SIMULATION EXAMPLES

PIPE FLOW SETUP

The governing equations are:

$$X\, dR/dt + Y\, R = H + D$$

$$A\, dH/dt + B\, H = Z\, Q$$

$$Q = K_P\, E + K_I \int E d\tau + K_D\, dE/dt$$

$$E = C - \mathbf{R}$$

Manipulation gives:

$$dR/dt = (H + D - Y\, R) / X$$

$$dH/dt = (Z\, Q - B\, H) / A$$

$$Q = K_P\, E + K_I \int E d\tau + K_D\, dE/dt$$

$$E = C - \mathbf{R}$$

Simple time stepping scheme gives:

$$R_{NEW} = R_{OLD} + \Delta t * (H_{OLD} + D_{OLD} - Y\, R_{OLD}) / X$$

$$H_{NEW} = H_{OLD} + \Delta t * (Z\, Q_{OLD} - B\, H_{OLD}) / A$$

$$Q_{OLD} = K_P\, E_{OLD} + K_I\, \Sigma\, E_{OLD}\, \Delta t + K_D\, \Delta E_{OLD}/\Delta t$$

$$E_{OLD} = C_{OLD} - \mathbf{R}_{OLD}$$

Let the parameters be:

$$X=0.25 \quad Y=1.0 \quad A=0.1 \quad B=1.0 \quad Z=1.0 \quad T=0.5$$

The Ziegler Nichols gains are:

$$K_P = 0.9 \quad K_I = 1.2 \quad K_D = 0.17$$

Let the command C be 5.0 and the disturbance D be 0.0. Let the control signal Q be saturated 10V high and 0V low. Let the time step Δt be 0.25. The time stepping equations become:

$$E_{OLD} = 5.0 - \mathbf{R}_{OLD}$$

$$Q_{OLD} = 0.9\, E_{OLD} + 1.2\, \Sigma\, E_{OLD}\, 0.25 + 0.17\, \Delta E_{OLD}/0.25$$

$$H_{NEW} = H_{OLD} + 0.25 * (1.0\, Q_{OLD} - 1.0\, H_{OLD}) / 0.1$$

$$R_{NEW} = R_{OLD} + 0.25 * (H_{OLD} + 0.0 - 1.0\, R_{OLD}) / 0.25$$

TIME STEP #1

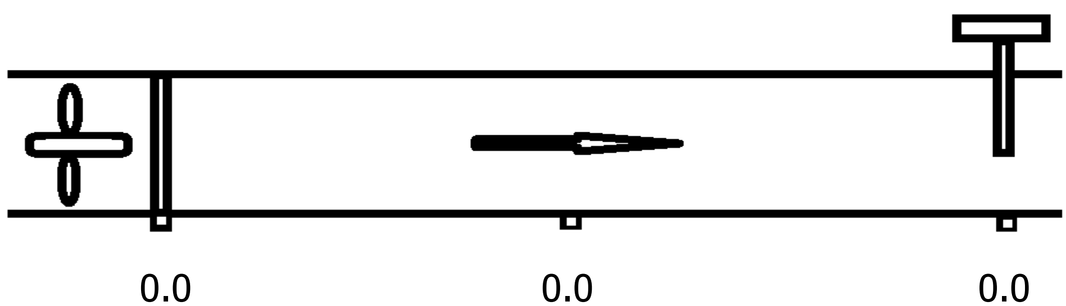
$R_{OLD} = 0.0$ $\mathbf{R}_{OLD} = 0.0$ $H_{OLD} = 0.0$

$E_{OLD} = 5.0 - 0.0 = 5.0$

$Q_{OLD} = 0.9*5.0 + 1.2*0.0*0.25 + 0.17*0.0/0.25 = 4.5$

$H_{NEW} = 0.0 + 0.25*(1.0*4.5-1.0*0.0)/0.1 = 11.25$

$R_{NEW} = 0.0 + 0.25*(0.0+0.0-1.0*0.0) / 0.25 = 0.0$



TIME STEP #2

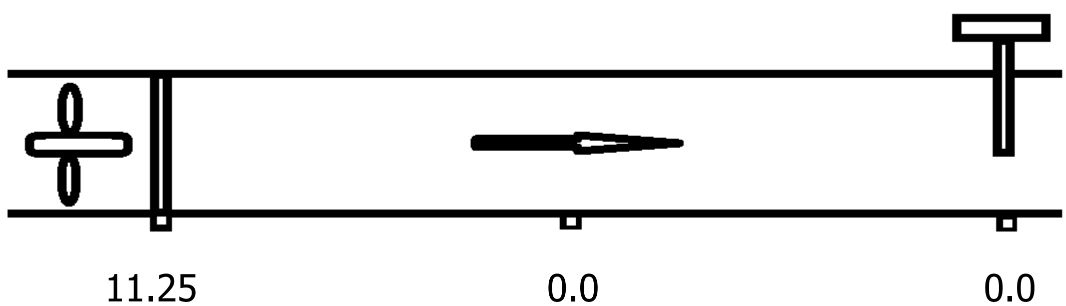
$R_{OLD} = 0.0$ $\mathbf{R}_{OLD} = 0.0$ $H_{OLD} = 11.25$

