MEMORIAL UNIVERSITY OF NEWFOUNDLAND
FACULTY OF ENGINEERING AND APPLIED SCIENCE

FLUID MECHANICS II
ENGINEERING 5913

FINAL EXAMINATION

WEDNESDAY 15 APRIL 2009 — INSTRUCTOR
9:00 AM TO 12:00 NOON — M. HINCHEY

Give a TRUE or FALSE answer to each of the following statements and briefly explain each answer: (1) Shock waves cannot occur in water. (2) Cooling a subsonic pipe flow can make it choke (3) Small eddies in a flow make it appear more viscous locally. (4) The momentum thickness of a boundary layer is a measure of wake drag. (5) A single Mach wave cannot be heard. (6) A falling body cannot move faster than the speed of sound. (7) A wave spectrum is measure of wave energy. (8) Friction can make a supersonic pipe flow become subsonic. (9) An expansion shock wave where M goes suddenly from subsonic to supersonic is not possible. (10) Flow in hydrodynamic lubrication bearings is turbulent. [THIS QUESTION IS WORTH 15%: EACH QUESTION PART IS WORTH 1.5%]

When one integrates conservation of momentum for flow in a capillary tube, one gets a parabolic velocity profile at each point along the tube. When the velocity profile is integrated, one gets the following equation for flow:

$$Q = [\pi R^4]/\ [8\mu]\ \Delta[\rho g H]/[\Delta S]$$

where Q is flow, H is vertical head and ΔS is tube length. Describe an experiment based on this equation that would allow you find viscosity μ. [THIS QUESTION IS WORTH 10%]

A certain journal bearing for a ship has a radius 0.25m and a width 0.5m. It has edge gaps 0.8mm and 0.4mm. The viscosity of its oil is 0.1 Ns/m^2. Its RPM is 240. Develop a CFD template for points within the bearing. [5] Use it to get equations for pressure at two points within the bearing. Use Gauss Seidel iteration to estimate pressure at the two points. [5] Use the pressures to estimate the load. [5] Where is the minimum gap located? [5] Describe a lab setup for studying a journal bearing. [5] [THIS QUESTION IS WORTH 25%: EACH QUESTION PART IS WORTH 5%]

Consider a circle with radius R equal to 1m and both offsets m and n equal to zero. Map 2 points on the circle to a Joukowsky foil plane. [5] Calculate the theoretical lift on the foil when it is moving through air at a speed S equal to 50m/s and angle of attack θ equal to 10°. Assume that density ρ equals $1 kg/m^3$. [5] Estimate pressure at 2 points on the foil. [5] Estimate the lift due to these pressures. [5] How could pressure measurements at various points on a lab foil be used to estimate the flow speed at these points? [5] [THIS QUESTION IS WORTH 25%: EACH QUESTION PART IS WORTH 5%]

What would be the lift on a flat plate supersonic foil travelling at Mach Number M=3 that has chord C=1m, span W=2m and angle of attack $\Theta=25^\circ$? [16] Describe briefly the function of the two waves attached to the leading edge of the foil. [3] How would you determine lift when the Mach Number M=0.3? [3] How could the lift on a foil be measured in a shock tube? [3] [THIS QUESTION IS WORTH 25%: THE FIRST PART IS WORTH 16%; THE OTHER THREE PARTS ARE EACH WORTH 3%]

NAME :

BONUS QUESTION

[5 MARKS]

According to potential flow theory the suction pressure at the top of a gopher hole relative to the pressure far upstream is: $-5/4\ [\rho S^2]/2$ where S is the wind speed and ρ is the air density. The suction pulls air through burrows and provides ventilation. Assume you know the geometry of a burrow and its friction factor f and its minor loss factors ΣK. How would you get the flow rate through the burrow?

5913 FINAL EXAM 2009

Students were given the formula sheets prior to the exam to use as a study guide. They were expected to show proper procedure and to use correct formulas. They were not expected to give accurate numerical answers. During exam they were told to outline procedure steps before doing calculations.

TRUE OR FALSE QUESTION

1) False: shock waves can occur in any fluid that has a speed of sound: water has a speed of sound.

2) False: heating will make subsonic flow choke.

3) True: small eddies mean flow is turbulent: small eddies diffuse momentum which is what viscosity does.

4) False : momentum thickness is a measure of wall drag.

