ZIEGLER NICHOLS GAINS

Ziegler Nichols gains are based on the proportional borderline gain $\mathbf{K_P}$ and borderline period $\mathbf{T_P}$ of a system. Ziegler Nichols PID gains are:

$$K_P = 0.6*\mathbf{K_P} \qquad K_I = K_P/T_I \qquad K_D = K_P*T_D$$

$$T_I = 0.5*\mathbf{T_P} \qquad T_D = 0.125*\mathbf{T_P}$$

Ziegler Nichols PI gains are:

$$K_P = 0.45*\mathbf{K_P} \qquad K_I = K_P/T_I$$

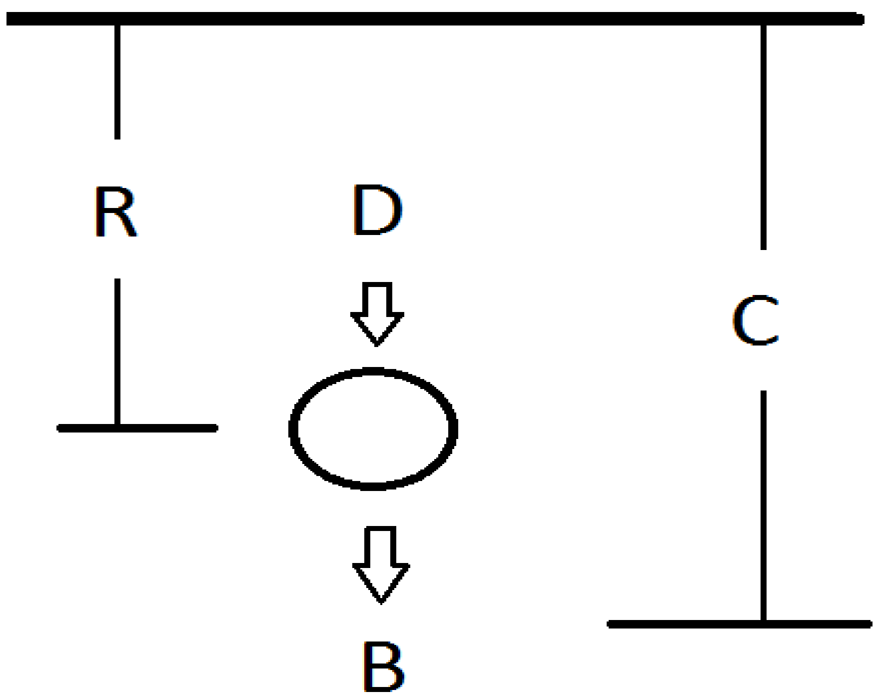
$$T_I = 0.83*\mathbf{T_P}$$

Ziegler Nichols gains only exist for systems that become unstable when the proportional gain is made bigger than the borderline proportional gain.

AUV DEPTH CONTROL

BORDERLINE GAIN AND PERIOD

Sketch below shows the geometry



The plant equation is:

$$M\ d^2R/dt^2 = B + D - W$$

$$W = X\, dR/dt\, |dR/dt| + Y\, dR/dt$$

$$W = N\, dR/dt$$

A drive equation is:

$$J\, dB/dt + I\, B = Q$$

The controller equations are:

$$Q = K\, E \qquad E = C - R$$

The plant equation gives

$$B = M\, d^2R/dt^2 + N\, dR/dt - D$$

$$dB/dt = M\, d^3R/dt^3 + N\, d^2R/dt^2 - dD/dt$$

Substitution into the drive equation gives

$$J\, [\, M\, d^3R/dt^3 + N\, d^2R/dt^2 - dD/dt]$$

$$+ I\, [\, M\, d^2R/dt^2 + N\, dR/dt - D\,] = K\, C - K\, R$$

During borderline stable operation

$$C = C_o \qquad D = D_o \qquad R = R_o + \Delta R\, \mathrm{Sin}\, [\omega t]$$

Substitution into the modified drive equation gives

$$- J\ M\ \boldsymbol{\omega}^3\ \Delta R\ \mathrm{Cos}[\boldsymbol{\omega} t] - J\ N\ \boldsymbol{\omega}^2\ \Delta R\ \mathrm{Sin}[\boldsymbol{\omega} t]$$

$$- I\ M\ \boldsymbol{\omega}^2\ \Delta R\ \mathrm{Sin}[\boldsymbol{\omega} t] + I\ N\ \boldsymbol{\omega}\ \Delta R\ \mathrm{Cos}[\boldsymbol{\omega} t]$$

$$- I\ D_o = \mathbf{K}\ C_o - \mathbf{K}\ R_o - \mathbf{K}\ \Delta R\ \mathrm{Sin}[\boldsymbol{\omega} t]$$

