HOMEWORK #2

Propellers are used to dynamically position a small boat near a target. The equations governing the dynamics of the boat are:

$$\text{PLANT} \quad M\, d^2R/dt^2 \ + \ N\, dR/dt \ = \ B \ + \ D$$

$$\text{DRIVE} \quad J\, dB/dt \ + \ I\, B \ = \ Z\, Q$$

$$\text{SENSOR} \quad X\, dP/dt \ + \ Y\, P \ = \ R$$

$$\text{CONTROLLER} \quad Q \ = \ K_P\, E \ + \ K_I \int E\, d\tau \ + \ K_D\, dE/dt$$

$$\text{ERROR} \quad E = C - P$$

where R is the actual position of the boat in meters, P is the actual position in volts, C is the command position in volts, E is position error, B is the propeller load, D is a disturbance load, Q is the control signal, K_P K_I K_D are controller gains and M N J I X Y Z are constants.

$$M=2 \ N=1 \quad J=1 \ I=5 \quad X=0.1 \ Y=2 \quad Z=2$$

Develop the overall block diagram for the boat. Reduce the overall block diagram down to the standard form with command as input and response as output. Reduce it down to the standard form with disturbance as input and response as output. Determine the transfer function connecting the response to the command and the transfer function connecting the response to the disturbance.

Determine the GH function for the boat when it being controlled by a proportional controller. Derive its characteristic equation. Use the Routh Hurwitz criteria to determine the borderline gain of the boat. Compare this borderline gain with homework #1 borderline gain.

Use the Root Locus concept to manually check the borderline gain and period obtained in homework #1. Sketch the proportional gain Root Locus plot using Matlab m code.

Determine the GH function for the boat when it being controlled by a PID controller. Derive its characteristic equation. Use Matlab m code to determine its roots when the controller has ZN Gains. Plot them in the S plane. Does the boat have sufficient damping?

Manually sketch the Nyquist Plot for the proportional controller case when the gain is half the borderline gain. Interpret the plot. Check the Nyquist plot using Matlab m code. Use m code to sketch the ZN Gains Nyquist plot. Interpret the plot. Estimate the gain and phase margins of the boat from the Matlab plot. Are they adequate?