$$E_{OLD} = 5.0 - 0.0 = 5.0$$

$$Q_{OLD} = 0.9*5.0 + 1.2*5.0*0.25 + 0.17*0.0/0.25 = 6.0$$

$$H_{NEW} = 11.25 + 0.25*(1.0*6.0-1.0*11.25)/0.1 = -1.875$$

$$R_{NEW} = 0.0 + 0.25*(11.25+0.0-1.0*0.0)/0.25 = 11.25$$



TIME STEP #3

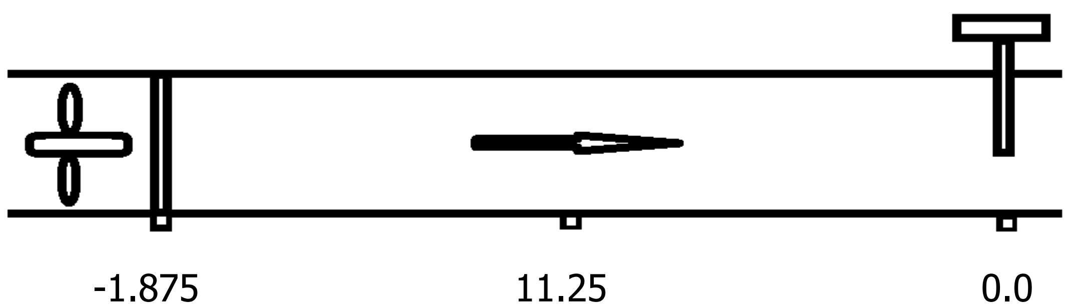
$R_{OLD} = 11.25$ $\mathbf{R}_{OLD} = 0.0$ $H_{OLD} = -1.875$

$$E_{OLD} = 5.0 - 0.0 = 5.0$$

$$Q_{OLD} = 0.9*5.0 + 1.2*(5.0+5.0)*0.25 + 0.17*0.0/0.25 = 7.5$$

$$H_{NEW} = -1.875 + 0.25*(1.0*7.5-1.0*[-1.875])/0.1 = 21.5625$$

$$R_{NEW} = 11.25 + 0.25*(-1.875+0.0-1.0*11.25)/0.25 = -1.875$$



TIME STEP #4

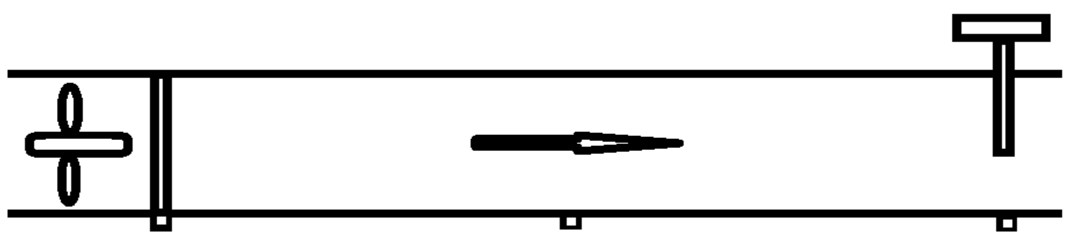
R_{OLD} = -1.875 $\mathbf{R}_{OLD}$ = 0.0 H_{OLD} = 21.5625

E_{OLD} = 5.0 - 0.0 = 5.0

Q_{OLD} = 0.9*5.0 + 1.2*(5.0+5.0+5.0)*0.25 + 0.17*0.0/0.25 = 9.0

H_{NEW} = 21.5625 + 0.25*(1.0*9.0-1.0*21.5625)/0.1 = -9.84375

R_{NEW} = -1.875 + 0.25*(21.5625+0.0-1.0*[-1.875])/0.25 =21.5625



21.5625 -1.875 11.25

TIME STEP #5

$R_{OLD} = 21.5625$ $\mathbf{R}_{OLD} = 11.25$ $H_{OLD} = -9.84375$

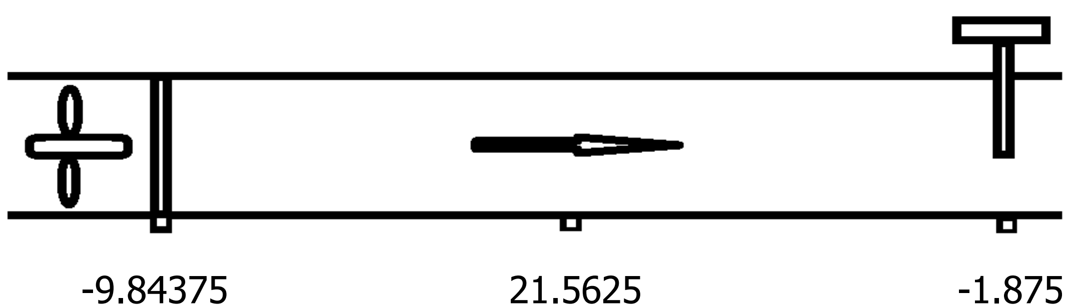
$E_{OLD} = 5.0 - 11.25 = -6.25$

$Q_{OLD} = 0.9*[-6.25] + 1.2*(5.0+5.0+5.0+5.0)*0.25 + 0.17*$
$[-6.25-5.0]/0.25 = -7.275 \quad <0 \qquad Q_{OLD} = 0.0$