5) True: a single Mach wave is created by an infinitesimal disturbance so it creates infinitesimal sound.

6) False: a heavy spear like body would have insufficient drag to keep it from travelling faster than sound.

7) True: a wave spectrum shows how energy is distributed over a range of wave frequencies.

8) False: friction can make flow go sonic but not subsonic.

9) True: it would violate the second law otherwise.

10) False: small gap in bearing suppress eddies.

CAPILLARY TUBE QUESTION

Manipulation of flow equation gives

$$\mu = [\pi R^4]/\ [8Q]\ \Delta[\rho g H]/[\Delta S]$$

For a capillary tube with known radius R and length ΔS one can set the ΔH between its ends and measure the flow Q by collecting discharge in a bucket for a set period of time. Substitution into the last equation then gives μ.

JOURNAL BEARING QUESTION

CFD TEMPLATE

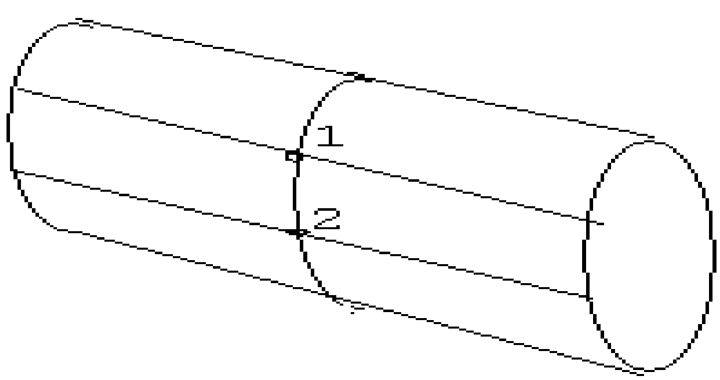
The curvature of the bearing does not influence the generation of pressure within the bearing. So we can roll the bearing out flat into a Cartesian geometry. The Cartesian Reynolds equation is:

$$\partial/\partial x \ (h^3/12\mu \ \partial P/\partial x) \ + \ \partial/\partial y \ (h^3/12\mu \ \partial P/\partial y)$$

$$= \ \partial[h(U_T+U_B)/2]/\partial x \ + \ \partial[h(V_T+V_B)/2]/\partial y \ + \ (W_T-W_B)$$

For the journal bearing this reduces to

$$\partial/\partial x \ (h^3 \ \partial P/\partial x) \ + \ \partial/\partial y \ (h^3 \ \partial P/\partial y) \ = \ 6\,\mu \ r\omega \ \partial h/\partial x$$



CFD applied to the first term in this PDE gives

$$[\ [(h_E+h_P)/2]^3\ \ [P_E-P_P]/\Delta x$$

$$-\ [(h_W+h_P)/2]^3\ \ [P_P-P_W]/\Delta x\ ]\ /\ \Delta x$$

CFD applied to the second term in the PDE gives

$$[\ [(h_N+h_P)/2]^3\ \ [P_N-P_P]/\Delta y$$

$$-\ [(h_S+h_P)/2]^3\ \ [P_P-P_S]/\Delta y\ ]\ /\ \Delta y$$

CFD applied to the last term in the PDE gives

$$6\mu\ \ r\omega\ \ (h_E-h_W)/[2\Delta x]$$

Substitution into the PDE gives:

$$P_P = \frac{(A\ P_E + B\ P_W + C\ P_N + D\ P_S + H)}{(A + B + C + D)}$$

where

$$A = [(h_E+h_P)/2]^3\ /\ [\Delta x]^2$$

$$B = [(h_W+h_P)/2]^3\ /\ [\Delta x]^2$$

$$C = [(h_N+h_P)/2]^3\ /\ [\Delta y]^2$$

$$D = [(h_S+h_P)/2]^3\ /\ [\Delta y]^2$$

$$H = -\ 6\mu\ S\ \ (h_E-h_W)/[2\Delta x]$$

Here Δx=0.26 and Δy=0.25 and $S=r\omega=2\pi=6.28$.

EQUATIONS FOR PRESSURE

Application of template at two points gives

$$P_1 = \frac{(A\ P_2 + H)}{(A + B + C + D)} = 0.18\ P_2 + 108989$$

where A=3.19 B=5.75 C=D=4.82 H=1957454.

$$P_2 = \frac{(B\ P_1 + H)}{(A + B + C + D)} = 0.34\ P_1 + 206265$$

where A=1.54 B=3.19 C=D=2.38 H=1957454.

