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ENGINEERING 6961

FLUID MECHANICS II

FINAL EXAMINATION

FALL 2011

MARKS IN SQUARE [] BRACKETS

INSTRUCTIONS

NO NOTES OR TEXTS ALLOWED

NO CALCULATORS ALLOWED

GIVE CONCISE ANSWERS

ASK NO QUESTIONS

Give a statement in words for each of the three conservation laws for fluid flow. [6]

Conservation of Mass: The time rate of change of the mass of an arbitrary specific group of fluid particles in a flow is zero.

Conservation of Momentum: The time rate of change of the momentum of an arbitrary specific group of fluid particles in a flow is equal to the net force acting on the group. The forces acting on the group can be of two types: body forces and surface forces. Body forces are generally due to gravity. Surface forces are due to pressure and viscous traction.

Conservation of Energy: The time rate of change of the energy of an arbitrary specific group of fluid particles in a flow is equal to the net work done on the group by the surroundings plus the net heat flux into the group from the surroundings.

The integral form of the conservation laws is given below. Identify each law. [3] The PDE form of each law is on the next page. Identify each PDE. [3]

CONSERVATION OF MASS

$$\frac{D}{Dt} \int_{V(t)} \rho \, dV = 0$$

$$\int_{V(t)} \left[\partial\rho/\partial t + \nabla.(\rho \mathbf{v}) \right] dV = 0$$

CONSERVATION OF MOMENTUM

$$\frac{D}{Dt} \int_{V(t)} \rho \mathbf{v} \, dV = \int_{S(t)} \boldsymbol{\sigma} \, dS + \int_{V(t)} \rho \mathbf{b} \, dV$$

$$\int_{V(t)} \left[\partial(\rho \mathbf{v})/\partial t + \nabla.(\rho \mathbf{v}\mathbf{v}) \right] dV = \int_{S(t)} \boldsymbol{\sigma} \, dS + \int_{V(t)} \rho \mathbf{b} \, dV$$

CONSERVATION OF ENERGY

$$\frac{D}{Dt} \int_{V(t)} \rho e \, dV = - \int_{S(t)} \mathbf{q.n} \, dS + \int_{S(t)} \mathbf{v}.\boldsymbol{\sigma} \, dS$$

$$e = u + \mathbf{v.v}/2 + gz$$

$$\int_{V(t)} \left[\partial(\rho e)/\partial t + \nabla.(\rho e \mathbf{v}) \right] dV = - \int_{S(t)} \mathbf{q.n} \, dS + \int_{S(t)} \mathbf{v}.\boldsymbol{\sigma} \, dS$$

CONSERVATION OF MASS

$$\partial U/\partial x + \partial V/\partial y + \partial W/\partial z = 0$$

CONSERVATION OF MOMENTUM

$$\rho\partial U/\partial t + \rho\ (U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z) = -\ \partial P/\partial x$$

$$+\ \mu\ (\partial^2 U/\partial x^2 + \partial^2 U/\partial y^2 + \partial^2 U/\partial z^2)$$

$$\rho\partial V/\partial t + \rho\ (U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z) = -\ \partial P/\partial y$$

$$+\ \mu\ (\partial^2 V/\partial x^2 + \partial^2 V/\partial y^2 + \partial^2 V/\partial z^2)$$

$$\rho\partial W/\partial t + \rho\ (U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z) = -\ \partial P/\partial z - \rho g$$

$$+\ \mu\ (\partial^2 W/\partial x^2 + \partial^2 W/\partial y^2 + \partial^2 W/\partial z^2)$$

CONSERVATION OF ENERGY

$$\rho C\ \partial T/\partial t + \rho C\ (U\partial T/\partial x + V\partial T/\partial y + W\partial T/\partial z) = \mu\ \Phi$$

$$+\ \partial/\partial x(k\partial T/\partial x) + \partial/\partial y(k\partial T/\partial y) + \partial/\partial z(k\partial T/\partial z)$$

The PDEs and AEs for Turbulent Wake Flows are given on the next few pages. Identify each PDE and AE. [6] Explain briefly how CFD can be used to get flows step by step in time. [3]

CONSERVATION OF MOMENTUM

$$\rho \left(\partial U/\partial t + U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z \right) + A = - \partial P/\partial x$$
$$+ \left[\partial/\partial x \left(\mu \, \partial U/\partial x\right) + \partial/\partial y \left(\mu \, \partial U/\partial y\right) + \partial/\partial z \left(\mu \, \partial U/\partial z\right) \right]$$

$$\rho \left(\partial V/\partial t + U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z \right) + B = - \partial P/\partial y$$
$$+ \left[\partial/\partial x \left(\mu \, \partial V/\partial x\right) + \partial/\partial y \left(\mu \, \partial V/\partial y\right) + \partial/\partial z \left(\mu \, \partial V/\partial z\right) \right]$$

$$\rho \left(\partial W/\partial t + U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z \right) + C = - \partial P/\partial z - \rho g$$
$$+ \left[\partial/\partial x \left(\mu \, \partial W/\partial x\right) + \partial/\partial y \left(\mu \, \partial W/\partial y\right) + \partial/\partial z \left(\mu \, \partial W/\partial z\right) \right]$$

CONSERVATION OF MASS

$$\partial P/\partial t + \rho \, c^2 \left(\partial U/\partial x + \partial V/\partial y + \partial W/\partial z \right) = 0$$

WATER SURFACE TRACKER

$$\partial F/\partial t + U\partial F/\partial x + V\partial F/\partial y + W\partial F/\partial z = 0$$

TURBULENCE KINETIC ENERGY

$$\partial k/\partial t + U\partial k/\partial x + V\partial k/\partial y + W\partial k/\partial z = T_P - T_D$$
$$+ \left[\partial/\partial x \left(\mu/a \, \partial k/\partial x\right) + \partial/\partial y \left(\mu/a \, \partial k/\partial y\right) + \partial/\partial z \left(\mu/a \, \partial k/\partial z\right) \right]$$

TURBULENCE DISSIPATION RATE

$$\partial \varepsilon/\partial t + U\partial \varepsilon/\partial x + V\partial \varepsilon/\partial y + W\partial \varepsilon/\partial z = D_P - D_D$$
$$+ \left[\partial/\partial x \left(\mu/b \, \partial \varepsilon/\partial x\right) + \partial/\partial y \left(\mu/b \, \partial \varepsilon/\partial y\right) + \partial/\partial z \left(\mu/b \, \partial \varepsilon/\partial z\right) \right]$$

PRODUCTION AND DISSIPATION

$$T_P = G \mu_t / \rho \qquad D_P = T_P C_1 \varepsilon / k$$

$$T_D = C_D \varepsilon \qquad D_D = C_2 \varepsilon^2 / k$$

VISCOSITIES

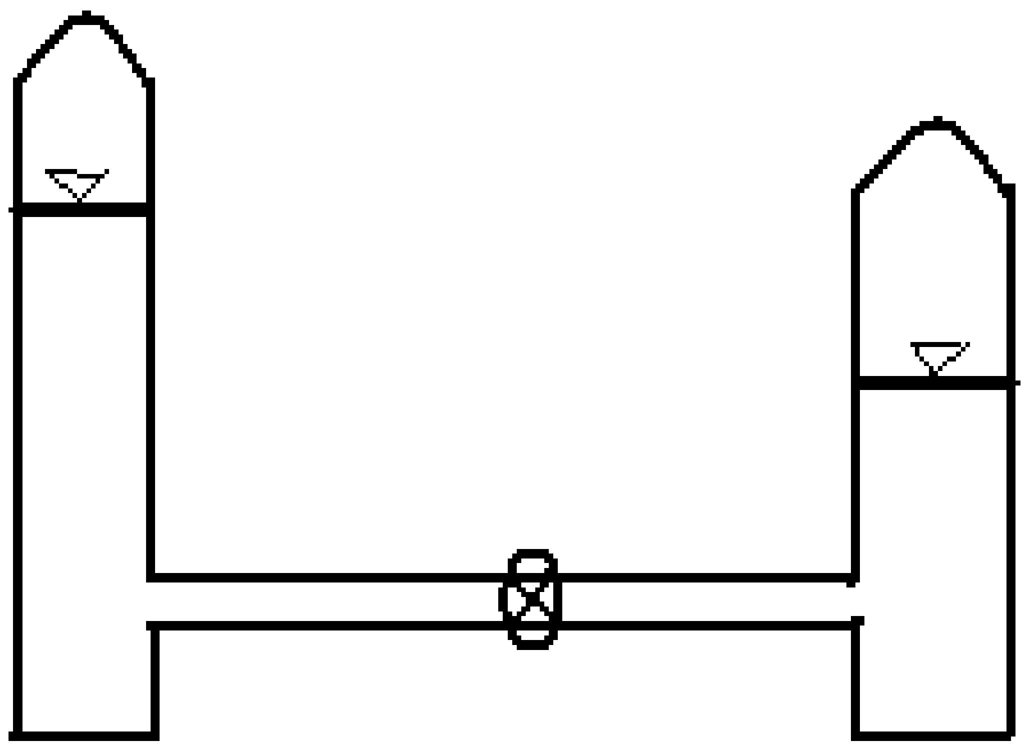
$$\mu_t = C_3 k^2 / \varepsilon \qquad \mu = \mu_t + \mu_l$$