$H_{NEW} = -9.84375 + 0.25*(1.0*0.0-1.0*[-9.84375])/0.1 = 14.765625$

$R_{NEW} = 21.5625 + 0.25*(-9.84375+0.0-1.0*[21.5625])/0.25 =$

-9.84375



TIME STEP #6

$R_{OLD} = -9.84375$ $\mathbf{R}_{OLD} = -1.875$ $H_{OLD} = 14.765625$

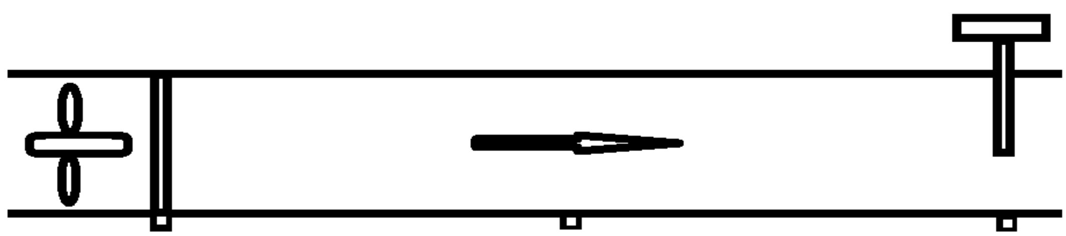
$E_{OLD} = 5.0 - [-1.875] = 6.875$

$Q_{OLD} = 0.9*[6.875] + 1.2*(5.0+5.0+5.0+5.0-6.25)*0.25 +$
$0.17*[6.875-[-6.25]]/0.25 = 19.2375 \ >10$ $Q_{OLD} = 10.0$

$H_{NEW} = 14.765625 + 0.25*(1.0*10.0-1.0*[14.765625])/0.1$
$=2.8515625$

$R_{NEW} = -9.84375 + 0.25*(14.765625+0.0-1.0*[-9.84375])/0.25 =$

14.765625



14.765625 -9.84375 21.5625

HYDRAULIC ACTUATOR

The governing equations for the actuator are:

Plant $A\, dR/dt = F + D$

Drive $F = B\, Q$

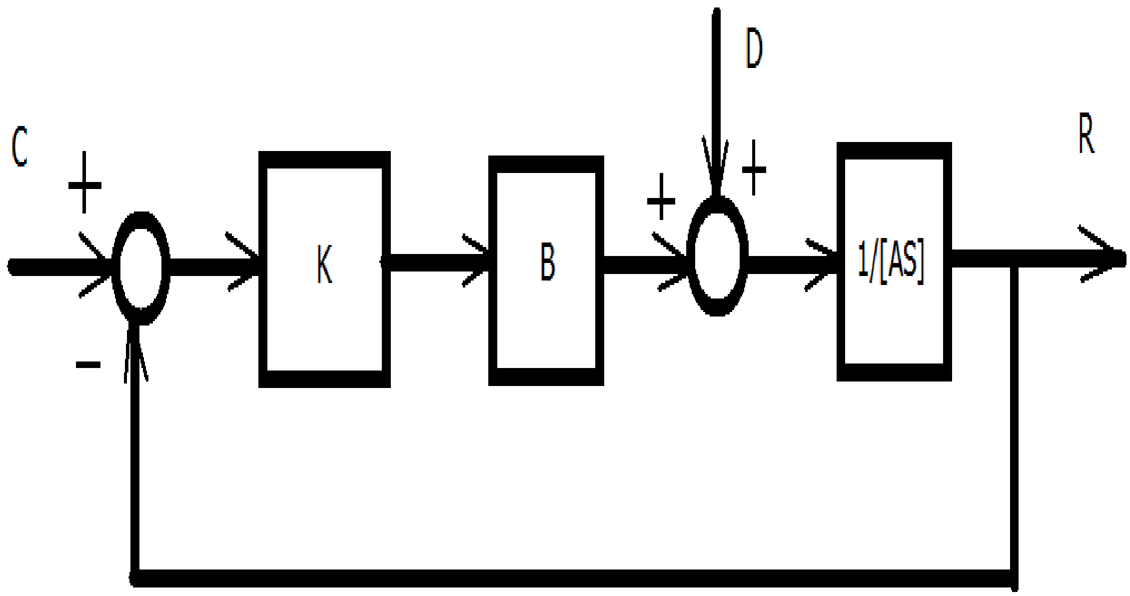
Controller $Q = K\, E$

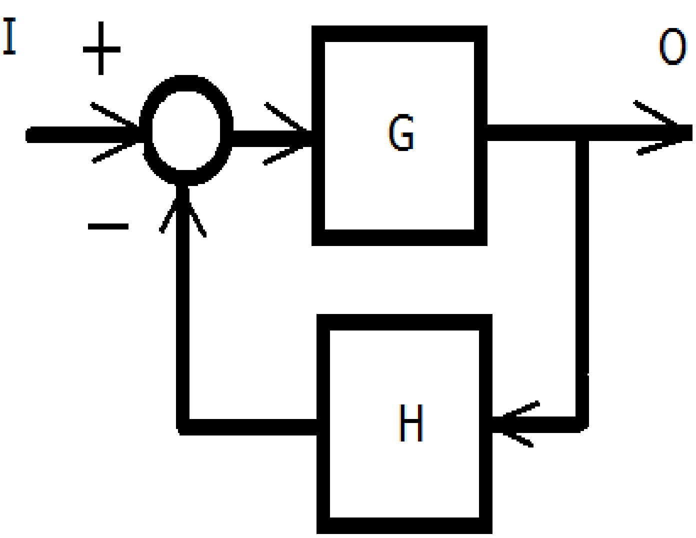
Error $E = C - R$

where C is the command position of the actuator rod, R is tht actual position, E is the position error, A is the piston area, F is the control flow, D is a leakage flow, B is a valve parameter, Q is the control signal and K is a proportional controller gain.

