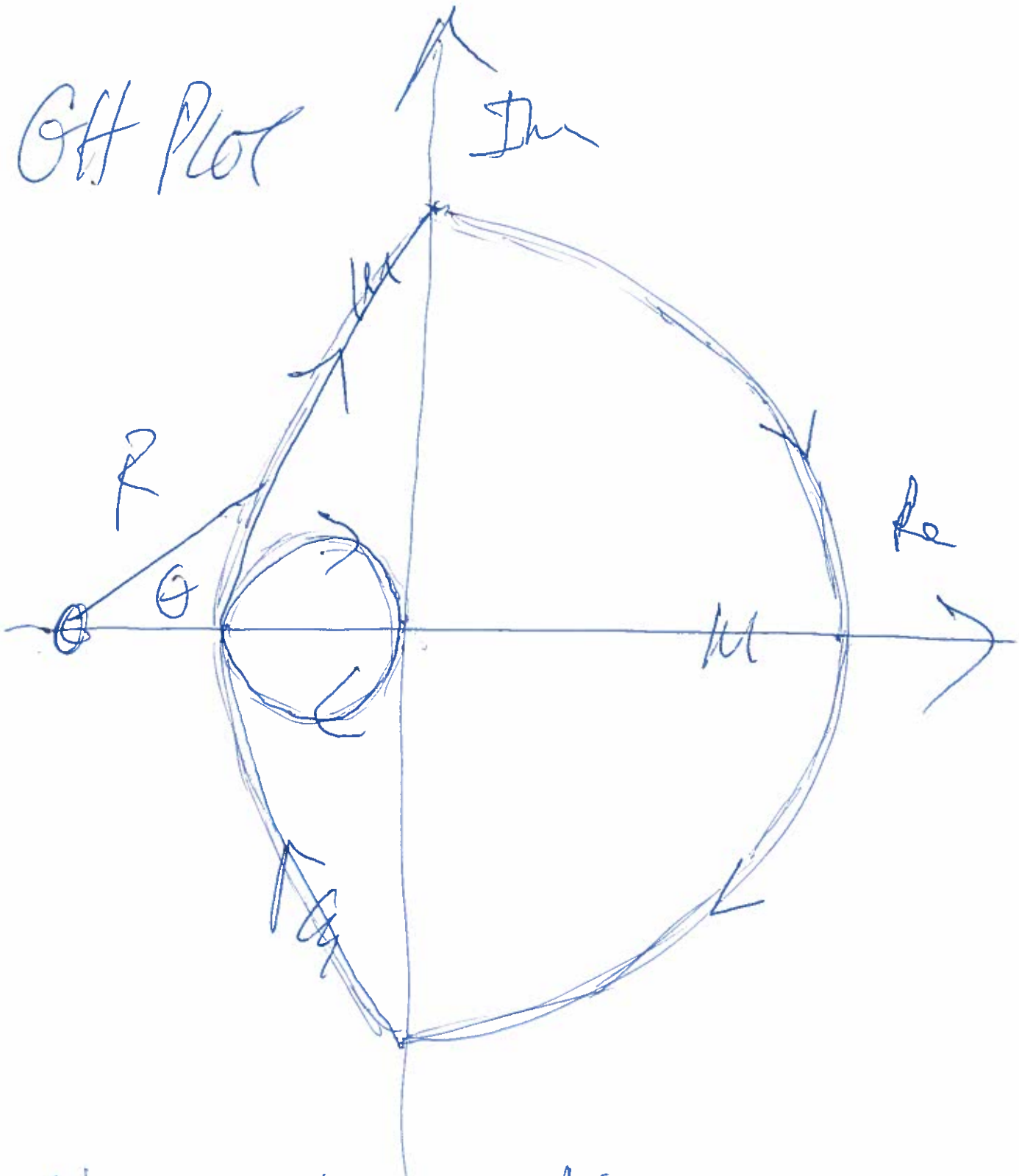
$$\Rightarrow \frac{0.5}{r\angle\theta} \Rightarrow +\infty\angle -\theta$$