CFD TEMPLATE

$$\partial M/\partial t = N \qquad M_{NEW} = M_{OLD} + \Delta t \, N_{OLD}$$

The region of interest is divided by a CFD grid. CFD cells surround each point where grid lines cross. Each PDE is put into the form: $\partial M/\partial t = N$. The template $M_{NEW} = M_{OLD} + \Delta t \, N_{OLD}$ is applied to each PDE at each point in the grid to get flows step by step in time. Finite differences are used to approximate the various derivatives in N. Central differences are used to approximate the diffusion terms. Upwind differences are used to approximate the convective terms. The eddy viscosity concept is used to model turbulence. The volume of fluid concept is used to track the water surface. The function F is 1 inside water and 0 outside it: cells with F between 1 and 0 contain the water surface. The Semi Implicit Method for Pressure Linked Equations or SIMPLE procedure is used to update pressure and correct velocities so that they satisfy mass and momentum.

Consider a pipe with a large pressurized tank at its upstream end and a large pressurized tank at its downstream end. Midway along its length is a valve. Using wave reflection concepts, briefly explain what happens inside the pipe downstream of the valve when there is a sudden valve closure. [5]



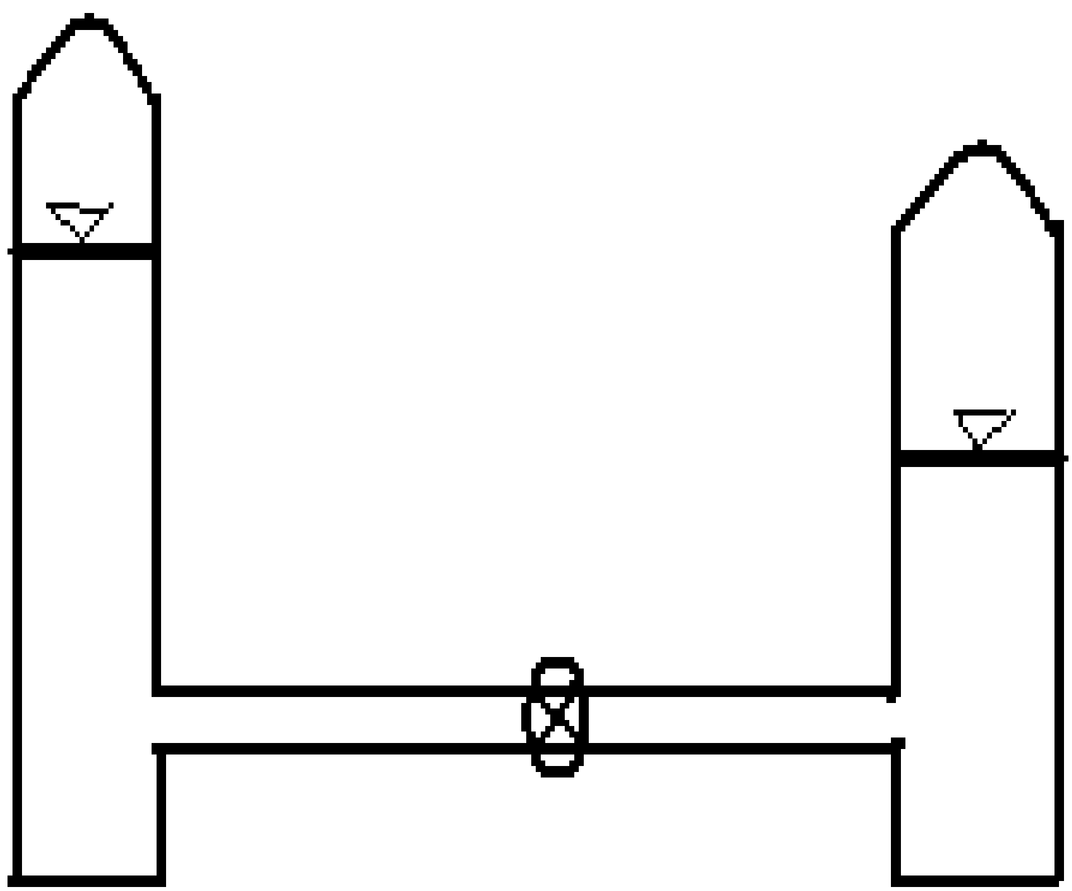
When the valve is suddenly closed, it creates a flow imbalance. A low pressure or suction wave is created which propagates down the pipe. As it does so, it brings the fluid to rest. The pipe has low pressure all along its length.

When the suction wave reaches the tank, it creates a pressure imbalance. A backflow wave is created. This propagates up the pipe restoring pressure everywhere to its original level.

When the backflow wave reaches the valve, it creates a flow imbalance. This causes a high pressure or surge wave to propagate down the pipe. As it does so, it brings the fluid to rest. The pipe has high pressure all along its length.

When the surge wave reaches the tank, it creates a pressure imbalance. An outflow wave is created. This travels up the pipe restoring pressure to its original level. Conditions in the pipe become what they were just before the valve was closed.

Consider a pipe with a large pressurized tank at its upstream end and a large pressurized tank at its downstream end. Midway along its length is a valve. Assume $P_o=2$ $U_o=1$ $\rho a=1$. Using the method of characteristics OR algebraic waterhammer analysis, calculate the pressure and flow velocity at the ends of the downstream pipe following a sudden valve closure. Do 4 transits of pipe. [8] Sketch the PU plot for the downstream pipe. [4]



ALBRBRAIC WATERMAMMER ANALYSIS

Move along an F wave from T to V. For an F wave:

$$\Delta P = + \rho a \ \Delta U$$

$$[P_V - P_T] = + \rho a \ [U_V - U_T]$$

In this equation, $U_V=0$; $P_T=P_O$; $U_T=U_o$. It gives P_V.

Move along an f wave from V to T. For an f wave:

$$\Delta P = - \rho a \ \Delta U$$

$$[P_T - P_V] = - \rho a \ [U_T - U_V]$$

In this equation, $U_V=0$; $P_T=P_o$; P_V known. It gives U_T.

METHOD OF CHARACTERISTICS ANALYSIS

At the tank, the C^+ characteristic gives

$$U_K - U_B + (P_K - P_B)/[\rho a] = 0$$

$$P_K = + [\rho a] \ U_B + P_B$$

At the valve, the C^- characteristic gives

$$U_I - U_B - (P_I-P_B)/[\rho a] = 0$$

$$U_I = U_B + (P_I-P_B)/[\rho a]$$

At the middle, the C^+ and C^- characteristics give

$$U_J = 0.5 \ [\ U_A + U_C + [P_A-P_C]/[\rho a] \ ]$$

$$P_J = 0.5 \ [P_A + P_B + [\rho a][U_A-U_C] \ ]$$

Here there is no middle point and the method of characteristics equations reduce to the algebraic waterhammer equations.

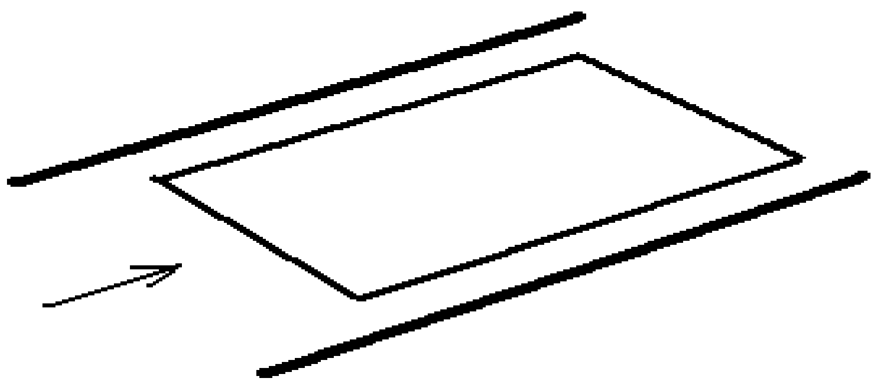
STEP#1 $[P_V - P_T] = + \rho a \ [U_V - U_T]$ $P_V = +1$

STEP#2 $[P_T - P_V] = - \rho a \ [U_T - U_V]$ $U_T = -1$

STEP#3 $[P_V - P_T] = + \rho a \ [U_V - U_T]$ $P_V = +3$

STEP#4 $[P_T - P_V] = - \rho a \ [U_T - U_V]$ $U_T = +1$

A certain low RE flow passageway has a 2 by 2 Cartesian geometry and a gap h=1 throughout. Its sides are blocked. The pressure P at its front entrance is 3 and at its back exit is 1. The speed of its moving surface is S=1. Its oil has viscosity μ=1. What is the pressure at the midpoint of the passageway? [8]