A block diagram representation of the system is given at the top of the next page. With C as input, this reduces to the standard form at the bottom of the page. The transfer function connecting R to C is

$$R/C = G/[1+GH] = Z/[S+Z] \qquad Z=KB/A$$


D
C
+
+
+
K
B
1/[AS]
R
-


I
+
G
O
–
H

For a step response this gives

$$R = Z/[S+Z] \;\; C_O/S$$

Partial Fraction Expansion gives

$$R = V/S + W/[S+Z]$$

$$= C_O/S \; - \; C_O/[S+Z]$$

Inverse Laplace Transformation gives

$$R = C_O - C_O \, e^{-Zt}$$

Let the parameters be: A=0.0005 B=0.0005

Let the command be C_O=0.1 and the gain be K=10.0.

In this case Z is 10 the time constant of the system is 0.1. The exact solution at time t=0.05 is

$$R = 0.1 - 0.1 \, e^{-10*0.05} = 0.0393$$

A simple time stepping scheme gives

$$A\ dR/dt = A\ \Delta R/\Delta t = A\ [R_{NEW} - R_{OLD}]/\Delta t = F_{OLD} - D_{OLD}$$

$$R_{NEW} = R_{OLD} + \Delta t\ [F_{OLD} - D_{OLD}]/A$$

$$F_{OLD} = B\ Q_{OLD} \quad Q_{OLD} = K\ E_{OLD}$$

$$E_{OLD} = C_{OLD} - R_{OLD}$$

$$R_{NEW} = R_{OLD} + \Delta t\ [\ KB/A\ [C_{OLD} - R_{OLD}] - D_{OLD}/A]$$

$$R_{NEW} = R_{OLD} + \Delta t\ [\ Z\ [C_{OLD} - R_{OLD}] - D_{OLD}/A]$$

For a single step to t=0.05 this gives

$$R_{NEW} = 0.0 + 0.05\ [\ 10\ [0.1 - 0.0]\ ] = 0.05$$

A more complex scheme goes across the step twice. The first step uses rates at the beginning of the step:

$$R_{ONE} = R_{OLD} + \Delta t\ [\ Z\ [C_{OLD} - R_{OLD}] - D_{OLD}/A]$$

$$R_{ONE} = 0.0 + 0.05\ [\ 10.0\ [0.1 - 0.0]\ ] = 0.05$$

The second step uses rates at the end of the step:

$$R_{TWO} = R_{OLD} + \Delta t\ [\ Z\ [C_{OLD} - R_{ONE}] - D_{OLD}/A]$$

$$R_{TWO} = 0.0 + 0.05\ [\ 10.0\ [0.1 - 0.05]\ \ ] = 0.025$$

The average of R_{ONE} and R_{TWO} gives R_{NEW}

$$R_{NEW} = [R_{ONE} + R_{TWO}]\ /2 = 0.0375$$

MATLAB m codes that perform the calculations are given below. They show that with small time steps the single step and double step give basically the same result. A SIMULINK Block Diagram for the actuator is also given below. It agrees with the m code predictions.

Imagine that the actuator is controlled by a computer that samples the sensor every n simulation time steps and gives out a control signal to the drive every m simulation time steps. A MATLAB M code for this case is given below. It shows that a slow computer can degrade performance.

Imagine that the actuator is on the Moon and is remotely controlled by a computer on the Earth. It takes approximately 1.25 seconds for the sensor signal to travel from the Moon to the Earth and for the drive signal to travel from the Earth to the Moon. A MATLAB M code for this case is given on the next page. It shows that that these transport lags can degrade performance.

```
%
% hydraulic actuator
%
clear all
ROLD=0.0;
LEAK=0.0;
COMMAND=0.1;
A=0.0005; B=0.0005;
GAIN=10.0; DELT=0.05;
NIT=10; TIME=0.0;
E(1)=0.0;R(1)=0.0;
for IT=2:NIT
TIME=TIME+DELT;
ERROR=COMMAND-ROLD;
SIGNAL=GAIN*ERROR;
FLOW=B*SIGNAL;
XYZ=FLOW+LEAK;
RNEW=ROLD+DELT*XYZ/A;
ROLD=RNEW;R(IT)=RNEW;
ABC=exp(-GAIN*B/A*TIME);
E(IT)=COMMAND*(1.0-ABC);
T(IT)=TIME;
end
plot(T,R,T,E)
xlabel('TIME')
ylabel('RESPONSE')
title('SINGLE')
```