This equation is of the form:

$$i\ \mathrm{Sin}[\boldsymbol{\omega} t] + j\ \mathrm{Cos}[\boldsymbol{\omega} t] + k = 0$$

Mathematics requires that i=0 j=0 k=0:

$$- J\ N\ \boldsymbol{\omega}^2 - I\ M\ \boldsymbol{\omega}^2 + \mathbf{K} = 0$$

$$- J\ M\ \boldsymbol{\omega}^3 + I\ N\ \boldsymbol{\omega} = 0$$

$$- I\ D_o - \mathbf{K}\ C_o + \mathbf{K}\ R_o = 0$$

Manipulation of these equations gives

$$R_o = C_o + I\ D_o / \mathbf{K}$$

$$\boldsymbol{\omega}^2 = [I\ N] / [J\ M]$$

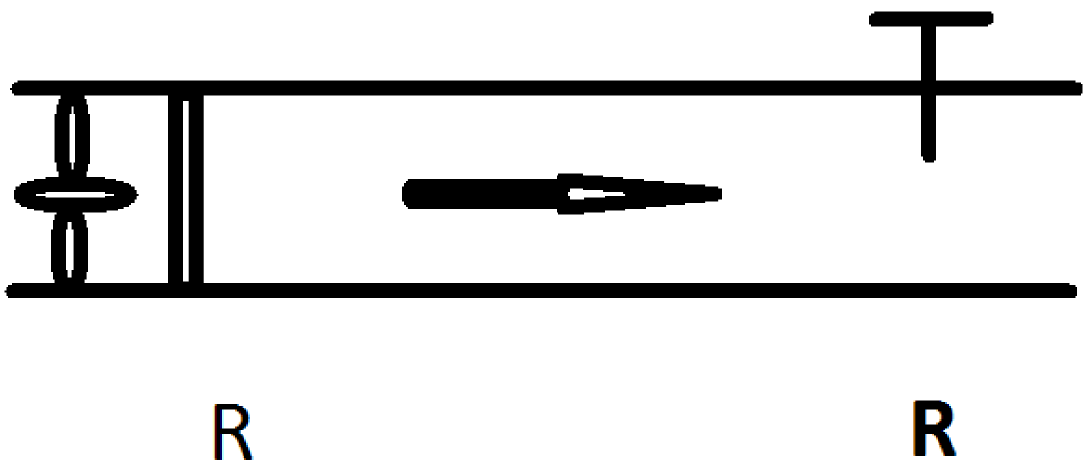
$$\mathbf{K} = [J\ N + I\ M]\ \boldsymbol{\omega}^2$$

$$= [J\ N + I\ M]\ [I\ N] / [J\ M]$$

PIPE FLOW SETUP

BORDERLINE GAIN AND PERIOD

Sketch below shows the geometry



The plant equation is:

$$X\ dR/dt + Y\ R = H + D$$

The drive equation is:

$$A\ dH/dt + B\ H = Z\ Q$$

The controller equations are:

$$Q = K\ E \qquad E = C - \mathbf{R}$$

$$\mathbf{R}(t) = R(t-T)$$

The plant equation gives

$$H = X\ dR/dt + Y\ R - D$$

$$dH/dt = X\ d^2R/dt^2 + Y\ dR/dt - dD/dt$$

Substitution into the drive equation gives

$$A\ [\ X\ d^2R/dt^2 + Y\ dR/dt - dD/dt]$$

$$+\ B\ (\ X\ dR/dt + Y\ R - D) = Z\ K\ C - Z\ K\ \mathbf{R}$$

During borderline stable operation

$$C = C_o \qquad D = D_o$$

$$R = R_o + \Delta R\ \mathrm{Sin}\ [\boldsymbol{\omega} t] \qquad \mathbf{R} = R_o + \Delta R\ \mathrm{Sin}\ [\boldsymbol{\omega}(t-T)]$$