This is a 1D problem. The governing equation is:

$$d/dx\ (h^3\ dP/dx) = 6\mu\ S\ dh/dx = H\ dh/dx$$

For a constant gap, this reduces to:

$$dP/dx = A$$

Integration gives:

$$P = A\ x + B$$

$$= [P_I - P_O]\ [x/d] + P_O$$

$$= [3-1]\ [d/2]/d + 1 = 2$$

A certain Joukowsky foil is obtained by mapping a circle with radius R=0.5 and offsets m=0 and n=0 to a foil plane. It has an angle of attack $\Theta=0$. It is moving through a fluid with $\rho=1$ at a speed S=1. What is the theoretical lift on the foil? [3] Map 4 points on the circle to the foil plane. [4] Calculate the pressure midway between consecutive points on the foil. [4] Calculate the lift from the calculated pressures. [4]

The theoretical lift is: $\rho S \Gamma$. The circulation is:

$$\Gamma = 4\pi SR \, \mathrm{Sin}\kappa$$

$$\kappa = \Theta + \varepsilon \qquad \varepsilon = \tan^{-1}[m/(n+a)]$$

Here Θ and ε are both zero. So Γ is zero and lift is zero.

Geometry gives:

$$\alpha = x + xa^2/(x^2+y^2) \qquad \beta = y - ya^2/(x^2+y^2)$$

$$x = \mathbf{X} - n \qquad y = \mathbf{Y} + m$$

$$\mathbf{X} = -R\,\mathrm{Cos}\Upsilon \qquad \mathbf{Y} = +R\,\mathrm{Sin}\Upsilon$$

Substitution into this gives a flat plate foil:

#1 $\alpha=-1$ $\beta=0$ #2 $\alpha=0$ $\beta=0$

#3 $\alpha=+1$ $\beta=0$ #4 $\alpha=0$ $\beta=0$

An application of the Bernoulli equation from a point far upstream of the foil to a point on the foil gives:

$$P = \rho/2\ [\ S^2 - (\partial\varphi/\partial c)^2\ ]$$

$$= \rho/2\ [\ S^2 - (\Delta\varphi/\Delta c)^2\ ]$$

Potential flow theory gives:

$$\Delta\varphi = 2S\ \Delta X + \Gamma/[2\pi]\ \Delta\sigma$$

Geometry gives:

$$\Delta c = \sqrt{[\Delta\alpha^2+\Delta\beta^2]}$$

$$X = \mathbf{X}\ \mathrm{Cos}\Theta + \mathbf{Y}\ \mathrm{Sin}\Theta$$

Substitution into this shows that $\Delta\varphi/\Delta c$ is equal to S at each point on the foil: so P is zero at each point.

The incremental lift on the foil is:

$$\Delta L = P\Delta c\ \mathrm{Sin}(\boldsymbol{\theta}-\Theta) = 0$$

The foil normal $\boldsymbol{\theta}$ is:

$$\boldsymbol{\theta} = \tan^{-1}[-\Delta\alpha/+\Delta\beta]$$

The total lift L is:

$$L = \Sigma\Delta L = 0$$

Explain how you would calculate the supersonic drift speed behind the shock wave generated by an explosion [7] and the pressure and temperature at the stagnation point of a blunt object in the flow. [7] Identify the important equations.

Assume that the pressure ratio across the shock wave generated by the explosion is known. In the shock frame, air upstream moves towards the shock at supersonic speed and air downstream moves away from it at subsonic speed. The pressure ratio equation gives M_U:

$$P_D/P_U = 1 + [2k/(k+1)]\ (M_U M_U - 1)$$

The wave speed equation gives a_U:

$$a_U = \sqrt{[kRT_U]}$$

The Mach Number equation gives U_U:

$$M_U = U_U/a_U$$

The Mach Number connection gives M_D:

$$M_D M_D = [(k-1)\ M_U M_U + 2]\ /\ [2k\ M_U M_U - (k-1)]$$

The temperature ratio equation gives T_D:

$$T_D/T_U = [\ (1 + [(k-1)/2]\ M_U M_U)\ /\ (1 + [(k-1)/2]\ M_D M_D)\ ]$$

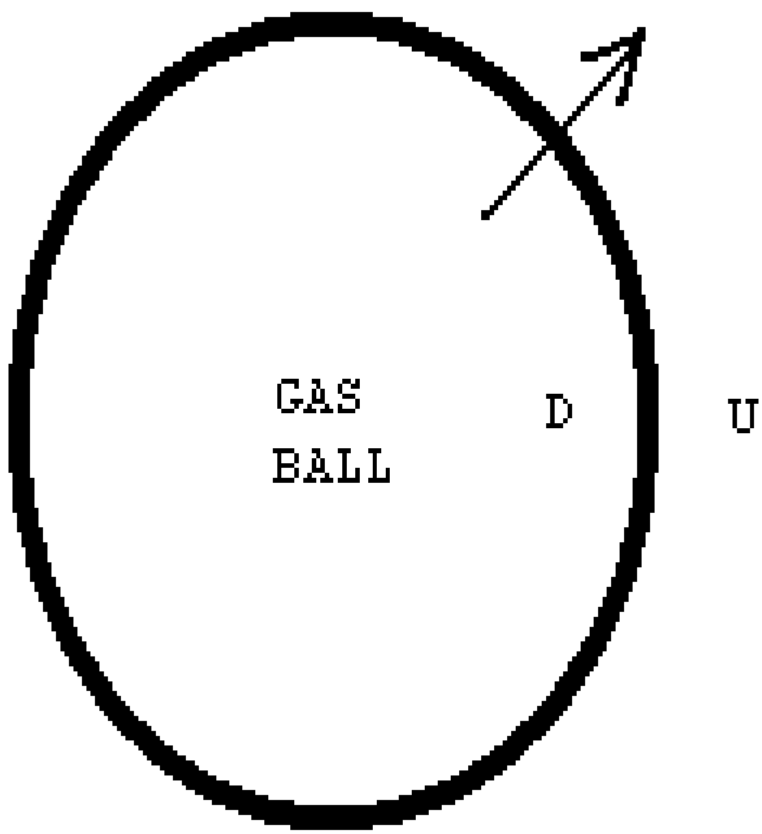
The wave speed equation gives a_D:

$$a_D = \sqrt{[kRT_D]}$$

The Mach Number equation gives U_D:

$$M_D = U_D/a_D$$

The drift speed is U_U minus U_D.



A bow shock wave forms directly in front of a blunt object in a supersonic flow. Let A be just upstream of the shock and B be just downstream. At the stagnation point on the object, M_S is zero. Conditions at A are known.

The temperature ratio equation based on energy gives the stagnation point temperature T_S:

$$T_S/T_A = [\ (1 + [(k-1)/2]\ M_A M_A)\ /\ (1 + [(k-1)/2]\ M_S M_S)\ ]$$

The pressure ratio equation gives the pressure downstream of the bow shock wave P_B:

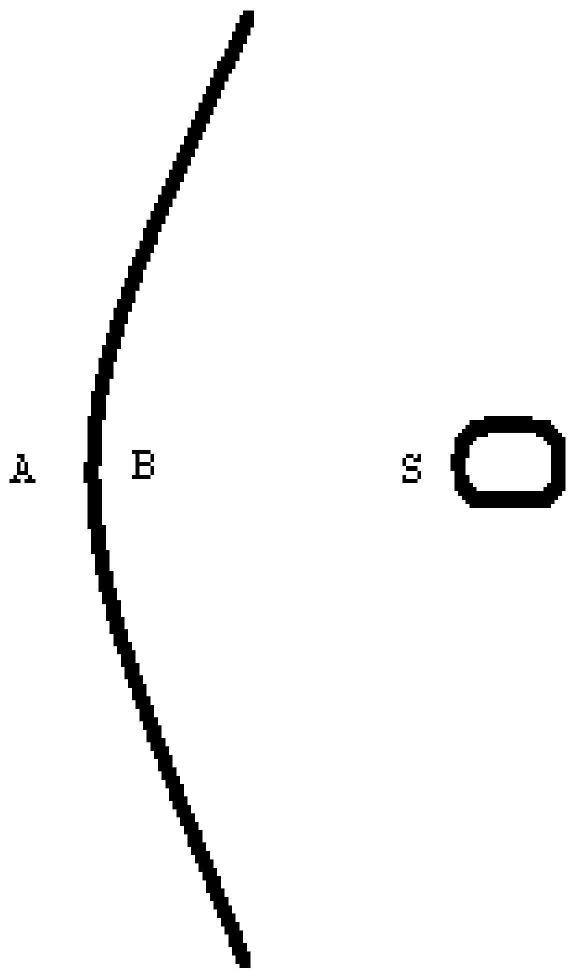
$$P_B/P_A = 1 + [2k/(k+1)]\ (M_A M_A - 1)$$