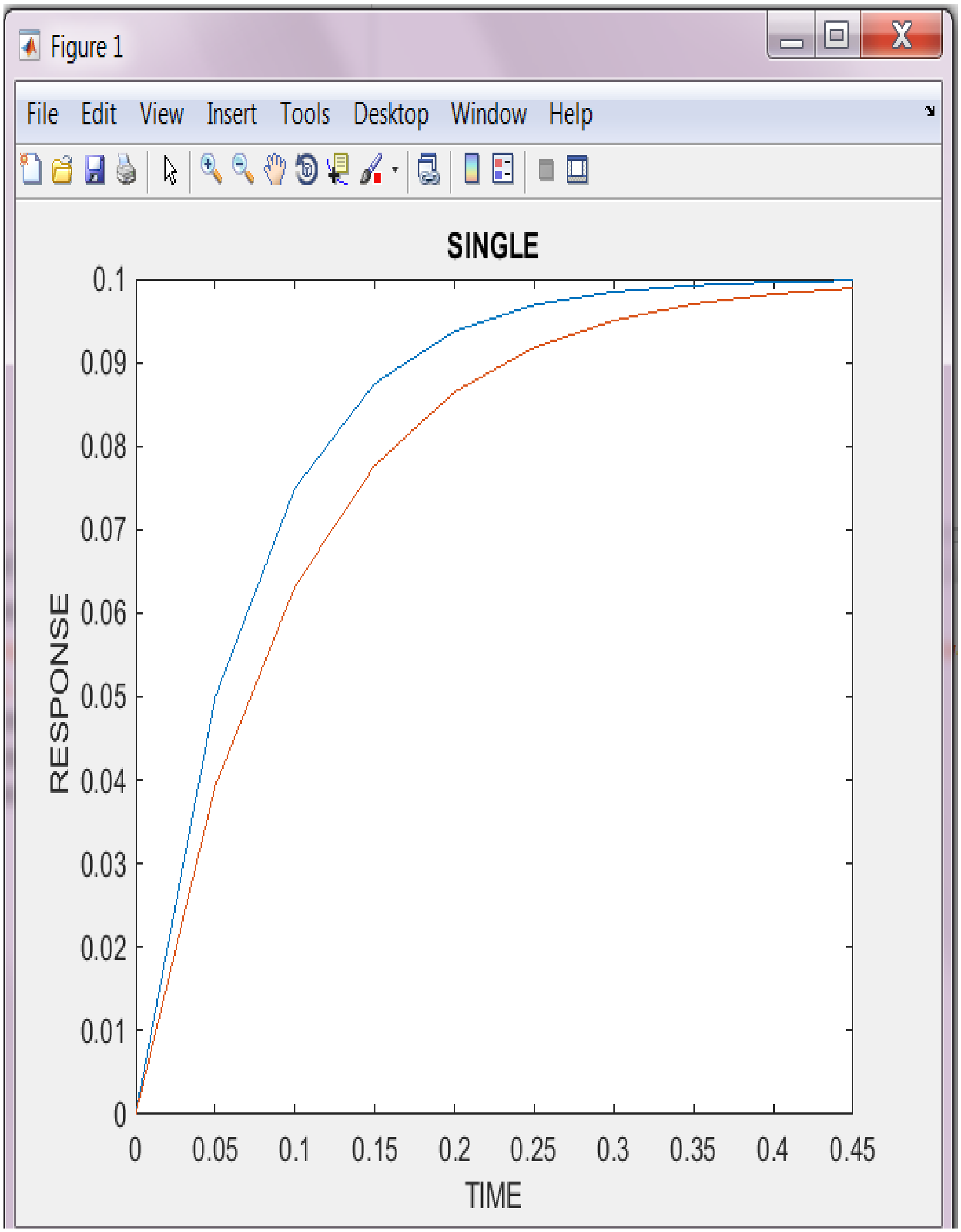


Figure 1
File Edit View Insert Tools Desktop Window Help
SINGLE
RESPONSE
TIME
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
0 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.1

```
%
% hydraulic actuator
%
clear all
ROLD=0.0;
LEAK=0.0;
COMMAND=0.1;
A=0.0005; B=0.0005;
GAIN=10.0; DELT=0.05;
NIT=10; TIME=0.0;
E(1)=0.0; R(1)=0.0;
for IT=2:NIT
TIME=TIME+DELT;
ERROR=COMMAND-ROLD;
SIGNAL=GAIN*ERROR;
FLOW=B*SIGNAL;
XYZ=FLOW+LEAK;
RONE=ROLD+DELT*XYZ/A;
ERROR=COMMAND-RONE;
SIGNAL=GAIN*ERROR;
FLOW=B*SIGNAL;
XYZ=FLOW+LEAK;
RTWO=ROLD+DELT*XYZ/A;
RNEW=(RONE+RTWO)/2.0;
ROLD=RNEW; R(IT)=RNEW;
ABC=exp(-GAIN*B/A*TIME);
E(IT)=COMMAND*(1.0-ABC);
T(IT)=TIME;
end
plot(T,R,T,E)
xlabel('TIME')
ylabel('RESPONSE')
title('DOUBLE')
```

