OVERVIEW OF NYQUIST

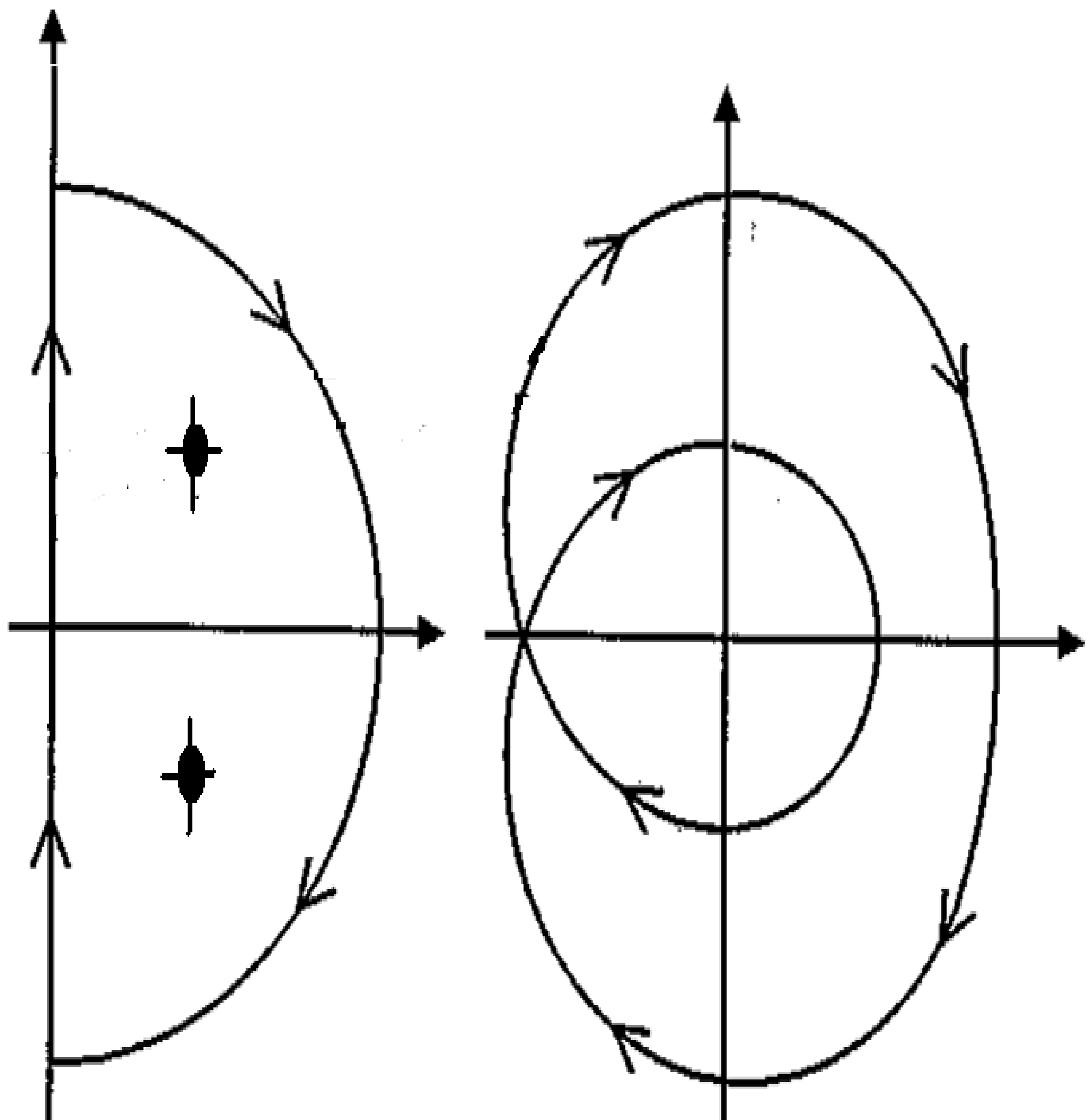
The Nyquist procedure is based on the 1+GH function:

$$1 + GH = 1 + N/D = (N+D)/D = \mathbf{D}/D$$

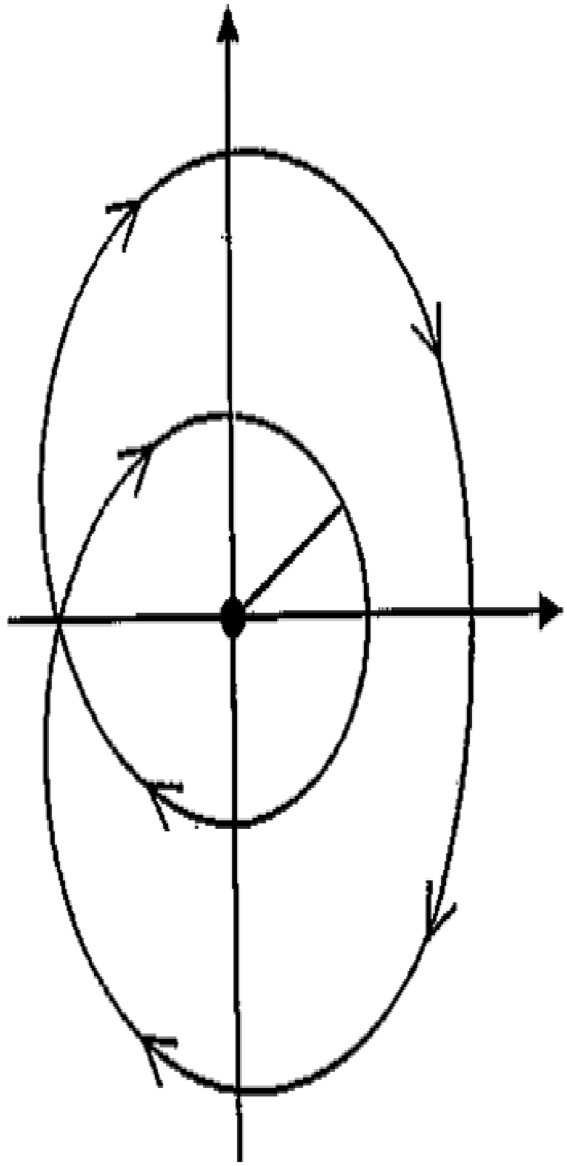
$$= \frac{\Gamma\ (S-Z_1)\ (S-Z_2)\ ::::: \ (S-Z_n)}{(S-P_1)\ (S-P_2)\ ::::: \ (S-P_m)} = R\angle\Theta$$

It is basically a vector made from zero and pole factors which are also vectors. When the tip of the S vector moves clockwise around the Nyquist contour, zeros Z inside it cause clockwise rotations of 1+GH while poles P inside it cause counterclockwise rotations. Only zeros and poles inside cause such rotations: zeros and poles outside only cause 1+GH to nod up and down. In the 1+GH plane R is drawn from the origin. In the GH plane, R is drawn from the minus one point. We want to find the number of unstable zeros N_Z. From the GH plot, one gets the net clockwise rotations of the GH vector N. The net clockwise rotations N must be equal to the number of unstable zeros N_Z minus the number of unstable poles N_P. From the GH function, one gets the number of unstable poles N_P. Manipulation gives N_Z:

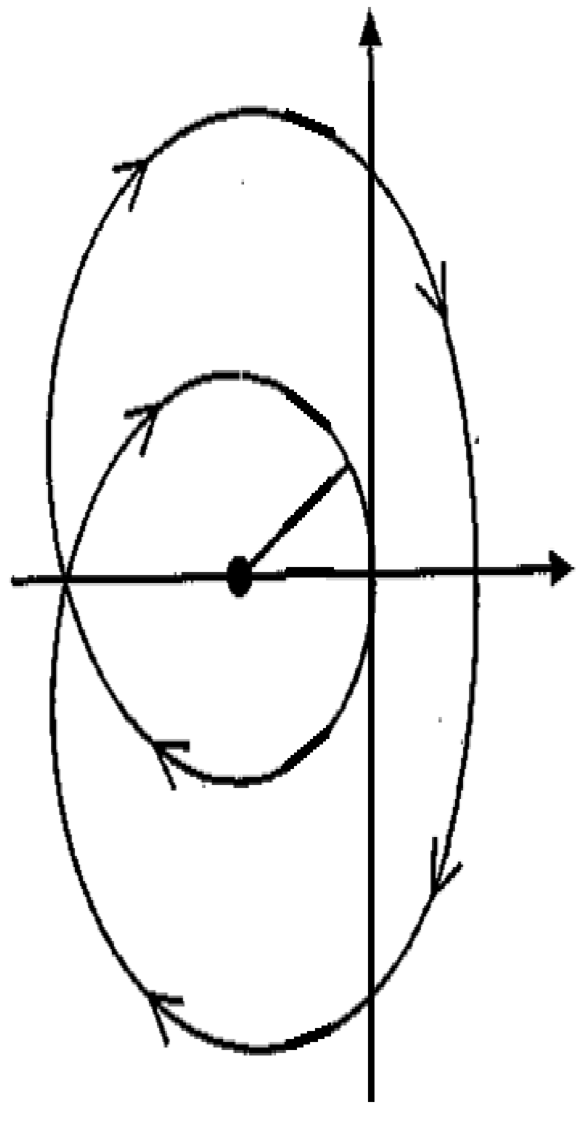
$$N = N_Z - N_P \qquad\qquad N_Z = N + N_P$$