The Mach Number connection gives M_B:

$$M_B M_B = [(k-1)\ M_A M_A + 2]\ /\ [2k\ M_A M_A - (k-1)]$$

The isentropic pressure ratio equation gives P_S:

$$P_S/P_B = [\ (1 + [(k-1)/2]\ M_B M_B)\ /\ (1 + [(k-1)/2]\ M_S M_S)\ ]^x$$


A
B
S

PROCESS: Explain how you would calculate the pressure and flow velocity changes which occur when a high speed flow moves down a pipe. Identify the important equations. [8]

Gas dynamics theory gives

$$\Delta M^2/M^2 = kM^2[1+[(k-1)/2]M^2] /[1-M^2] f\Delta x/D$$

$$\Delta P/P = -kM^2[1+(k-1) M^2]/[2(1-M^2)] f\Delta x/D$$

$$\Delta T/T = -k(k-1)M^4/[2(1-M^2)] f\Delta x/D$$

These can be put into the form:

$$\Delta M^2 = A \Delta x$$

$$\Delta P = B \Delta x$$

$$\Delta T = C \Delta x$$

For moving a step down a pipe, these equations give:

$$[M^2]_{NEW} = [M^2]_{OLD} + A_{OLD} \Delta x$$

$$P_{NEW} = P_{OLD} + B_{OLD} \Delta x$$

$$T_{NEW} = T_{OLD} + C_{OLD} \Delta x$$

Note that A B C is each a function of M^2 so they must be updated after each step down the pipe.

MECHANICAL: Explain how you would calculate the lift on a supersonic diamond foil. Identify the important equations. [8]

The supersonic foil has waves as shown in the sketch.

With known upstream Mach Number M_U and local attack angle ε, the oblique shock plot gives the shock angle β. Substitution into the normal Mach Number equation gives N_U:

$$N_U = M_U \ \mathrm{Sin}\beta \qquad N_D = M_S \ \mathrm{Sin}\,\kappa$$

Substitution into the pressure ratio equation gives P_B:

$$P_D/P_U = 1 + [2k/(k+1)]\ (N_U N_U - 1)$$

$$N_D N_D = [(k-1)\ N_U N_U + 2]\ /\ [2k\ N_U N_U - (k-1)]$$

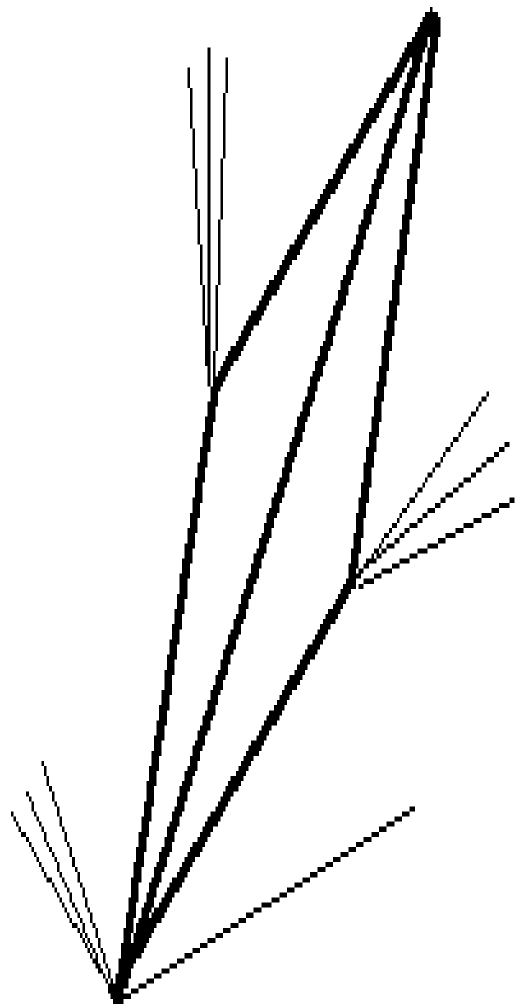
With known upstream Mach Number M_U and local attack angle ε, the expansion wave plot gives M_D for each expansion. The isentropic pressure ratio equation then gives P_D.

$$T_D/T_U = [\ (1 + [(k-1)/2]\ M_U M_U)\ /\ (1 + [(k-1)/2]\ M_D M_D)\ ]$$

$$P_D/P_U = [T_D/T_U]^x \qquad x = k/(k-1)$$

The lift is:

$$\Sigma\ P_B\ C\ \mathrm{Cos}\,\boldsymbol{\theta}\ -\ \Sigma\ P_T\ C\ \mathrm{Cos}\,\boldsymbol{\theta}$$



LAB QUESTIONS [9]

DO ANY 3 OUT OF 4 QUESTIONS

In the shock tube lab you used two high speed or ballistic pressure sensors to measure pressure at two points along the tube spaced a known distance apart. Explain how you would determine the Mach Number of the shock wave from just one pressure sensor oscilloscope trace and theory. [3] Speculate on what would happen to the shock wave pressure traces on the oscilloscope if the exit from the tube was blocked. [3]

In the foil lab, you measured pressure at points on the foil. You also measured the static pressure and the flow speed far upstream of the foil. Explain how you would determine the speed of the flow at points on the foil from these measurements. [3]

In the waterhammer lab, you used a pressure sensor to measure the pressure generated by a sudden valve closure. How could the pressure trace be used to calibrate the sensor? Assume that the pressure transient causes the water to vaporize. [3]

Shock Tube Lab: Explain how you would determine the Mach Number of the shock wave from just one pressure sensor oscilloscope trace and theory. Substitute the pressure ratio measured by the sensor into the pressure ratio equation for a normal shock wave that contains only the upstream Mach Number.

Shock Tube Lab: Speculate on what would happen to the shock wave pressure traces on the oscilloscope if the exit from the tube was blocked. The pressure traces would remain the same. The initial shock wave that goes down the tube is not affected by the boundary condition at the exit until it hits it.

Joukowsky Foil Lab: Explain how you would determine the speed of the flow at points on the foil from lab measurements. Application of Bernoulli from a point far upstream to any point on the foil gives an equation for the speed there.

Waterhammer Lab: How could the pressure trace be used to calibrate the sensor? The pressure on the sensor when there is no flow is known. The pressure when there is vaporization is also known. By recording the voltages at these states, the sensor can be calibrated: $CF = \Delta P/\Delta V$.

IDENTIFY AND OUTLINE THE FOLLOWING DERIVATION [4]

REYNOLDS EQUATION FOR PRESSURE

Conservation of Mass is:

$$\partial U/\partial x + \partial V/\partial y + \partial W/\partial z = 0$$

Simplified Conservation of Momentum is:

$$\partial P/\partial x = \mu \partial^2 U/\partial z^2$$

$$\partial P/\partial y = \mu \partial^2 V/\partial z^2$$

$$0 = \mu \partial^2 W/\partial z^2$$

Integration of mass gives:

$$\int [\partial U/\partial x + \partial V/\partial y + \partial W/\partial z]\, dz = 0$$

Manipulation gives:

$$\partial I/\partial x + \partial J/\partial y + K = 0$$

$$I = \int U dz \qquad J = \int V dz \qquad K = \int \partial W/\partial z\, dz$$

Double integration of momentum gives:

$$\partial P/\partial x \ (z^2 - zh)/2\mu + (U_T - U_B) z/h + U_B$$

$$\partial P/\partial y \ (z^2 - zh)/2\mu + (V_T - V_B) z/h + V_B$$

$$(W_T - W_B) z/h + W_B$$

Substitution into the I J K equations gives:

$$\partial P/\partial x \ (-h^3/12\mu) + (U_T - U_B) h/2 + U_B h$$

$$\partial P/\partial y \ (-h^3/12\mu) + (V_T - V_B) h/2 + V_B h$$

$$W_T - W_B$$

Substitution into integrated mass gives:

$$\partial/\partial x \ (h^3/12\mu \ \partial P/\partial x) + \partial/\partial y \ (h^3/12\mu \ \partial P/\partial y)$$

$$= \partial[h(U_T + U_B)/2]/\partial x + \partial[h(V_T + V_B)/2]/\partial y + (W_T - W_B)$$

This is Reynolds Equation for Pressure.