(U) $+\infty\angle +90$ $+\infty j$

(V) $+\infty\angle 0$ $+\infty$

(W) $+\infty\angle -90$ $-\infty j$

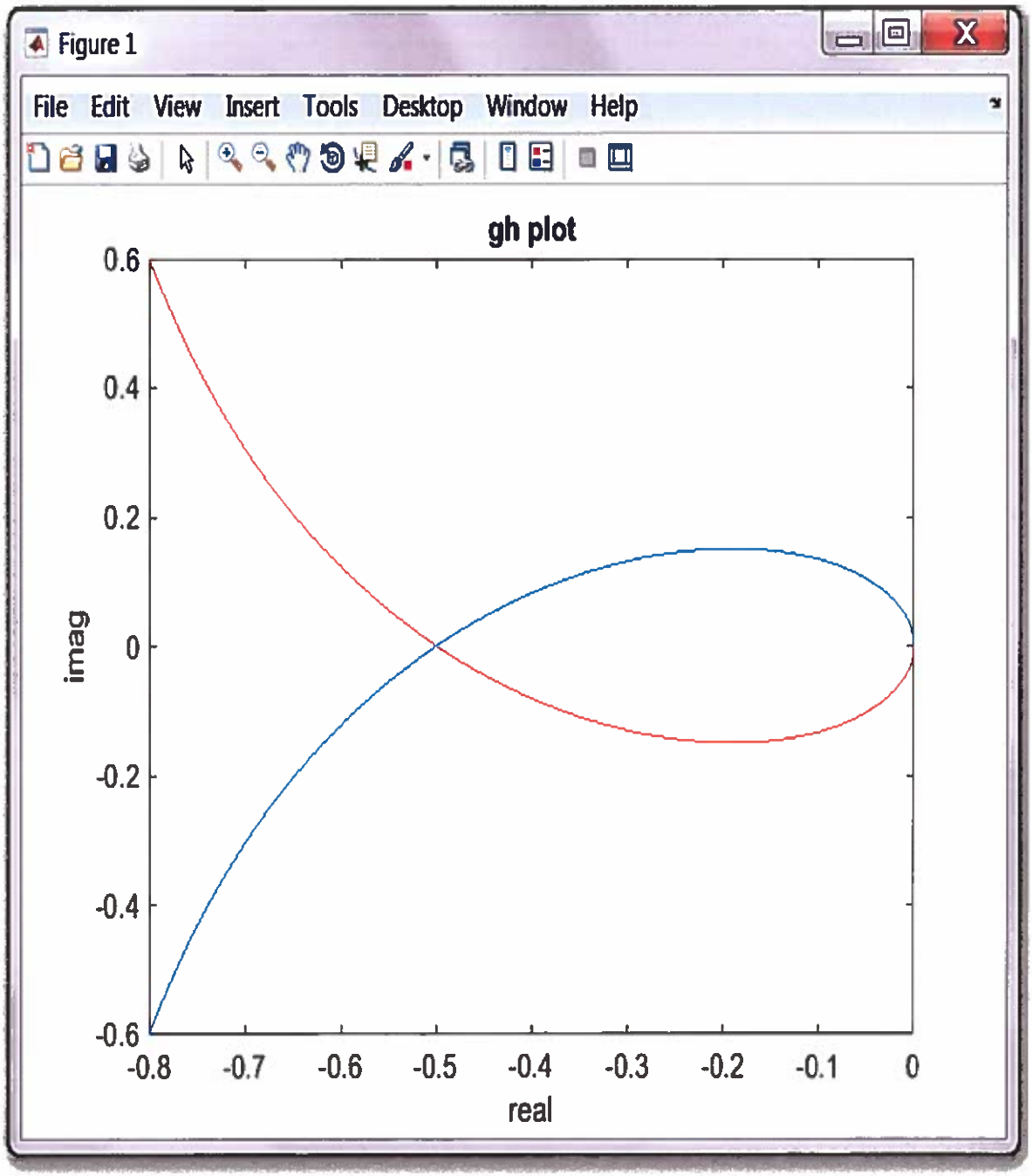
GH Plot



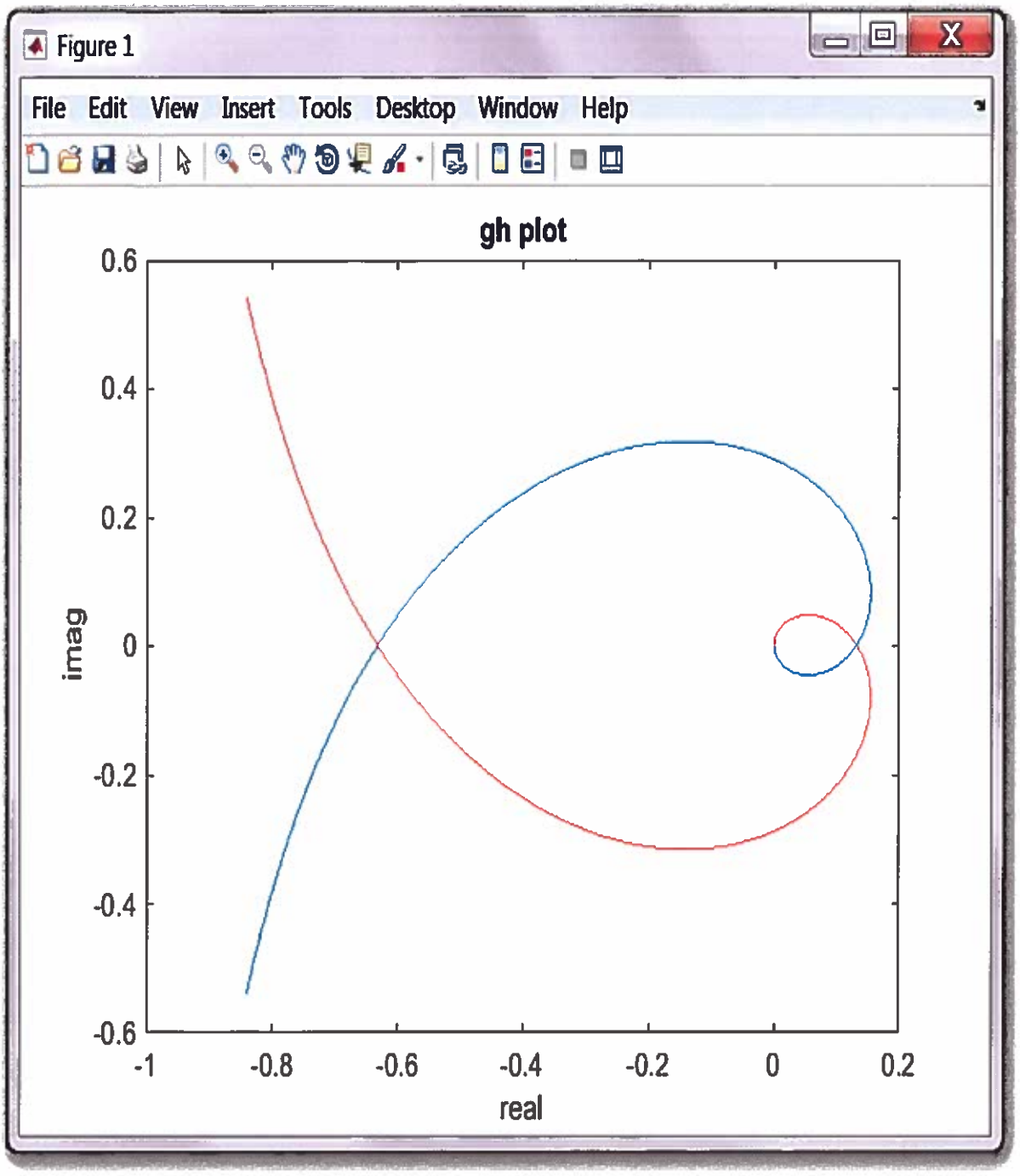
$N = 0 \quad N_p = 0 \quad N_z = 0$

STABLE

```
%
% STEEL SHEET GH PLOT
%
clear all
A=1.0; B=1.0; T=2.0;