S PLANE 1+GH PLANE



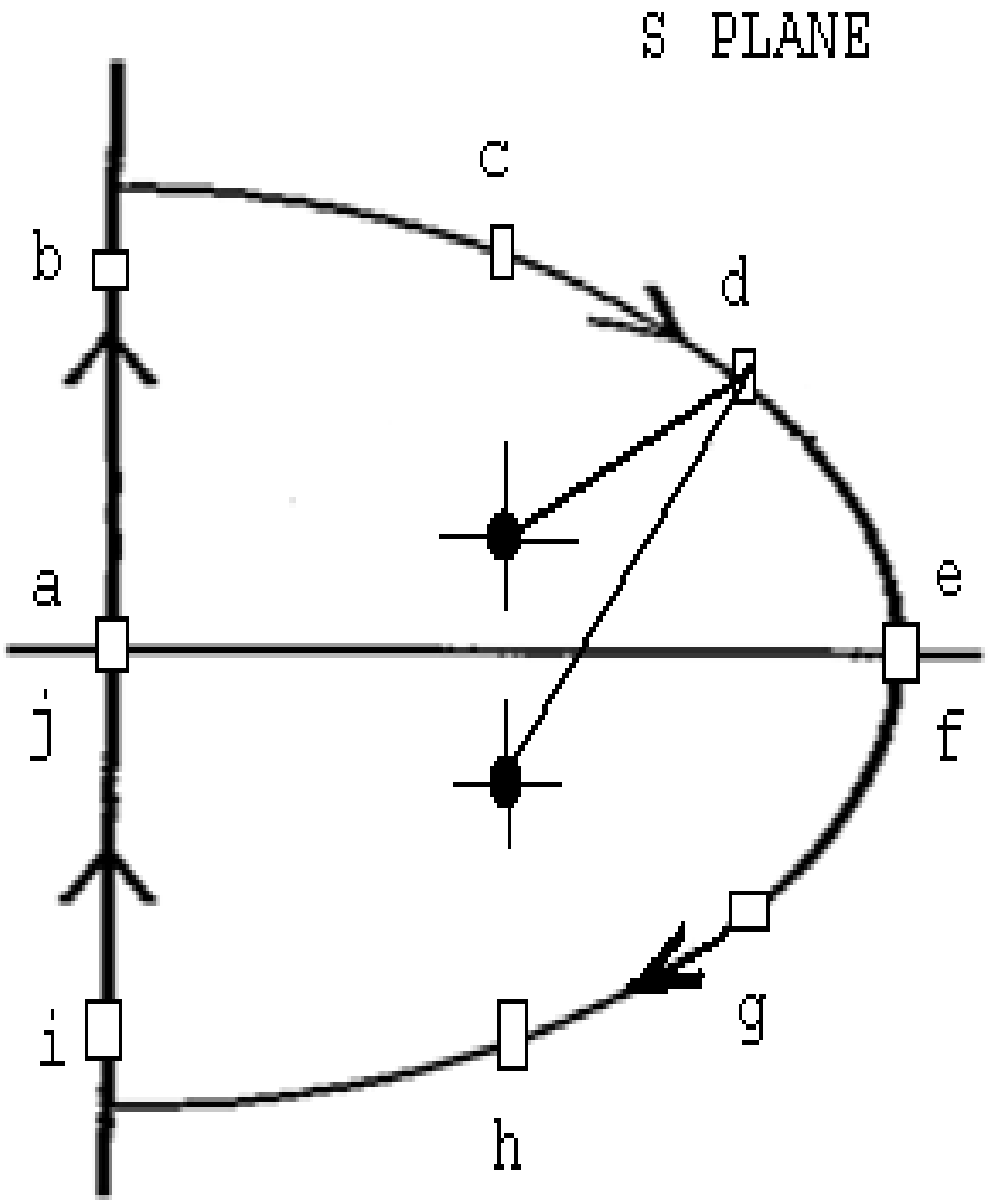
1+GH PLOT

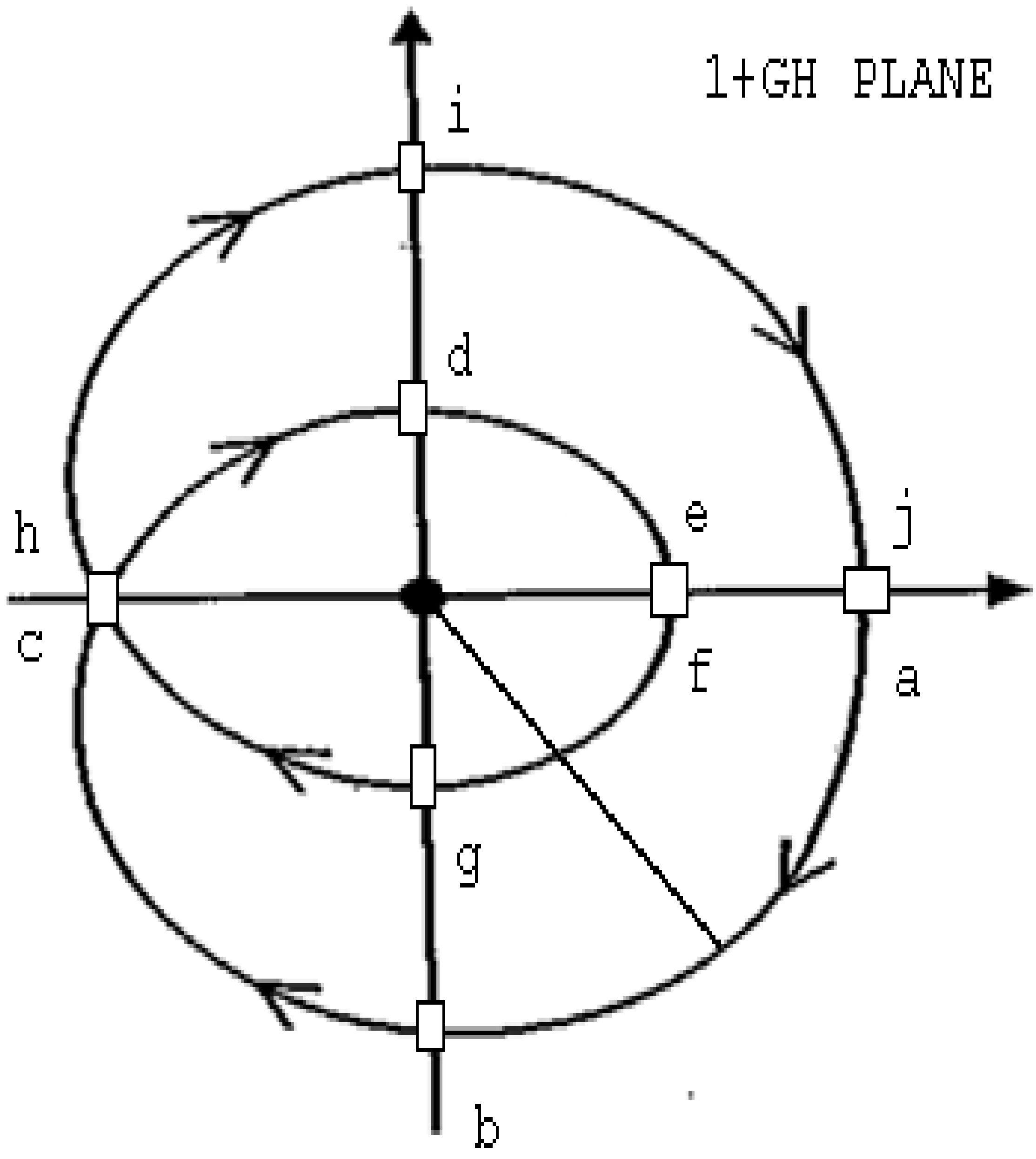


GH PLOT

NYQUIST ILLUSTRATION

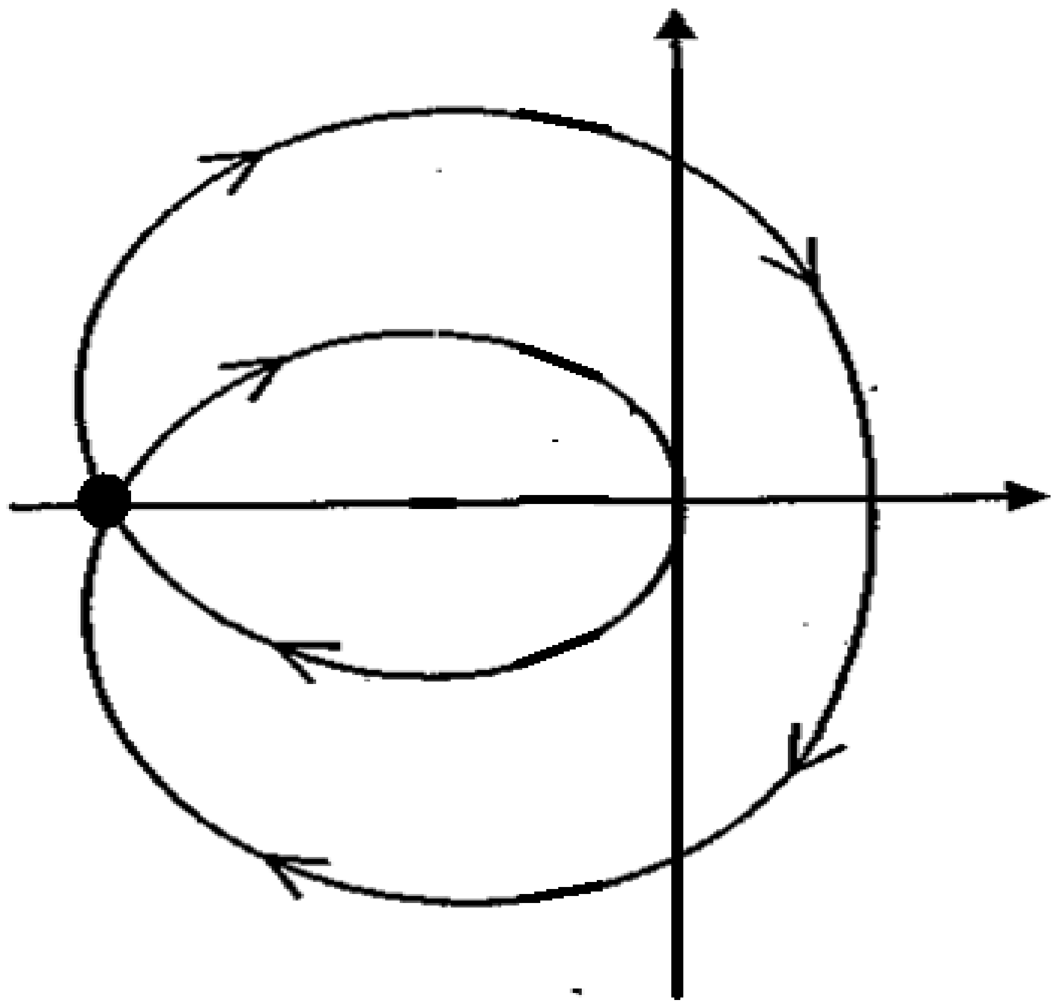
Consider the case where there two unstable zeros in the right half of the S plane and all other zeros and poles are far into the left half of the S plane. Now surround the unstable zeros by a clockwise contour as shown on the next page. When we map points on this contour to the 1+GH plane, we get the contour two pages over. When we draw a vector with radius R and angle Θ to the contour in the 1+GH plane and count the number of times it rotates clockwise as we move around the contour in the S plane, we get two clockwise rotations. These rotations are caused by the unstable zeros. Nyquist allows us to determine the number of unstable zeros without having to find their exact locations.