IDENTIFY AND OUTLINE THE FOLLOWING DERIVATION [4]

ALGRBRAIC WATERHAMMER

Conservation of Momentum is:

$$\rho\ \partial U/\partial t + \rho U\ \partial U/\partial x + \partial P/\partial x - \rho\ C = 0$$

$$C = f/D\ U|U|/2 - g\ \mathrm{Sin}\alpha$$

Conservation of Mass is:

$$\partial P/\partial t + U\ \partial P/\partial x + \rho a^2\ \partial U/\partial x = 0$$

Simplification gives:

$$\rho\ \partial U/\partial t + \partial P/\partial x = 0$$

$$\partial P/\partial t + \rho a^2\ \partial U/\partial x = 0$$

Manipulation gives the wave equations:

$$\partial^2 P/\partial t^2 = a^2\ \partial^2 P/\partial x^2$$

$$\partial^2 U/\partial t^2 = a^2\ \partial^2 U/\partial x^2$$

The General Solutions are:

$$P - P_o = f(N) + F(M)$$

$$U - U_o = [f(N) - F(M)] / \rho a$$

where N and M are wave fixed frames

$$M = t + x/a \qquad N = t - x/a$$

Multiplication of the U equation by ρa followed by subtraction from the P equation gives:

$$P - P_o - \rho a [U - U_o] = 2 F(M)$$

$$\Delta P = + \rho a \Delta U$$

This equation connects the downstream end of the pipe now to the upstream end one transit time back in time.

Multiplication of the U equation by ρa followed by addition to the P equation gives:

$$P - P_o + \rho a [U - U_o] = 2 f(N)$$

$$\Delta P = - \rho a \Delta U$$

This equation connects the upstream end of the pipe now to the downstream end one transit time back in time.

NAME: ____

$$T_D/T_U = [\ (1 + [(k-1)/2]\ M_UM_U) / (1 + [(k-1)/2]\ M_DM_D)\ ]$$

$$P_D/P_U = [T_D/T_U]^x \qquad x = k/(k-1)$$

$$P_D/P_U = [\ 1 + k\ M_UM_U\ ] / [\ 1 + k\ M_DM_D\ ]$$

$$P_D/P_U = 1 + [2k/(k+1)]\ (M_UM_U - 1)$$

$$M_DM_D = [(k-1)\ M_UM_U + 2] / [2k\ M_UM_U - (k-1)]$$

$$N_DN_D = [(k-1)\ N_UN_U + 2] / [2k\ N_UN_U - (k-1)]$$

$$P_D/P_U = 1 + [2k/(k+1)]\ (N_UN_U - 1)$$

$$N_U = M_U \sin\beta \qquad N_D = M_D \sin\kappa$$

$$\kappa = \beta - \Theta \qquad \upsilon_D = \upsilon_U + \Theta$$

$$\tan(\beta)/\tan(\kappa) = [(k+1)\ N_U\ N_U\ ] \ /\ [\ (k-1)\ N_U\ N_U + 2\ ]$$

$$\upsilon = \sqrt{[(k+1)/(k-1)]}\ \tan^{-1}\sqrt{[[(k-1)/(k+1)](M^2-1)]} - \tan^{-1}\sqrt{[M^2-1]}$$

$$\dot{M} = \rho A U \qquad M = U/C \qquad C = \sqrt{kRT}$$

$$\rho = P/[RT] \qquad \dot{M}\,U + \Delta P\ A$$

$$d\rho/\rho + dA/A + dU/U = 0$$

$$UdU + c^2 d\rho/\rho = 0$$

$$dU = UdA\ /\ [A(M^2-1)]$$

$$\Delta M^2/M^2 = kM^2[1+[(k-1)/2]M^2] /[1-M^2] f\Delta x/D$$

$$\Delta P/P = -kM^2[1+(k-1) M^2]/[2(1-M^2)] f\Delta x/D$$

$$\Delta T/T = -k(k-1)M^4/[2(1-M^2)] f\Delta x/D$$

$$\Delta\rho/\rho = -kM^2/[2(1-M^2)] f\Delta x/D$$

$$fL^*/D = (1-M^2)/(kM^2) + [(k+1)/(2k)] \ln[(k+1)M^2/(2+(k-1)M^2)]$$

$$fL^*/D = (1-kM^2)/(kM^2) + \ln[kM^2]$$

$$S_\eta = A/\omega^5 e^{-B/\omega 4} \qquad A=346H^2/T^4 \qquad B=691/T^4$$

$$S_R = RAO^2 S_\eta \qquad M_n = 1/2 \int S_R(\omega) \omega^n d\omega$$

$$H_R = 4 \sqrt{M_0} \qquad T_S = 2\pi M_0/M_1$$

$$P(R_\circ>R_\bullet) = e^{-X} \qquad X = R_\bullet R_\bullet/[2M_0]$$

$$CF = \sqrt{[1-\varepsilon\varepsilon]} \qquad \varepsilon = [M_0M_4-M_2M_2]/[M_0M_4]$$

$$\partial/\partial x\ (h^3/12\mu\ \partial P/\partial x) + \partial/\partial y\ (h^3/12\mu\ \partial P/\partial y)$$

$$= \partial[h(U_T+U_B)/2]/\partial x + \partial[h(V_T+V_B)/2]/\partial y + (W_T-W_B)$$

$$A = [(h_E+h_P)/2]^3 / [\Delta x^2]$$

$$B = [(h_W+h_P)/2]^3 / [\Delta x^2]$$

$$C = [(h_N+h_P)/2]^3 / [\Delta y^2]$$

$$D = [(h_S+h_P)/2]^3 / [\Delta y^2]$$

$$H = -\ 6\mu\ S\ (h_E-h_W)/[2\Delta x]$$

$$\partial/\partial r\ (rh^3/12\mu\ \partial P/\partial r) + r\ \partial/\partial c\ (h^3/12\mu\ \partial P/\partial c)$$

$$= \partial[rh(U_T+U_B)/2]/\partial r + \partial[h(V_T+V_B)/2]/\partial\Theta + r(W_T-W_B)$$

$$A = [(h_E+h_P)/2]^3\ r_P / [\Delta c^2]$$

$$B = [(h_W+h_P)/2]^3\ r_P / [\Delta c^2]$$

$$C = [(h_N+h_P)/2]^3\ [(r_N+r_P)/2] / [\Delta r^2]$$

$$D = [(h_S+h_P)/2]^3\ [(r_S+r_P)/2] / [\Delta r^2]$$

$$H = -\ 6\mu\ r_P\omega\ (h_E-h_W)/[2\Delta\Theta]$$

$$P_P = \frac{(A\ P_E + B\ P_W + C\ P_N + D\ P_S + H)}{(A + B + C + D)}$$

$$d/dx\ (h^3\ dP/dx) = 6\mu\ S\ dh/dx = H\ dh/dx$$

$$h^3\ dP/dx = H\ h + A \qquad dP/dx = H/h^2 + A/h^3$$

$$dP/dx = H/(sx+b)^2 + A/(sx+b)^3 \qquad s=[a-b]/d$$

$$P = -H/[s(sx+b)] - A/[2s(sx+b)^2] + B$$

$$A = \mathbf{[P_I\text{-}P_0}][2sa^2b^2]/[b^2-a^2] - 2Hba/[b+a]$$

$$B = \mathbf{[P_I}b^2\mathbf{-P_0}a^2]/[b^2-a^2] + H/[s(b+a)]$$

$$\Delta[h^3\ dP/dx] = H\ \Delta h$$

$$a^3\ [P_O-\mathbf{P}]/v - b^3\ [\mathbf{P}-P_I]/w = H\ [a-b]$$

$$\mathbf{P} = [\ a^3/v\ P_O + b^3/w\ P_I + H\ [b-a]\ ]\ /\ [\ a^3/v + b^3/w\ ]$$

$$d/dy\ (h^3\ dP/dy) = 6\mu\ S\ dh/dx = H\ dh/dx$$

$$d/dy\ (dP/dy) = H/h^3\ dh/dx = G$$

$$P = G/2\ y^2 + Ay + B$$

$$\varphi = S \, [\, X + XR^2/(X^2+Y^2) \,] \; + \; \Gamma/[2\pi] \, \sigma$$

$$\varphi = 2 \, S \, X \; + \; \Gamma/[2\pi] \, \sigma$$

$$\rho/2 \, [\, S^2 - (\partial\varphi/\partial c)^2 \,] \qquad \rho/2 \, [\, S^2 - (\Delta\varphi/\Delta c)^2 \,]$$

$$\alpha = x + xa^2/(x^2+y^2) \qquad \beta = y - ya^2/(x^2+y^2)$$

$$\Gamma = 4\pi SR \, \mathrm{Sin}\kappa \qquad \Delta c = \sqrt{[\Delta\alpha^2+\Delta\beta^2]} \qquad \rho S\Gamma$$

$$X = \mathbf{X} \, \mathrm{Cos}\Theta + \mathbf{Y} \, \mathrm{Sin}\Theta \qquad Y = \mathbf{Y} \, \mathrm{Cos}\Theta - \mathbf{X} \, \mathrm{Sin}\Theta$$