GP=1.0; GI=0.0;GD=0.0;
NIT=1500; W=0.5;DW=0.01;
for K=1:NIT
S=complex(0.0,W);
PID=GD*S+GP+GI/S;
EXACT=exp(-T*S);
APPROX=(1-S*T/2)/(1+S*T/2);
%APPROX=(S*S-3*S+3)/(S*S+3*S+3);
num=PID*B; den=A*S;
map=num/den;
%map=map*EXACT;
map=map*APPROX;
p(K)=real(map);
g(K)=imag(map);
W=W+DW;
end
plot(p,q,p,-q);
title('gh plot')
xlabel('real')
ylabel('imag')
grid
```



PADE 1ST ORDER



PADE 2ND ORDER

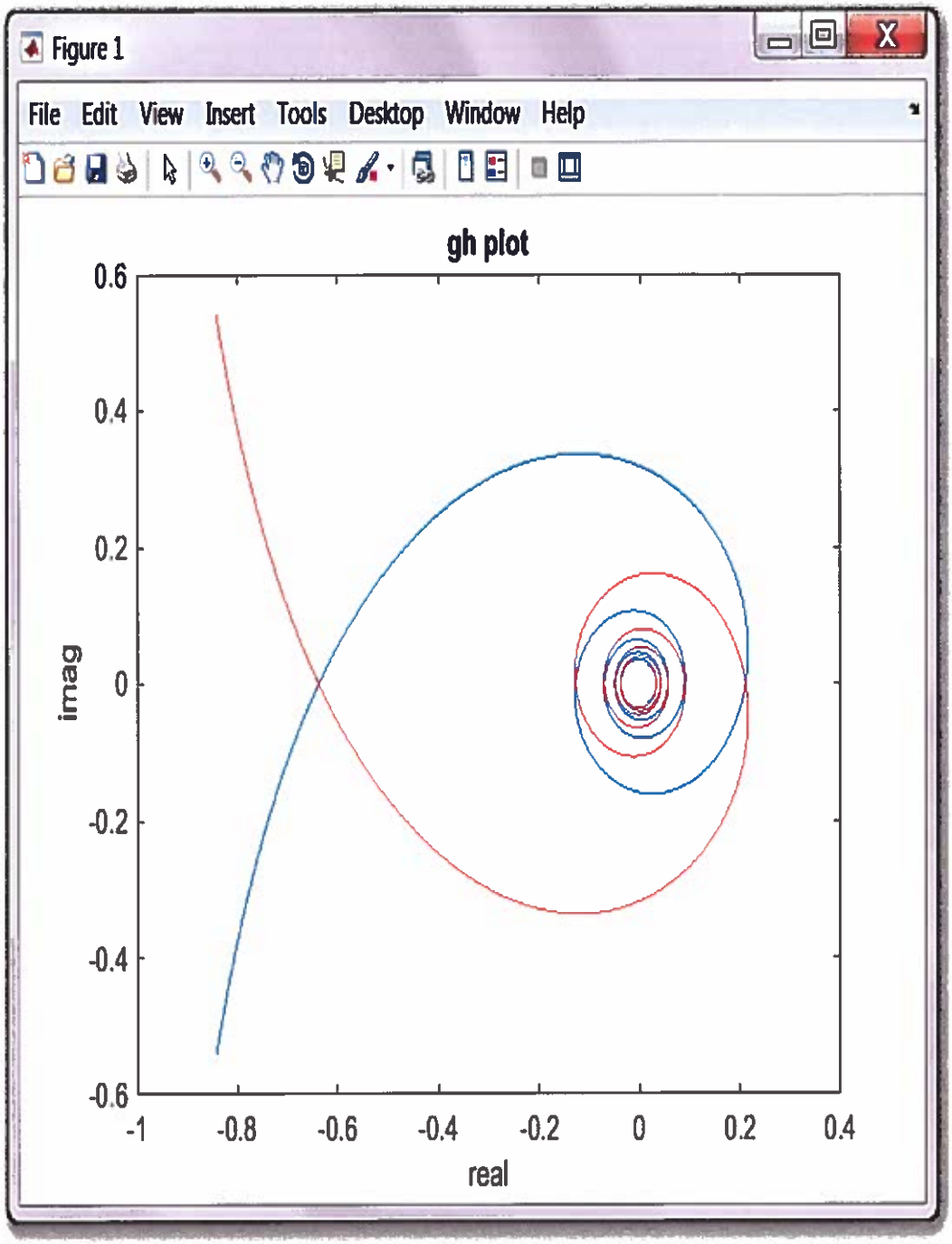


Figure 1
File Edit View Insert Tools Desktop Window Help
gh plot
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
imag
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
real