Substitution into the modified drive equation gives

$$A\ [\ -\ X\ \boldsymbol{\omega}^2\ \Delta R\ \mathrm{Sin}[\boldsymbol{\omega} t] + Y\ \boldsymbol{\omega}\ \Delta R\ \mathrm{Cos}[\boldsymbol{\omega} t]\ ]$$

$$+\ B\ [\ +\ X\ \boldsymbol{\omega}\ \Delta R\ \mathrm{Cos}[\boldsymbol{\omega} t] + Y\ R_o + Y\ \Delta R\ \mathrm{Sin}[\boldsymbol{\omega} t] - D_o\ ]$$

$$= Z\ \mathbf{K}\ C_o - Z\ \mathbf{K}\ R_o - Z\ \mathbf{K}\ \Delta R\ \mathrm{Sin}[\boldsymbol{\omega}(t-T)]$$

A trigonometric identity gives

$$\mathrm{Sin}[\omega(t-T)] = \mathrm{Sin}[\omega t]\ \mathrm{Cos}[\omega T] - \mathrm{Cos}[\omega t]\ \mathrm{Sin}[\omega T]$$

Substitution into the modified drive equation gives an equation of the form

$$i\ \mathrm{Sin}[\omega t] + j\ \mathrm{Cos}[\omega t] + k = 0$$

Mathematics requires that i=0 j=0 k=0

$$- A\ X\ \omega^2 + B\ Y + Z\ \mathbf{K}\ \mathrm{Cos}[\omega T] = 0$$

$$A\ Y\ \omega + B\ X\ \omega - Z\ \mathbf{K}\ \mathrm{Sin}[\omega T] = 0$$

$$B\ Y\ R_o - B\ D_o - Z\ \mathbf{K}\ C_o + Z\ \mathbf{K}\ R_o = 0$$

Manipulation of the first two equations gives

$$\mathbf{K_P} = [A\ X\ \omega^2 - B\ Y] / [Z\ \mathrm{Cos}[\omega T]]$$

$$\mathbf{K_P} = [A\ Y\ \omega + B\ X\ \omega] / [Z\ \mathrm{Sin}[\omega T]]$$

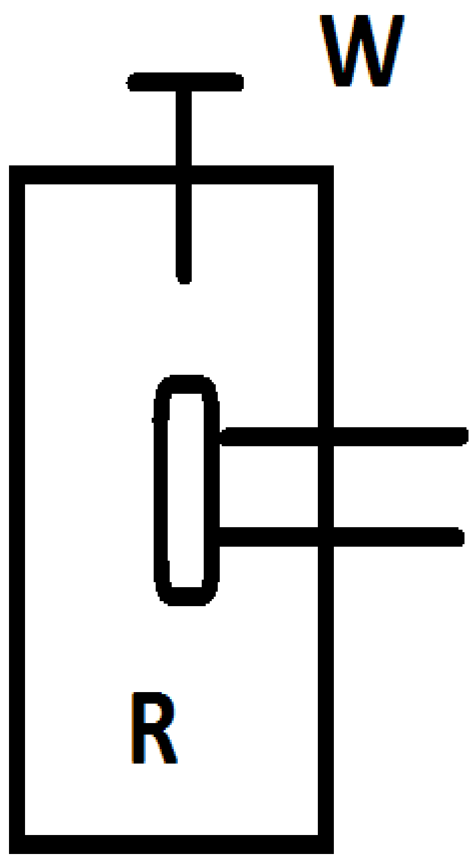
$$\mathrm{Sin}[\omega T]/\mathrm{Cos}[\omega T] = \mathrm{Tan}[\omega T]$$

$$= [A\ Y\ \omega + B\ X\ \omega] / [A\ X\ \omega^2 - B\ Y]$$

PROCESS TEMPERATURE CONTROL

BORDERLINE GAIN AND PERIOD

Sketch below shows the geometry



The plant equation is:

$$X\ dR/dt + Y\ R = P + D$$

The drive equation is:

$$J\ dP/dt + I\ P = Q$$

The sensor equation is:

$$A\ dW/dt + B\ W = R$$

The controller equations are:

$$Q = K\ E \qquad E = C - W$$

The sensor equation gives

$$R = A\ dW/dt + B\ W$$
$$dR/dt = A\ d^2W/dt^2 + B\ dW/dt$$

Substitution into the plant equation gives:

$$X\ (A\ d^2W/dt^2 + B\ dW/dt)$$
$$+\ Y\ (A\ dW/dt + B\ W) = P + D$$

The modified plant equation gives

$$P = X\ (A\ d^2W/dt^2 + B\ dW/dt)$$
$$+\ Y\ (A\ dW/dt + B\ W) - D$$

$$dP/dt = X\ (A\ d^3W/dt^3 + B\ d^2W/dt^2)$$
$$+\ Y\ (A\ d^2W/dt^2 + B\ dW/dt) - dD/dt$$