$$\mathbf{X} = x + n \qquad \mathbf{Y} = y - m \qquad \kappa = \Theta + \varepsilon$$

$$\mathbf{X} = - R \, \mathrm{Cos}\,\sigma \qquad \mathbf{Y} = + R \, \mathrm{Sin}\,\sigma$$

$$\varepsilon = \tan^{-1} [m/(n+a)] \qquad m^2 + (a+n)^2 = R^2$$

$$P\Delta c \, \mathrm{Sin}(\boldsymbol{\theta}-\Theta) \qquad P\Delta c \, \mathrm{Cos}(\boldsymbol{\theta}-\Theta)$$

$$\boldsymbol{\theta} = \tan^{-1}[-\Delta\alpha/+\Delta\beta] \qquad L=\Sigma\Delta L \qquad D=\Sigma\Delta D$$

$$\delta^* = \int (1-U/\mathbf{U})\,dy \qquad \Theta = \int U/\mathbf{U}(1-U/\mathbf{U})\,dy \qquad \mathbf{D}=\int \tau b d\mathbf{c}=\Sigma\tau b\Delta\mathbf{c}$$

$$\tau = C\ \rho\mathbf{U}^2/(\mathbf{U}\delta/\upsilon)^{1/k} \qquad \tau = \rho\ d[\mathbf{U}^2\Theta]/dc + \rho\delta^*\ \mathbf{U}d\mathbf{U}/dc$$

$$d\delta/dc = H \qquad \delta_{NEW} = \delta_{OLD} + \Delta c\ H_{OLD} \qquad U/\mathbf{U} = (y/\delta)^{1/n}$$

$$\mathbf{P} = [\mathbf{D}+\mathbf{W}]\mathbf{U} \qquad \delta = A\ x\ R_{EX}^{-1/a} \qquad \delta^* = I\delta \qquad \Theta = J\delta$$

$$\mathbf{D} = M\ bx\ R_{EX}^{-1/m}\ \rho\mathbf{U}^2 \qquad \mathbf{D} = N\ bx\ R_{EX}^{-1/2}\ \rho\mathbf{U}^2$$

$$\mathbf{D} = K\ bx\ \rho\mathbf{U}^2 \qquad \mathbf{W} = C\ B\ \rho\mathbf{U}^2/2$$

$$\rho\ \partial U/\partial t + \rho U\ \partial U/\partial x + \partial P/\partial x - \rho g\ \mathrm{Sin}\alpha + f/D\ \rho U|U|/2 = 0$$

$$\partial P/\partial t + U\ \partial P/\partial x + \rho a^2\ \partial U/\partial x = 0$$

$$\rho\ \partial U/\partial t + \rho U\ \partial U/\partial x + \partial P/\partial x + \rho C$$
$$+ \lambda\ (\partial P/\partial t + U\ \partial P/\partial x + \rho a^2\ \partial U/\partial x) = 0$$

$$C = f/D\ U|U|/2 - g\ \mathrm{Sin}\alpha$$

$$\rho \; (\partial U/\partial t + [U+\lambda a^2] \; \partial U/\partial x)$$

$$+ \; \lambda \; (\partial P/\partial t + [1/\lambda+U] \; \partial P/\partial x) + \rho C = 0$$

$$dx/dt = U + \lambda a^2 = 1/\lambda + U \qquad \lambda = \pm \; 1/a$$

$$dU/dt + \lambda/\rho \; dP/dt + C = 0$$

$$dU/dt + 1/\rho a \; dP/dt + C = 0 \qquad dx/dt = U + a$$

$$dU/dt - 1/\rho a \; dP/dt + C = 0 \qquad dx/dt = U - a$$

$$U_P - U_L + (P_P-P_L)/[\rho a] + C_L(t_P-t_L) = 0 \qquad x_P-x_L = (U_L+a)(t_P-t_L)$$

$$U_P - U_R - (P_P-P_R)/[\rho a] + C_R(t_P-t_R) = 0 \qquad x_P-x_R = (U_R-a)(t_P-t_R)$$

$$U_P = 0.5 \; (U_L + U_R + [P_L-P_R]/[\rho a] - \Delta t(C_L+C_R))$$

$$P_P = 0.5 \; (P_L + P_R + [\rho a][U_L-U_R] - \Delta t[\rho a](C_L-C_R))$$

$$\Delta P = + \; \rho a \; \Delta U$$

$$\Delta P = - \; \rho a \; \Delta U$$

$$U = U_\circ \, M/M_\circ \, \zeta \, a$$

$$U_\circ = D/\mathbf{T} \qquad M_\circ = \rho D^2$$

$$U = \beta/\mathbf{T} \, \sqrt{[M\delta/\rho]}$$

$$U = \beta U_\circ \, \sqrt{[\delta M/M_\circ]}$$

$$U = D/[ST]$$

$$T = \mathbf{T}$$

$$U^2 = [\, EI/[\rho A] \, \pi^2/L^2 + T/[\rho A] - P/\rho \,]$$

$$U = [4 + 14 \, M_\circ/M] \, U_\circ$$

$$U_\circ = \sqrt{[EI]/[M_\circ L^2]} \qquad M_\circ = \rho A$$

$$\mathbf{T}_n = [2L/n] \, \sqrt{[m/T]}$$

$$\mathbf{T}_n = [L/n]^2 \, [2/\pi] \, \sqrt{[m/EI]}$$

$$\mathbf{T}_n = 2\pi L^2/K_n \, \sqrt{[m/EI]}$$

$$C_P = \sqrt{(g/k \ \mathrm{Tanh}[kh])}$$

$$\omega = \sqrt{(gk \ \mathrm{Tanh}[kh])}$$

$$U = + H/2 \ 2\pi/T \ \mathrm{Cosh}[k(z+h)]/\mathrm{Sinh}[kh] \ \mathrm{Sin}(kX)$$

$$W = - H/2 \ 2\pi/T \ \mathrm{Sinh}[k(z+h)]/\mathrm{Sinh}[kh] \ \mathrm{Cos}(kX)$$

$$dU/dt = - H/2 \ (2\pi/T)^2 \ \mathrm{Cosh}[k(z+h)]/\mathrm{Sinh}[kh] \ \mathrm{Cos}(kX)$$

$$dW/dt = - H/2 \ (2\pi/T)^2 \ \mathrm{Sinh}[k(z+h)]/\mathrm{Sinh}[kh] \ \mathrm{Sin}(kX)$$

$$x_p = x_o + H/2 \ \mathrm{Cosh}[k(z+h)]/\mathrm{Sinh}[kh] \ \mathrm{Cos}(kX)$$

$$Z_p = z_o + H/2 \ \mathrm{Sinh}[k(z+h)]/\mathrm{Sinh}[kh] \ \mathrm{Sin}(kX)$$

$$\eta = \eta_O \ \mathrm{Sin}(kX) \qquad \Delta P = \rho g \ \eta \ \mathrm{Cosh}[k(z+h)]/\mathrm{Cosh}[kh]$$

$$C_G = d\omega/dk = C_P \ (1/2 + [kh]/\mathrm{Sinh}[2kh])$$

$$\mathbf{E} = 1/8 \ \rho g \ H^2 \qquad \mathbf{P} = C_G \ \mathbf{E}$$

NAME: ____

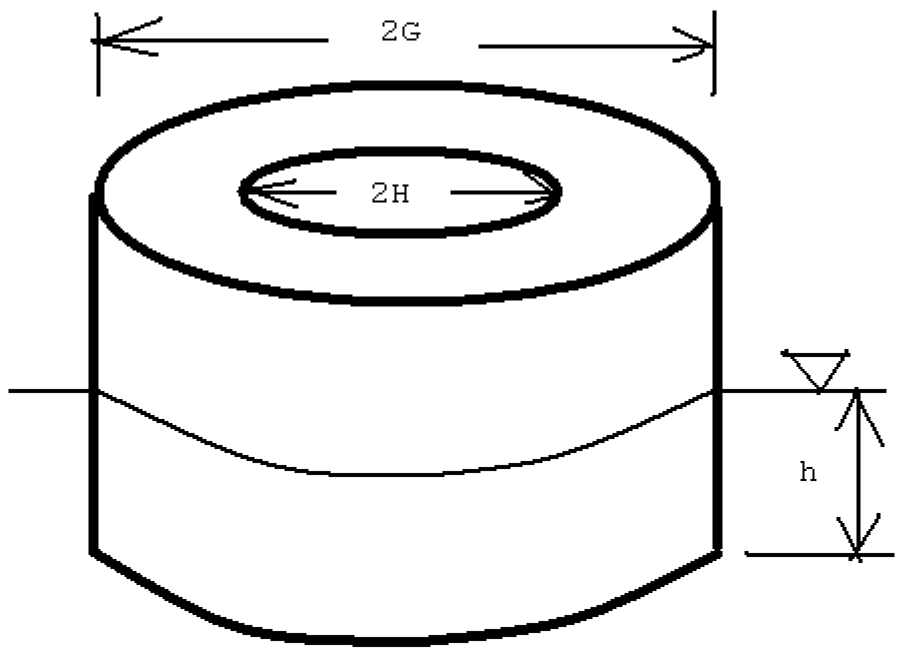
ENGINEERING 6961

FLUID MECHANICS II

QUIZ #1

QUESTIONS	MARKS
HYDROSTATIC STABILITY [30]	_______
TURBOMACHINES [30]	_______
TURBULENT WAKE FLOWS [20]	_______
COMPUTATIONAL FLUID DYNAMICS [20]	_______
BONUS : CONSERVATION LAWS [5]	_______
TOTAL [105]	_______