EXACT

ROOT LOCUS CHECK

$$\frac{K(1-s)}{s(1+s)}$$

$$= \frac{-K(s-1)}{s(s+1)}$$

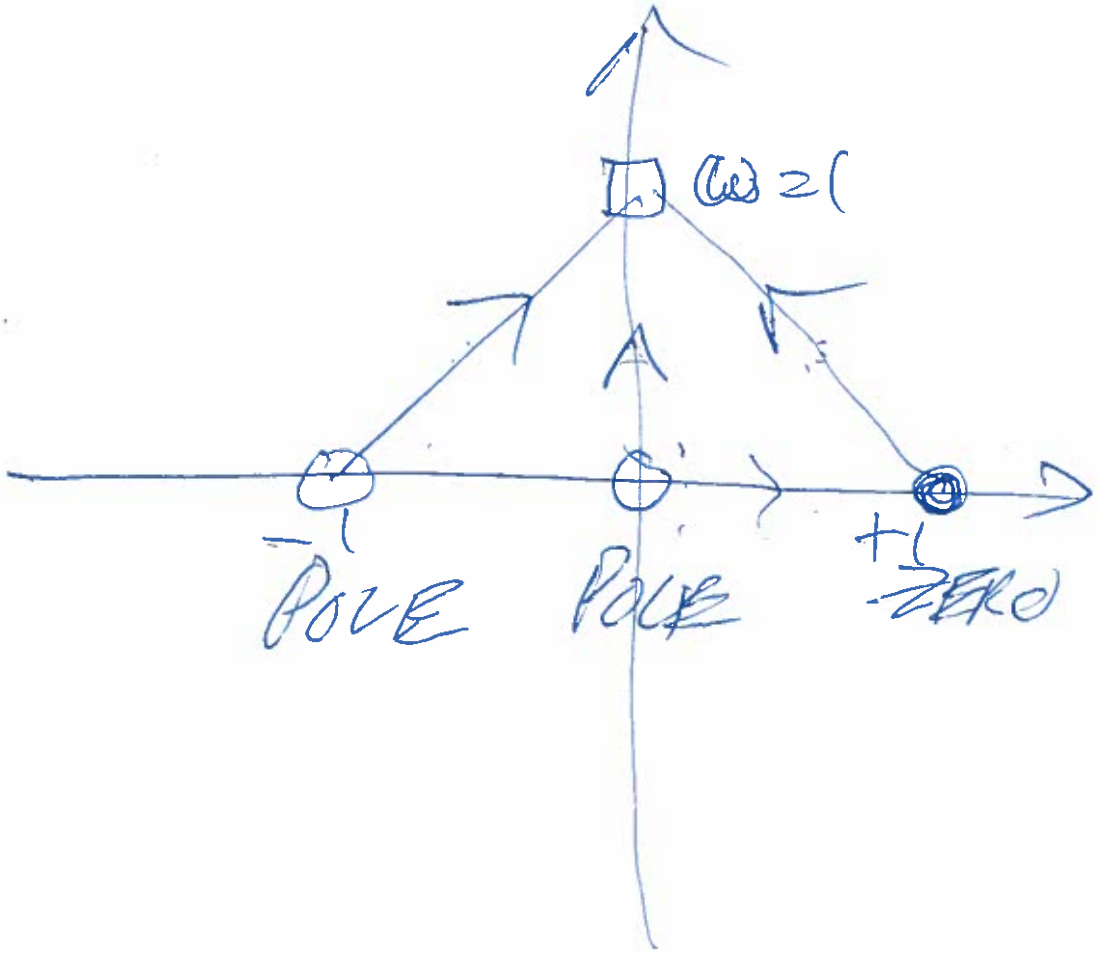
POLES $s=0$ $s=-1$ ZERO $s=+1$

ANGLES

$+135 - 90 - 45$

$+180$ ✓

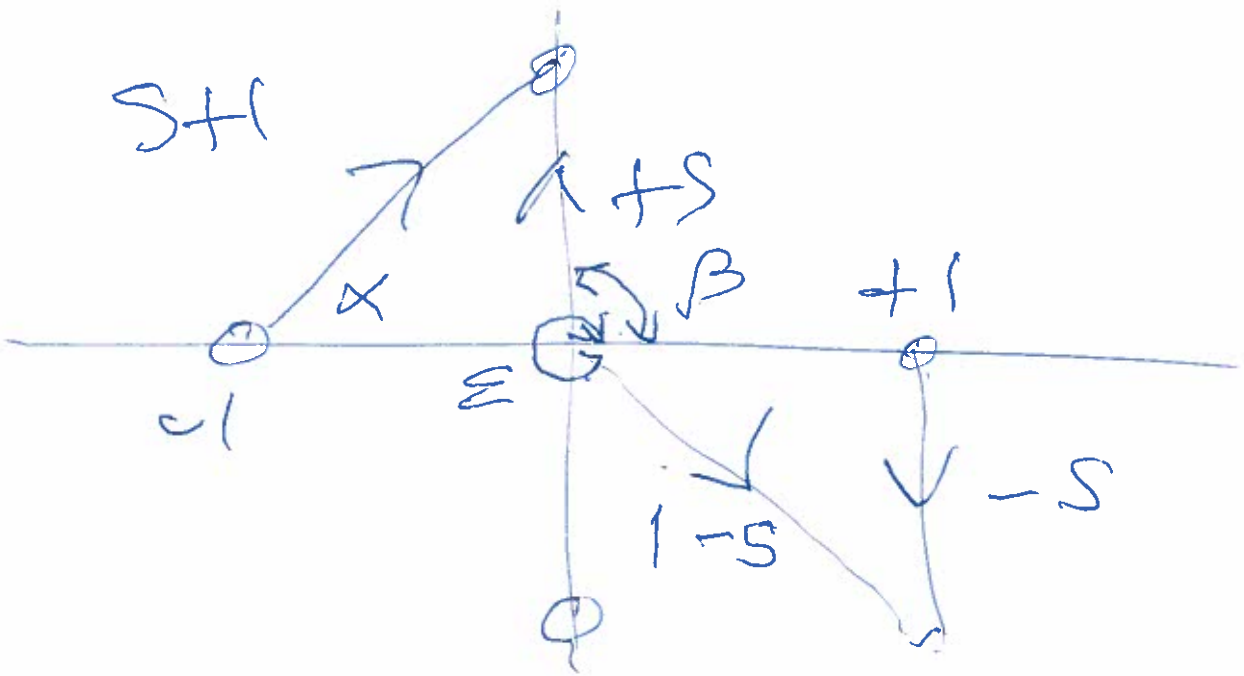
$= +180$



LENGTHS

$\frac{K\sqrt{2}}{C\sqrt{2}} = 1$

$K = C$ ✓



$$\varepsilon = 180 + 90 + 45$$

$$\beta = -90 \qquad \alpha = -45$$

$$\alpha + \beta + \varepsilon = 180$$

BORDERLINE
OPEN PERIOD

$$A\frac{dR}{dt} = M + N$$

$$M = Bq$$

$$q = KE$$

$$E = C - P$$

$$P(t) = R(t - T)$$

PADE APPROXIMANT

$$\frac{P}{R} = \frac{1 - \frac{T}{2}s}{1 + \frac{T}{2}s}$$

$$P + \frac{T}{2}\frac{dP}{dt} = R - \frac{T}{2}\frac{dR}{dt}$$

$$\frac{dP}{dt} + \frac{I}{2}\frac{d^2P}{dt^2} = \frac{dR}{dt} - \frac{I}{2}\frac{d^2R}{dt^2}$$