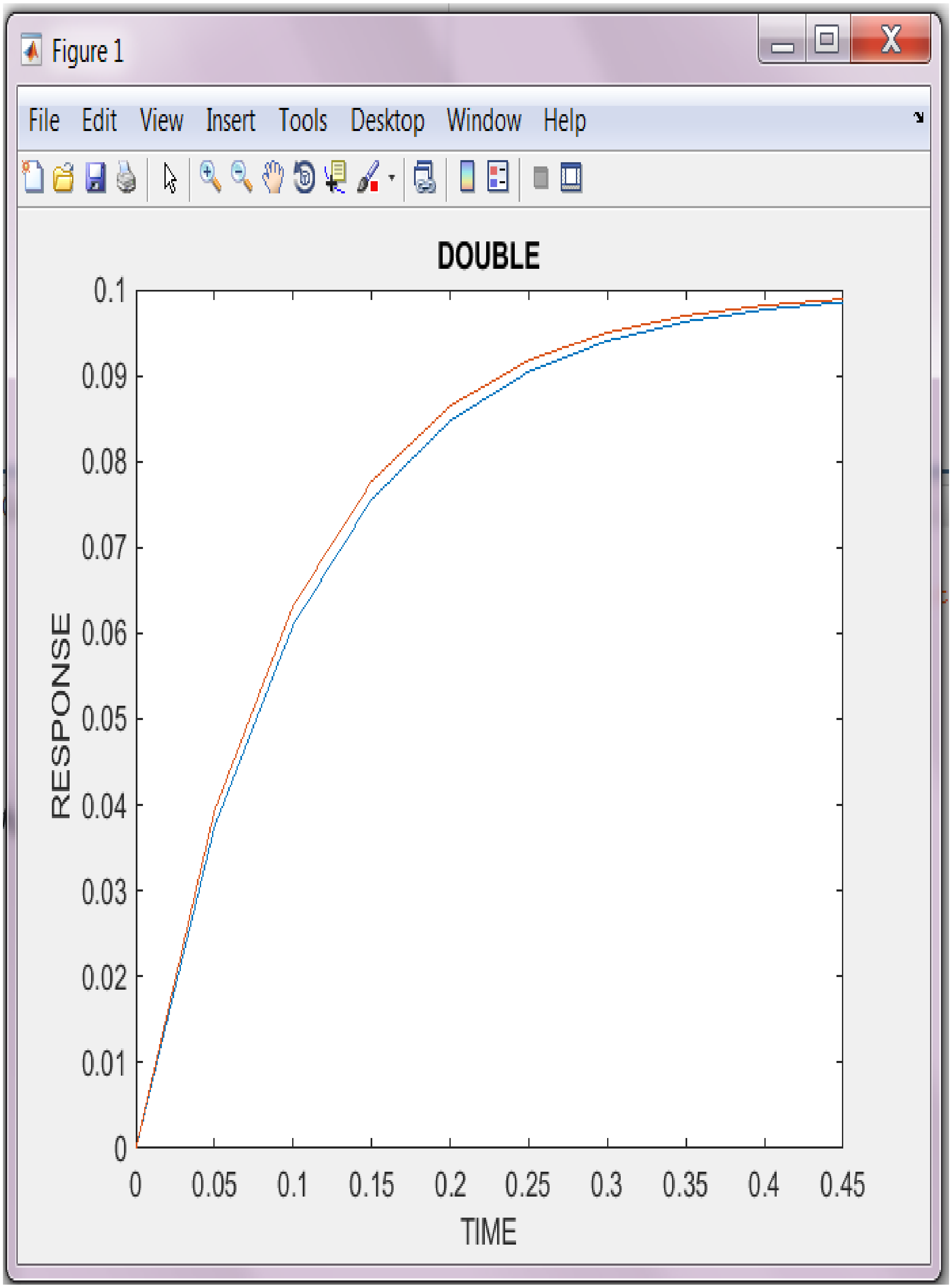


Figure 1
File Edit View Insert Tools Desktop Window Help
DOUBLE
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
RESPONSE
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
TIME

```
%
% hydraulic actuator
%
clear all
ROLD=0.0;
LEAK=0.0;
FLOW=0.0;
COMMAND=0.1;
E(1)=0.0; R(1)=0.0;
A=0.0005; B=0.0005;
GAIN=10.0; DELT=0.005;
NIT=100; MIT=0;
IS=2; ID=8;
TIME=0.0;
for IT=2:NIT
MIT=MIT+1;
TIME=TIME+DELT;
if(MIT==IS)
 SENSOR=ROLD; end;
if(MIT==ID)
ERROR=COMMAND-SENSOR;
SIGNAL=GAIN*ERROR;
FLOW=B*SIGNAL;
MIT=0; end;
XYZ=FLOW+LEAK;
RNEW=ROLD+DELT*XYZ/A;
ROLD=RNEW;R(IT)=RNEW;
ABC=exp(-GAIN*B/A*TIME);
E(IT)=COMMAND*(1.0-ABC);
T(IT)=TIME;
end
plot(T,R,T,E)
xlabel('TIME')
ylabel('RESPONSE')
title('DIGITAL')
```