S PLANE
a
b
c
d
e
f
g
h
i
j


1+GH PLANE
i
d
e
j
h
c
f
a
g
b

SIGNIFICANCE OF GH EQUAL TO MINUS ONE

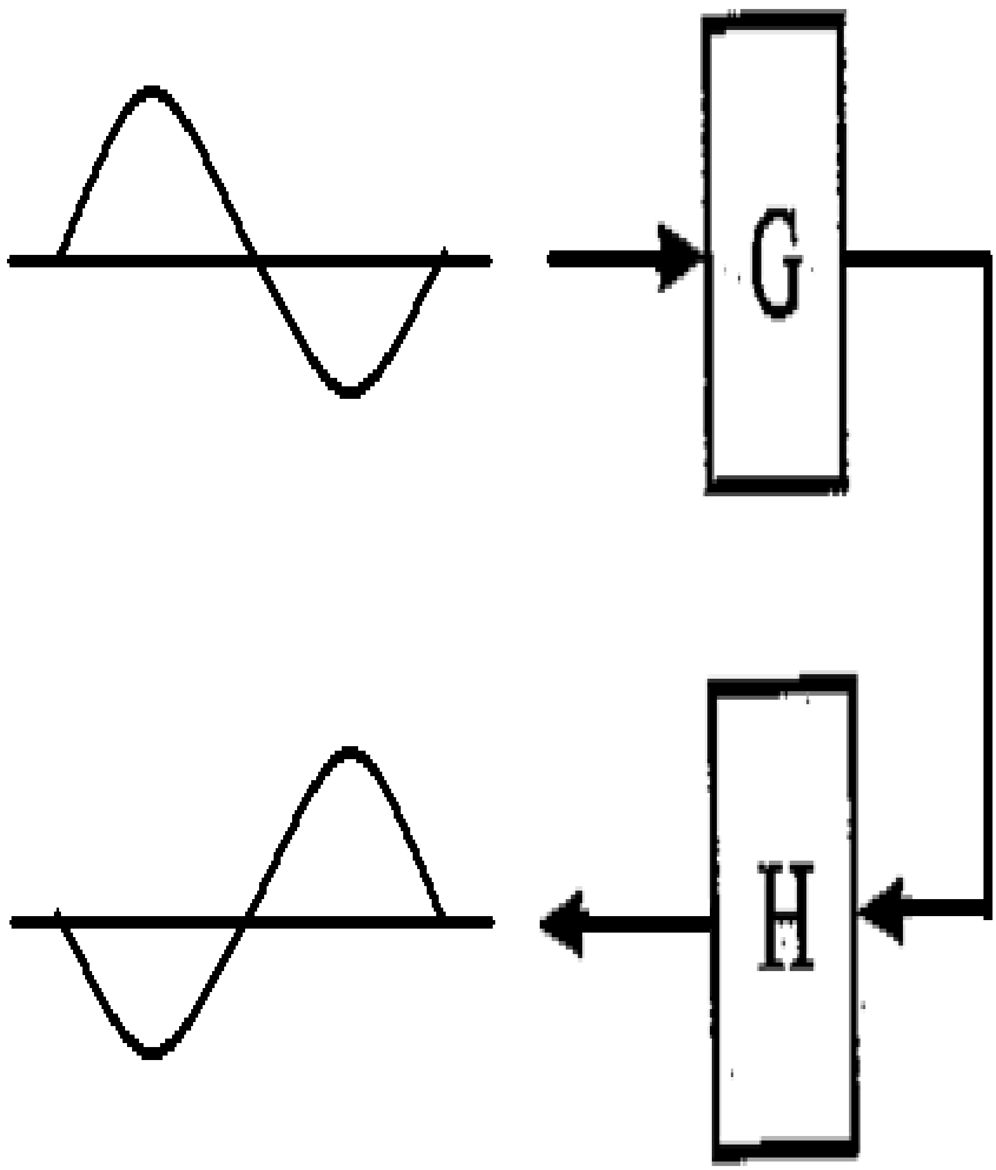
Consider the case where only proportional control is being used and the GH plot passes through the minus one point in the GH plane. If $GH=-1$ then $1+GH=0$. This implies that at this point $S=Z$: in other words, it is a root of the overall characteristic equation. But along the GH plot $S=\pm j\omega$. So $Z=\pm j\omega$. So there is a complex conjugate pair of roots of the overall characteristic equation on the imaginary axis in the S plane. This means the system is borderline stable and the gain K is the borderline stable gain **K** and the frequency ω is the borderline stable frequency **ω**. The borderline stable period is $\mathbf{T}=2\Pi/\boldsymbol{\omega}$.



GH PLOT

OPEN LOOP FREQUENCY RESPONSE

A GH plot is basically a polar open loop frequency response plot. Consider the case where only proportional control is being used. When GH=-1, a command sine wave produces a response which has the same magnitude as the command but is 180° out of phase. If the command was suddenly removed and the loop was suddenly closed, the negative of the response would take the place of the command and keep the system oscillating. The system would be borderline stable with gain **K**. If the gain was bigger than **K**, the command would produce a response bigger than itself. When this takes over, it would produce growing or unstable oscillations. If the gain was smaller than **K**, the command would produce a response smaller than itself. When this takes over, it would produce decaying or stable oscillations.


G
H