$$A\frac{dR}{dt} = M + N$$

$$\frac{dR}{dt} = \frac{M+N}{A} = \frac{BK(C-P)}{A} + \frac{N}{A}$$

$$\frac{d^2R}{dt^2} = \frac{BK}{A}\left(\frac{dC}{dt} - \frac{dP}{dt}\right) + \frac{1}{A}\frac{dN}{dt}$$

$$\frac{dP}{dt} + \frac{T}{2}\frac{d^2P}{dt^2}$$

$$=$$

$$\frac{BK(C-P)}{A} + \frac{N}{A}$$

$$-\frac{I}{2}\left(\frac{BK}{A}\left(\frac{dC}{dt} - \frac{dP}{dt}\right) + \frac{1}{A}\frac{dN}{dt}\right)$$

$$C = C_0 \qquad N = N_0$$

$$P = P_0 + \delta P \sin \omega t$$

$$\frac{I}{2}\frac{d^2p}{dt^2} + \left(1 - \frac{I}{2}\frac{BK}{A}\right)\frac{dP}{dt} + \frac{BKP}{A} = \{\}$$

$$\cos \omega t \Rightarrow \quad 1 - \frac{I}{2}\frac{BK}{A} = 0 \qquad K \Rightarrow \frac{2A}{IB}$$

$$\sin \omega t \Rightarrow \quad -\frac{I}{2}\omega^2 + \frac{BK}{A} = 0 \qquad \omega^2 \Rightarrow \frac{2BK}{AI}$$

$$\omega^2 \Rightarrow \frac{4}{T^2} \Rightarrow \omega = \frac{2}{T}$$

$$P(t) = R(t - \tau)$$

$$R(t) = R_0 + \Delta R \sin \omega t$$

$$R(t - \tau) = R_0 + \Delta R \sin \omega (t - \tau)$$

$$\sin \omega (t - \tau)$$

$$=$$

$$\sin [\omega t] \cos [\omega \tau] - \cos [\omega t] \sin [\omega \tau]$$

Simulation

$$\frac{I}{2}\frac{d^2P}{dt^2} + \left(1 - \frac{I}{2}\frac{BK}{A}\right)\frac{dP}{dt} + \frac{BK}{A}P$$

$$=$$

$$\frac{BKC}{A} + \frac{N}{A} - \frac{I}{2}\frac{BK}{A}\frac{dC}{dt} - \frac{T}{2A}\frac{dN}{dt}$$

$$U = \frac{dP}{dt} \qquad P_{NEW} = P_{OLD} + \Delta t\, U_{OLD}$$

$$W = \frac{dU}{dt} \qquad U_{NEW} = U_{OLD} + \Delta t\, W_{OLD}$$

```
%
% sheet thickness code
%
clear all
ROLD=0.0; SENSOR=ROLD;
UOLD=0.0; POLD=0.0;
COMMAND=10.0;LEAK=0.0;
A=1.0; B=1.0; GAIN=1.0;
NIT=10000; MIT=1000;
TIME=0.0;DELT=0.002;
PERIOD=MIT*DELT;
for IT=2:NIT
TIME=TIME+DELT;
if(IT>MIT)
 SENSOR=R(IT-MIT);
end
ERROR=COMMAND-SENSOR;
SIGNAL=GAIN*ERROR;
FLOW=B*SIGNAL;
XYZ=FLOW+LEAK;
RNEW=ROLD+DELT*XYZ/A;
ROLD=RNEW;R(IT)=RNEW;
PNEW=POLD+DELT*UOLD;
RATE=(B*GAIN/A*(COMMAND-POLD) ...
  -(1.0-PERIOD/2*B*GAIN/A)*UOLD) ...
  /(PERIOD/2);
UNEW=UOLD+DELT*RATE;
POLD=PNEW;UOLD=UNEW;
P(IT)=PNEW;T(IT)=TIME;
end
plot(T,R,T,P)
xlabel('TIME')
ylabel('RESPONSE')
title('STEEL SHEET')
```