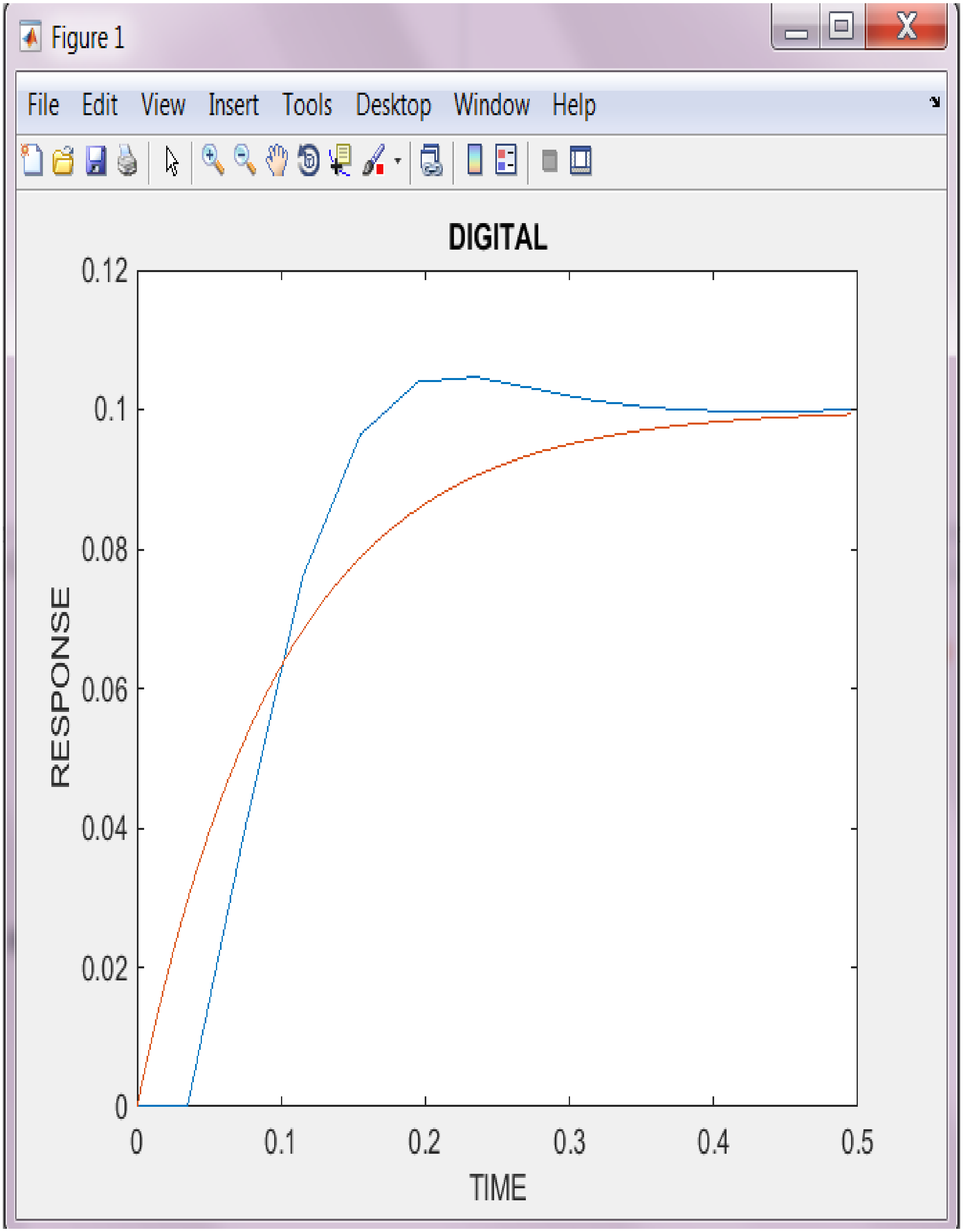


Figure 1
File Edit View Insert Tools Desktop Window Help
DIGITAL
0.12
0.1
0.08
0.06
0.04
0.02
0
RESPONSE
0
0.1
0.2
0.3
0.4
0.5
TIME

```
%
% hydraulic actuator
%
clear all
ROLD=0.0;
LEAK=0.0;
FLOW=0.0;
COMMAND=0.1;
SENSOR=ROLD;
A=0.0005; B=0.0005;
GAIN=10.0; DELT=0.005;
NIT=100; IS=8; ID=16;
IM=ID-IS; TIME=0.0;
E(1)=0.0; R(1)=0.0;
for IT=2:NIT
TIME=TIME+DELT;
if(IT>IS)
 SENSOR=R(IT-IS);
 S(IT)=SENSOR; end;
if(IT>ID)
ERROR=COMMAND-S(IT-IM);
SIGNAL=GAIN*ERROR;
FLOW=B*SIGNAL; end;
XYZ=FLOW+LEAK;
RNEW=ROLD+DELT*XYZ/A;
ROLD=RNEW;R(IT)=RNEW;
ABC=exp(-GAIN*B/A*TIME);
E(IT)=COMMAND*(1.0-ABC);
T(IT)=TIME;
end
plot(T,R,T,E)
xlabel('TIME')
ylabel('RESPONSE')
title('DELAY')
```