The exact solution is: P_1 = 155642 P_2 = 259183

GAUSS SEIDEL ITERATION

Start iteration by assuming P_1 and P_2 are both zero. The first iteration gives:

$$P_1 = 0.18\ P_2 + 108989 = 108989$$

$$P_2 = 0.34\ P_1 + 206265 = 243321$$

The second iteration gives:

$$P_1 = 0.18\ P_2 + 108989 = 152786$$

$$P_2 = 0.34\ P_1 + 206265 = 258212$$

LOAD ON BEARING

The load due to each pressure is : $\Delta F = P\ \Delta x\ \ \Delta y$. These loads must be rolled back on to the bearing and broken up in to vertical and horizontal components:

$$\Delta F_V = \Delta F\ \text{Cos}\theta \qquad \qquad \Delta F_H = \Delta F\ \text{Sin}\theta$$

For point 1 angle θ is 120°: for point 2 it is 60°. Substitution into the component equations gives:

$$F_V = (155642*\text{Cos}120 + 259183*\text{Cos}60)*0.065 = 3365\ \text{N}$$

$$F_H = (155642*\text{Sin}120 + 259183*\text{Sin}60)*0.065 = 23351\ \text{N}$$

MINIMUM GAP LOCATION

The location of the total force relative to the bottom of the bearing based on components is:

$$\Theta = \tan^{-1}\ [F_H\ /\ F_V] = 82°$$

The total force must be vertical so the minimum gap must be located Θ up the opposite wall.

LAB SETUP

This could be a Cartesian setup like the wedge bearing used in the lubrication lab or it could be an actual journal bearing with pressure probes in the sleeve.

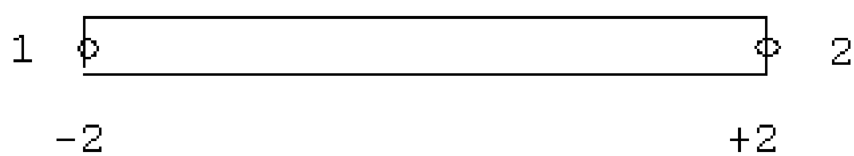
JOUKOWSKY FOIL QUESTION

MAPPING FROM CIRCLE TO FOIL PLANE

Points in the circle plane are mapped to foil plane using:

$$\alpha = x + xa^2/(x^2+y^2) \qquad \beta = y - ya^2/(x^2+y^2)$$

With n and m both zero the mapping gives a flat plate foil.



THEORETICAL LIFT

The theoretical lift is $\rho S\Gamma$ where Γ is the circulation needed to make the flow look realistic at the trailing edge of the foil. This circulation is $4\pi SR \operatorname{Sin}\Theta$.

$$\Gamma = 4\pi SR \operatorname{Sin}\Theta = 4\pi * 50 * 1 * \operatorname{Sin} 10 = 109$$

The theoretical lift is

$$\rho S\Gamma = 1 * 50 * 109 = 5450 \text{ N}$$

PRESSURE AT POINTS ON FOIL

The pressure midway between two points on the foil is:

$$\rho/2\ [\ S^2 - (\Delta\varphi/\Delta c)^2\ ]$$

where Δc is the distance between the points

$$\Delta c = \sqrt{[\Delta\alpha^2+\Delta\beta^2]} = 4$$

The difference in potential between the points is:

$$\Delta\varphi = 2\ S\ \Delta X + \Gamma/[2\pi]\ \Delta\sigma$$

where, for a flat plate, geometry gives

$$X = \mathbf{X}\ \mathrm{Cos}\Theta + \mathbf{Y}\ \mathrm{Sin}\Theta \qquad \mathbf{X} = x \qquad \mathbf{Y} = y$$

Substitution into these equations gives:

$$P_T = 1/2\ [\ 50^2 - [+251/4]^2\ ] = -719$$

$$P_B = 1/2\ [\ 50^2 - [-141/4]^2\ ] = +629$$

LIFT DUE TO PRESSURES

The lift due to pressures is given by:

$$\Sigma \ P\Delta c \ \mathrm{Sin}(\boldsymbol{\theta}-\Theta)$$

where $\boldsymbol{\theta}$ is the foil normal. This is 270° on top of the foil and 90° on the bottom of the foil. Substitution into the lift equation gives:

$$-719 * 4 * \mathrm{Sin}(270-10) + 629 * 4 * \mathrm{Sin}(90-10)$$
$$= 2832 + 2478 = 5310 \text{ N}$$