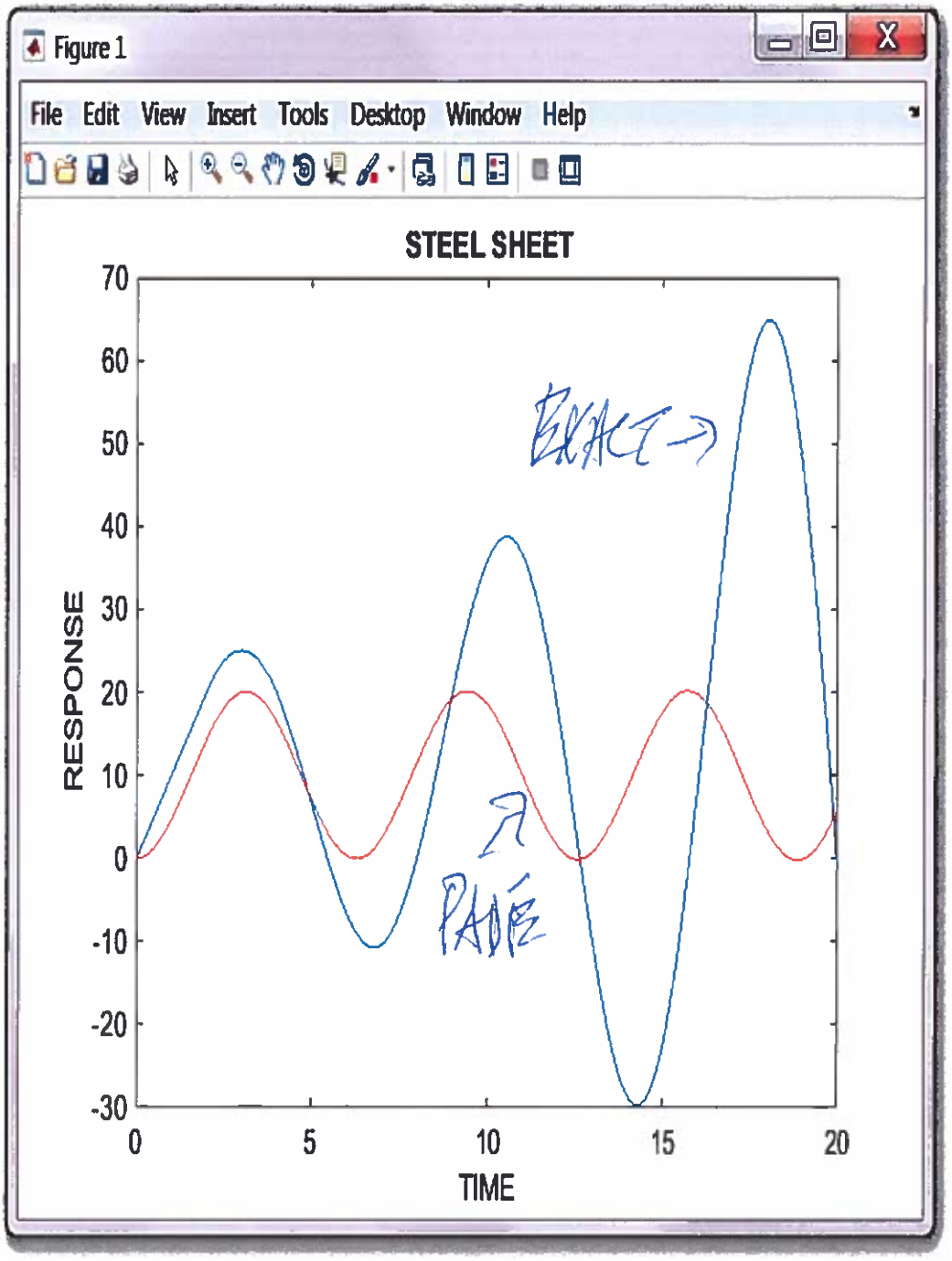


Figure 1
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STEEL SHEET
RESPONSE
TIME
70
60
50
40
30
20
10
0
-10
-20
-30
0
5
10
15
20