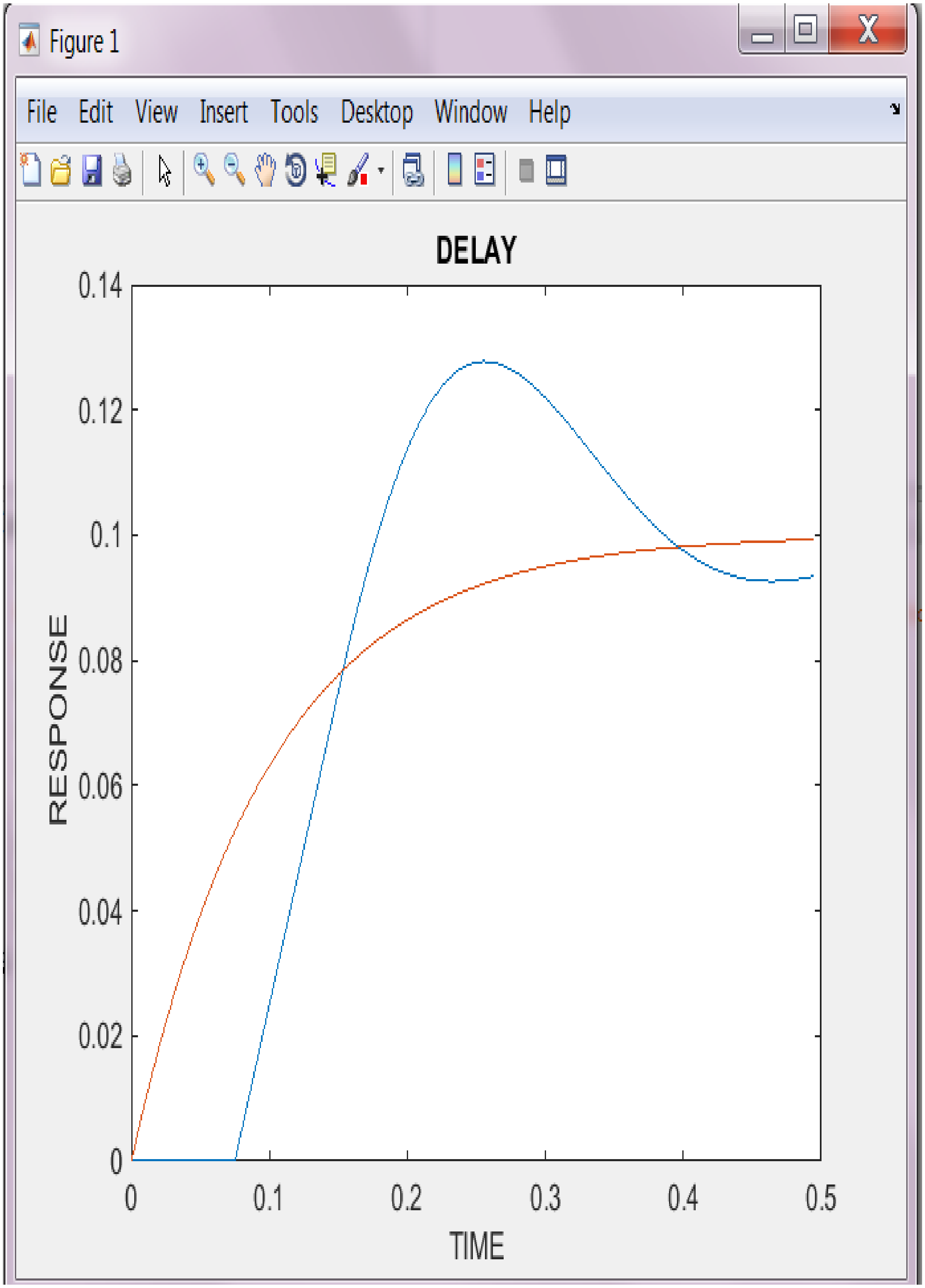
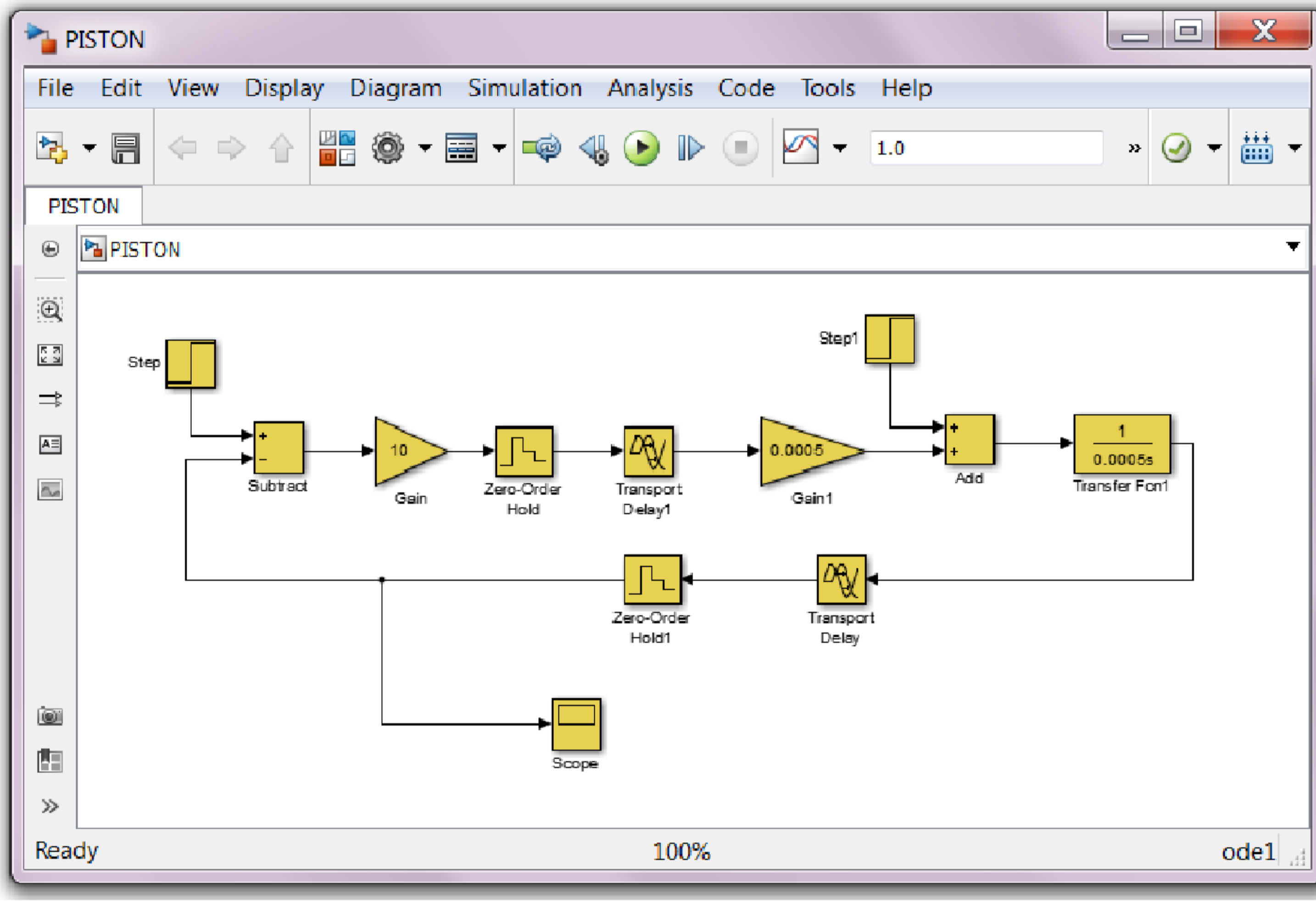


Figure 1
File Edit View Insert Tools Desktop Window Help
DELAY
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
RESPONSE
0
0.1
0.2
0.3
0.4
0.5
TIME


PISTON
File Edit View Display Diagram Simulation Analysis Code Tools Help
1.0
PISTON
PISTON
Step
Subtract
10
Gain
Zero-Order Hold
Transport Delay1
0.0005
Gain1
Step1
Add
1
0.0005s
Transfer Fcn1
Zero-Order Hold1
Transport Delay
Scope
Ready
100%
ode1