FLOW SPEED ON FOIL

Application of Bernoulli from a point far upstream of the foil to a point on the foil gives:

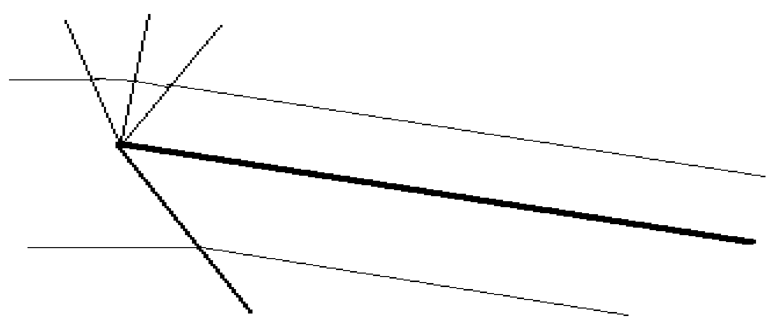
$$S^2/2 \ = \ C^2/2 \ + P/\rho$$

where C is the flow speed on the foil. Manipulation gives:

$$C \ = \sqrt{\ [S^2 \ - 2 \ P/\rho]}$$

SUPERSONIC FOIL QUESTION

The expansion wave plot would give the Mach number on top of the foil. Isentropic pressure ratio equation would then give pressure there. The oblique shock plot would give the shock angle. Substitution into the oblique shock pressure ratio equation would then give the pressure on the bottom of the foil. Lift is just pressure bottom minus pressure top times the chord times the Cos of the angle of attack.



The expansion wave plot gives υ=55 for upstream Mach Number M=3. Adding attack angle 25 to this gives υ=80 on top of the foil. The plot gives M=5.5 for top of foil. Substitution into the isentropic pressure ratio equation gives:

$$P_T/P_U = [(1 + (k-1)/2\ M_U M_U) \ / \ (1 + (k-1)/2\ M_T M_T)]^{(k/(k-1))}$$

$$P_T = P_U * (2.8/7.1)^{3.5} = 100000 * 0.039 = 3700$$

The oblique shock wave plot gives the shock angle β=45 for upstream Mach Number M=3 and attack angle θ=25. The normal Mach Number upstream is $N_U = M_U \sin\beta = 3 \sin 45 = 2.1$.

The oblique shock pressure ratio equation is:

$$P_B/P_U = 1 + 2k/(k+1)\ (N_U N_U - 1)$$

Substitution into this gives

$$P_B = P_U * 4.98 = 498000$$

The lift is

$$(P_B - P_T) * C * \cos\theta = (498000-3700) * 1 * \cos 25$$
$$= 447988 \text{ N}$$

The waves turn flow so that it is parallel to the plate. Oblique shock wave theory and expansion wave theory are both based on this fact. When crossing a wave the tangential component of speed does not change but the normal component changes to make the flow parallel.

One would use subsonic foil theory to estimate lift at M=0.3 because it is subsonic. This gives the lift: $\rho S \Gamma$.

By putting a pressure sensor on top and a pressure sensor on bottom of the foil one can measure pressures. Knowing pressures one can calculate lift as outlined above.

BONUS QUESTION

[5 MARKS]

According to potential flow theory the suction pressure at the top of a gopher hole relative to the pressure far upstream is: $- 5/4\ [\rho S^2]/2$ where S is the wind speed and ρ is the air density. The suction pulls air through burrows and provides ventilation. Assume you know the geometry of a burrow and its friction factor f and its minor loss factors ΣK. How would you get the flow rate through the burrow?

This question makes use of pipe flow friction theory. The head loss along a pipe is

$$h_L = (fL/D + \Sigma K)\ C^2/2g$$

It is also

$$h_L = [\ 5/4\ [\rho S^2]/2\ ]\ /\ [\rho g]$$

Manipulation gives the burrow speed C. Then flow rate Q equals C times A where A is the area of the burrow.