Substitution into the drive equation gives

$$JXA\ d^3W/dt^3 + (JXB + JYA + IXA)\ d^2W/dt^2$$
$$+\ (JYB + IXB + IYA)\ dW/dt - J\ dD/dt$$
$$= K\ C - (K+IYB)\ W + I\ D$$

During borderline stable operation

$$C = C_o \qquad D = D_o \qquad W = W_o + \Delta W\ \mathrm{Sin}(\omega t)$$

Substitution into the modified drive equation gives

$$- JXA\,\boldsymbol{\omega}^3\,\Delta W\,\mathrm{Cos}(\boldsymbol{\omega}t) - (JXB+JYA+IXA)\,\boldsymbol{\omega}^2\,\Delta W\,\mathrm{Sin}(\boldsymbol{\omega}t)$$

$$+ (JYB+IXB+IYA)\,\boldsymbol{\omega}\,\Delta W\,\mathrm{Cos}(\boldsymbol{\omega}t)$$

$$= \mathbf{K}\,C_o - (\mathbf{K}+IYB)\,W_o - (\mathbf{K}+IYB)\,\Delta W\,\mathrm{Sin}(\boldsymbol{\omega}t) + I\,D_o$$

This equation is of the form

$$i\,\mathrm{Sin}[\boldsymbol{\omega}t] + j\,\mathrm{Cos}[\boldsymbol{\omega}t] + k = 0$$

$$i = -(JXB+JYA+IXA)\,\boldsymbol{\omega}^2 + (\mathbf{K}+IYB)$$

$$j = -JXA\,\boldsymbol{\omega}^3 + (JYB+IXB+IYA)\,\boldsymbol{\omega}$$

$$k = -\mathbf{K}\,C_o + (\mathbf{K}+IYB)\,W_o - I\,D_o$$

Mathematics requires that i=0 j=0 k=0.

$$\boldsymbol{\omega}^2 = (JYB+IXB+IYA) / (JXA)$$

$$\mathbf{K} = (JXB+JYA+IXA)\,(JYB+IXB+IYA) / (JXA) - IYB$$

$$W_o = (\mathbf{K}\,C_o + I\,D_o) / (\mathbf{K}+IYB)$$

ROCKET ATTITUDE CONTROL

The governing equation is:

$$J\ d^2R/dt^2 - I\ R = +\ P\ K\ (C-R) + M\ N\ (dC/dt-dR/dt)$$

The proportional control torque is due to the lift engine while the derivative control torque is due to the gas rockets. Proportional control is spring like while derivative control is drag like.

Manipulation gives

$$J\ d^2R/dt^2 + M\ N\ dR/dt + (P\ K - I)\ R$$
$$= +\ P\ K\ C + M\ N\ dC/dt$$

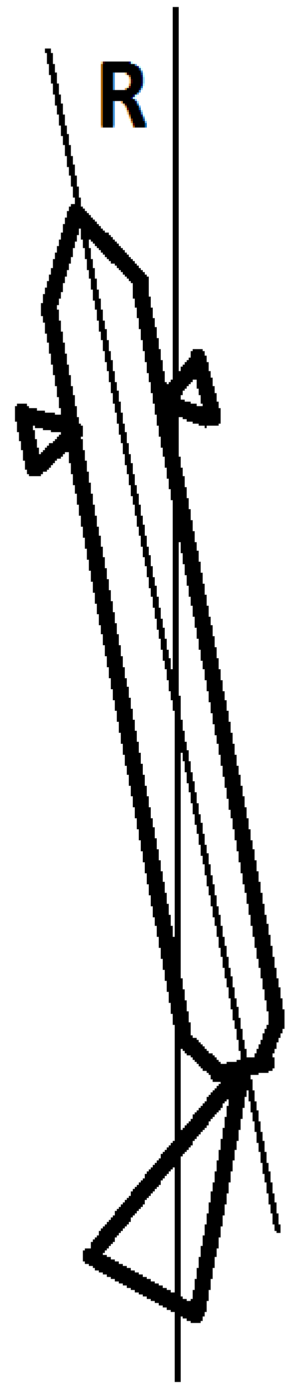
This is of the form

$$m\ d^2R/dt^2 + c\ dR/dt + k\ R = p$$

The borderline gain and period are:

$$\mathbf{K} = I/P \qquad \boldsymbol{\omega} = 0 \qquad \mathbf{T} = \infty$$

Note that for stable operation K must be greater than **K**. So this system does not have ZN Gains.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SATELLITE ATTITUDE CONTROL

The governing equation is:

$$J\, d^2R/dt^2 = M\, K\, (E + N\, dE/dt)$$

The controller acts on the attitude error plus N times the attitude error rate. The error control is spring like and the rate control is drag like.