A GBS type of rig with a moon pool is sketched below. Derive an equation for its metacentric radius. [20]



Note that the K and V of a basic GBS rig are:

$$K = \pi G^4 / 4$$

$$V = \pi G^2 h$$

GBS with Moon Pool

$$R = [K_O - K_I) / (V_O - V_I)$$

$$= (\pi G^4/4 - \pi H^4/4) / (\pi G^2 h - \pi H^2 h)$$

The equation governing the hydrostatic stability of floating bodies is given below. Explain it. [10] Use sketches to illustrate your answer.

$$S \rho g V = \int_{-G}^{+G} x \rho g x\Theta w \, dx$$

Slice volume is: $dV = x\Theta w \, dx$

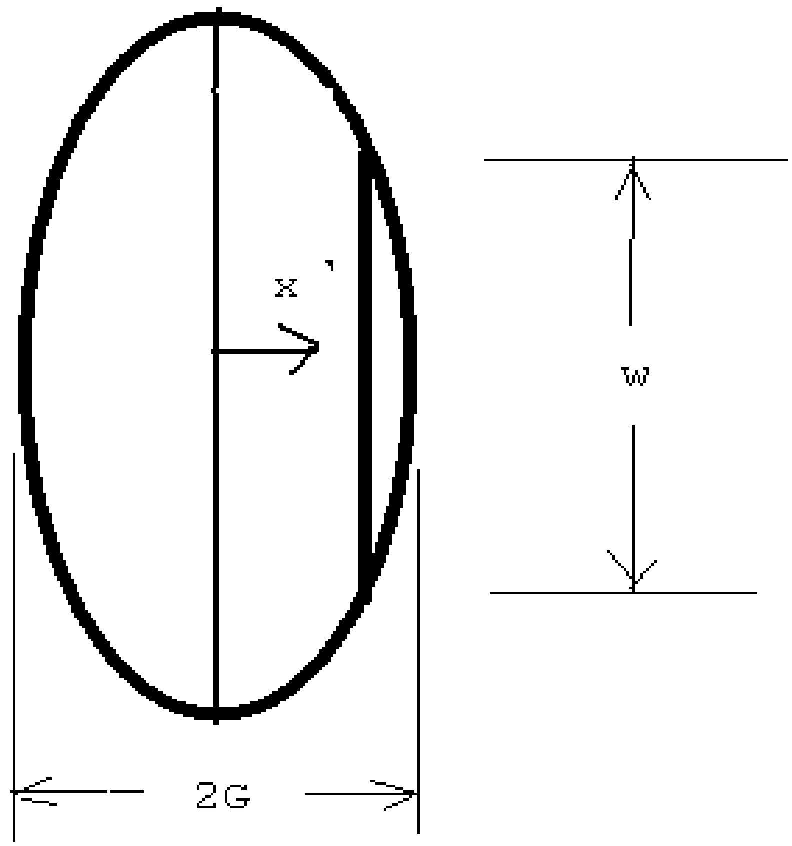
Slice Weight is: $dW = \rho g \, dV$

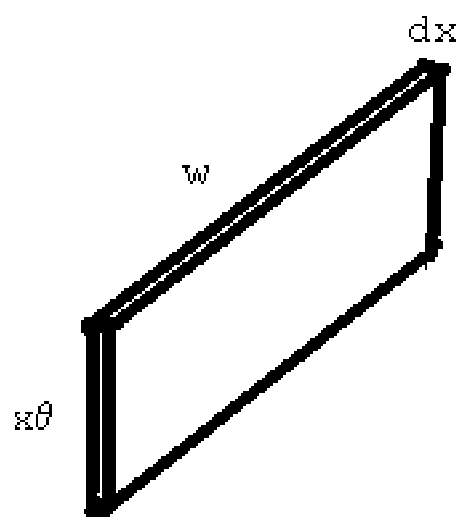
Slice Moment is: $x \, dW$

Integration gives: $\rho g K \Theta$

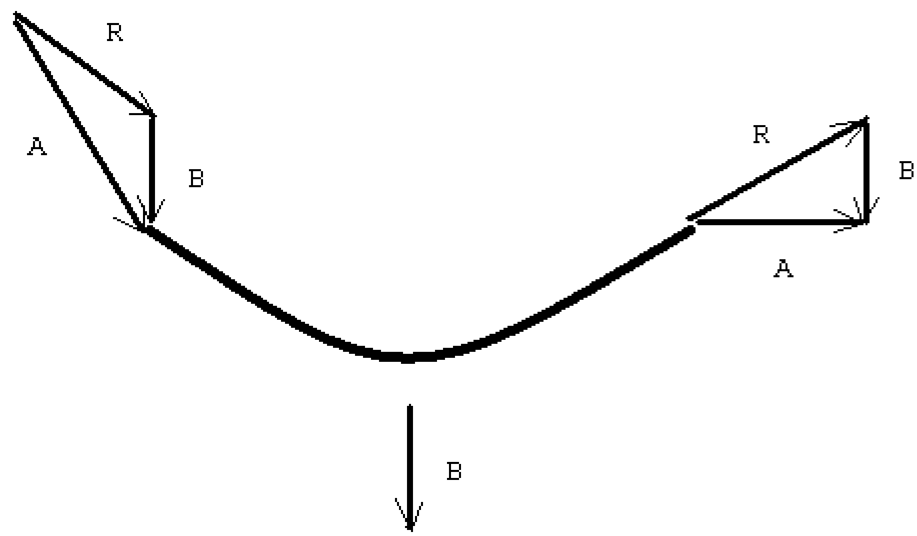
Manipulation gives: $S = K/V \Theta = R \Theta$

Metacentric Radius: R


x '
w
2G


dx
w
xθ

A Dentist Drill consists of a number of buckets attached to the rim of a small wheel or rotor. A sketch of one bucket for an optimum power flow configuration is shown below. In the sketch, B is the velocity of the bucket, R is the velocity of the fluid relative to the bucket and A is the absolute velocity of the fluid. Derive an equation for the power output of the drill. State all assumptions. [15]



Power **P** = ρQ 2B B

The equations governing the power of a turbomachine are given below. Explain them. [15]

$$F = \rho Q \, V_T \qquad\qquad S = R\,\omega$$

$$\mathbf{P} = F\,S = F\,R\,\omega = T\,\omega$$

$$\mathbf{P} = \Delta\,[\,\rho Q \, V_T \, R\,\omega\,]$$

Momentum Force: $F = \rho Q \, V_T$

Force Speed: $S = R\,\omega$

Power: $\mathbf{P} = F\,S = T\,\omega$

$$\mathbf{P} = \Delta\,[\,\rho Q \, V_T \, R\,\omega\,]$$

PDEs for Turbulent Flows are given below. Write brief notes on each PDE. [10] Write brief notes on Turbulence. [10]

Conservation of X Momentum

$$\rho\ (\partial U/\partial t + U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z) + A = - \partial P/\partial x$$

$$+\ [\partial/\partial x\ (\mu\ \partial U/\partial x) + \partial/\partial y\ (\mu\ \partial U/\partial y) + \partial/\partial z\ (\mu\ \partial U/\partial z)]$$

Conservation of Y Momentum

$$\rho\ (\partial V/\partial t + U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z) + B = - \partial P/\partial y$$

$$+\ [\partial/\partial x\ (\mu\ \partial V/\partial x) + \partial/\partial y\ (\mu\ \partial V/\partial y) + \partial/\partial z\ (\mu\ \partial V/\partial z)]$$

Conservation of Z Momentum

$$\rho\ (\partial W/\partial t + U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z) + C = - \partial P/\partial z - \rho g$$

$$+\ [\partial/\partial x\ (\mu\ \partial W/\partial x) + \partial/\partial y\ (\mu\ \partial W/\partial y) + \partial/\partial z\ (\mu\ \partial W/\partial z)]$$

Conservation of Mass

$$\partial P/\partial t + \rho\ c^2\ (\partial U/\partial x + \partial V/\partial y + \partial W/\partial z) = 0$$

Water Surface Tracker

$$\partial F/\partial t + U\partial F/\partial x + V\partial F/\partial y + W\partial F/\partial z = 0$$