Manipulation gives

$$J\, d^2R/dt^2 + MKN\, dR/dt + MK\, R = MK\, C + MKN\, dC/dt$$

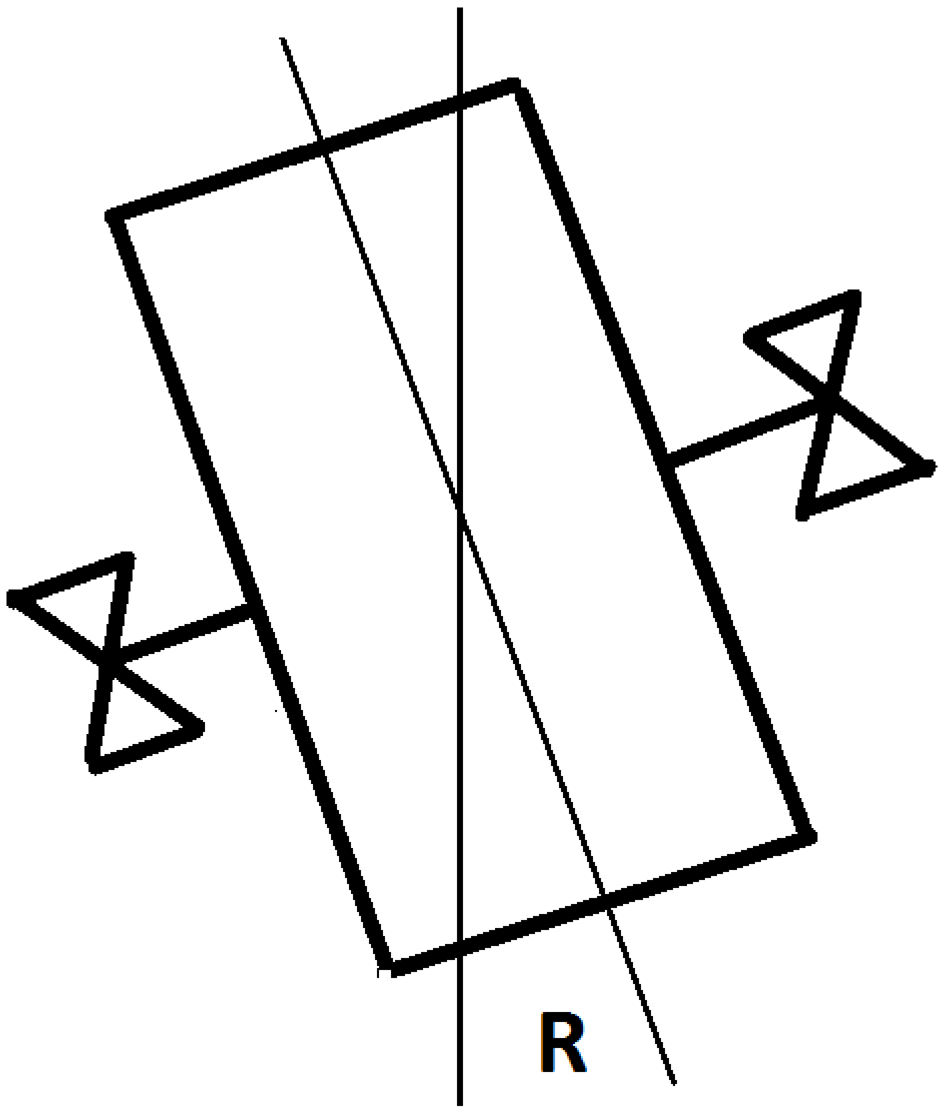
This is of the form

$$m\, d^2R/dt^2 + c\, dR/dt + k\, R = p$$

The borderline gain and period are:

$$\mathbf{K} = 0 \qquad \boldsymbol{\omega} = 0 \qquad \mathbf{T} = \infty$$

Note that for stable operation K must be greater than **K**. So, this system does not have ZN Gains.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