Kinetic Energy of Turbulence

$$\partial k/\partial t + U\partial k/\partial x + V\partial k/\partial y + W\partial k/\partial z = T_P - T_D$$

$$+ \left[\partial/\partial x\ (\mu/a\ \partial k/\partial x) + \partial/\partial y\ (\mu/a\ \partial k/\partial y) + \partial/\partial z\ (\mu/a\ \partial k/\partial z) \right]$$

Dissipation Rate of Turbulence

$$\partial \varepsilon/\partial t + U\partial \varepsilon/\partial x + V\partial \varepsilon/\partial y + W\partial \varepsilon/\partial z = D_P - D_D$$

$$+ \left[\partial/\partial x\ (\mu/b\ \partial \varepsilon/\partial x) + \partial/\partial y\ (\mu/b\ \partial \varepsilon/\partial y) + \partial/\partial z\ (\mu/b\ \partial \varepsilon/\partial z) \right]$$

Time Stepping Template

$$\partial M/\partial t = N \qquad M_{NEW} = M_{OLD} + \Delta t\ N_{OLD}$$

Turbulence Functions

$$T_P = G\ \mu_t / \rho \qquad D_P = T_P\ C_1\ \varepsilon / k$$

$$T_D = C_D\ \varepsilon \qquad D_D = C_2\ \varepsilon^2 / k$$

Viscosities

$$\mu_t = C_3\ k^2 / \varepsilon \qquad \mu = \mu_t + \mu_l$$

Write brief notes on CFD for Turbulent Flows. [10]

The region of interest is divided by a CFD grid. CFD cells surround each point where grid lines cross. Each PDE is put into the form:

$$\partial M/\partial t = N$$

Application of simple time stepping gives:

$$M_{NEW} = M_{OLD} + \Delta t \ N_{OLD}$$

This template is applied to each PDE at each point in the grid. Finite differences are used to approximate the various derivatives in N. Central differences are used to approximate the diffusion terms. Upwind differences are used to approximate the convective terms. The eddy viscosity concept is used to model turbulence. The volume of fluid concept is used to track the water surface. The function F is 1 inside water and 0 outside it: cells with F between 1 and 0 contain the water surface. The Semi Implicit Method for Pressure Linked Equations or SIMPLE procedure is used to update pressure and correct velocities so that they satisfy mass and momentum.

Give an illustration of upwind differencing. [5]

$$W \quad \partial W/\partial z$$

Flow North to South

$$(W_N + W_P)/2 \quad (W_N - W_P)/\Delta z$$

Give an illustration of the water surface tracker. [5]

Water Surface Tracker

$$\partial F/\partial t + U\,\partial F/\partial x + V\,\partial F/\partial y + W\,\partial F/\partial z = 0$$

Flow South to North

$$\partial F/\partial t + \partial[FW]/\partial z = 0$$

$$\Delta F = \Delta t\ ([FW]_{OUT} - [FW]_{IN})\ /\ \Delta z$$

BONUS [5]

The integral form of the conservation laws is given below. Identify each law. The PDE form of each law is on the next page. Identify each PDE.

Conservation of Mass

$$\frac{D}{Dt} \int_{V(t)} \rho \, dV = 0$$

$$\int_{V(t)} [\partial\rho/\partial t + \nabla . (\rho \mathbf{v})] \, dV = 0$$

Conservation of Momentum

$$\frac{D}{Dt} \int_{V(t)} \rho \mathbf{v} \, dV = \int_{S(t)} \boldsymbol{\sigma} \, dS + \int_{V(t)} \rho \mathbf{b} \, dV$$

$$\int_{V(t)} [\partial(\rho \mathbf{v})/\partial t + \nabla . (\rho \mathbf{v}\mathbf{v})] \, dV = \int_{S(t)} \boldsymbol{\sigma} \, dS + \int_{V(t)} \rho \mathbf{b} \, dV$$

Conservation of Energy

$$\frac{D}{Dt} \int_{V(t)} \rho e \, dV = - \int_{S(t)} \mathbf{q}.\mathbf{n} \, dS + \int_{S(t)} \mathbf{v}.\boldsymbol{\sigma} \, dS$$

$$e = u + \mathbf{v}.\mathbf{v}/2 + gz$$

$$\int_{V(t)} [\partial(\rho e)/\partial t + \nabla . (\rho e \mathbf{v})] \, dV = - \int_{S(t)} \mathbf{q}.\mathbf{n} \, dS + \int_{S(t)} \mathbf{v}.\boldsymbol{\sigma} \, dS$$

Conservation of Mass

$$\partial U/\partial x \; + \; \partial V/\partial y \; + \; \partial W/\partial z \; = \; 0$$

Conservation of Momentum

$$\rho \partial U/\partial t + \rho \; (U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z) = - \; \partial P/\partial x$$

$$+ \; \mu \; (\partial^2 U/\partial x^2 + \partial^2 U/\partial y^2 + \partial^2 U/\partial z^2)$$

$$\rho \partial V/\partial t + \rho \; (U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z) = - \; \partial P/\partial y$$

$$+ \; \mu \; (\partial^2 V/\partial x^2 + \partial^2 V/\partial y^2 + \partial^2 V/\partial z^2)$$

$$\rho \partial W/\partial t + \rho \; (U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z) = - \; \partial P/\partial z - \rho g$$

$$+ \; \mu \; (\partial^2 W/\partial x^2 + \partial^2 W/\partial y^2 + \partial^2 W/\partial z^2)$$

Conservation of Energy

$$\rho C \; \partial T/\partial t \; + \; \rho C \; (U\partial T/\partial x + V\partial T/\partial y + W\partial T/\partial z) \; = \; \mu \; \Phi$$

$$+ \; \partial/\partial x(k\partial T/\partial x) \; + \; \partial/\partial y(k\partial T/\partial y) \; + \; \partial/\partial z(k\partial T/\partial z)$$

FORMULA SHEET

$$S \; \rho g \; V = \int_{-G}^{+G} x \; \rho g \; x\Theta \; w \; dx$$

$$K = \int_{-G}^{+G} x^2 \; w \; dx \qquad V = A \; h$$

$$S = K/V \; \Theta = R \; \Theta$$

$$T = \Delta[\; \rho Q \; V_T \; R \;]$$

$$\mathbf{P} = T \; \omega = \Delta[\; \rho Q \; V_T \; R \; \omega \;]$$

$$P \; Q \qquad V_B = R \; \omega$$

$$P = \rho \; [V_J]^2 / 2 \qquad Q = V_J \; A$$

NAME:

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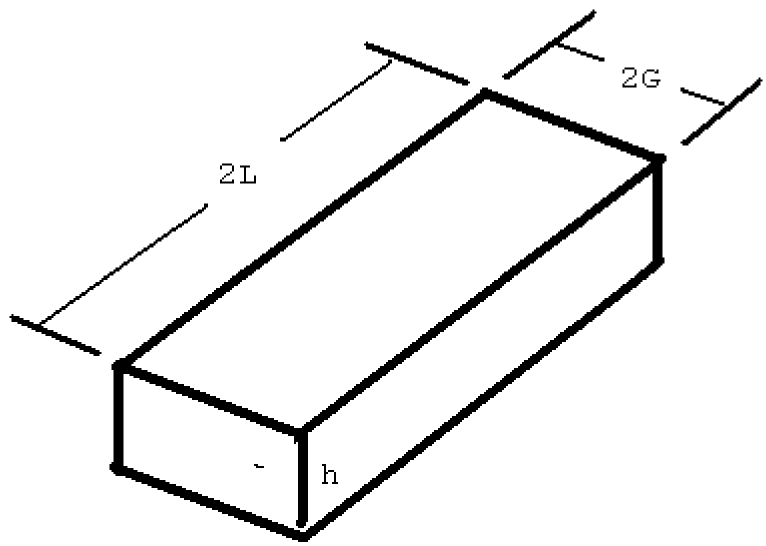
ENGINEERING 6961

FLUID MECHANICS II

QUIZ #1

QUESTIONS	MARKS
HYDROSTATIC STABILITY [30]	_______
TURBOMACHINES [30]	_______
CONSERVATION LAWS [20]	_______
TURBULENT WAKE FLOW [20]	_______
BONUS : CFD [5]	_______
TOTAL [105]	_______

A rectangular barge is sketched below. Derive an equation for its roll metacentric radius. [20] By analogy write down an equation for its pitch metacentric radius. [10]



The buoyancy of a floating body is

$$\rho \; g \; V$$

The shift in the center of buoyancy caused by rotation is S. The moment of the buoyancy force is:

$$S \; \rho \; g \; V$$

This moment must be equal to the sum of the moments of all of the slices that make up the wedge shaped volumes created by rotation. For roll of the barge this moment is

$$\int_{-G}^{+G} x \; \rho g \; x\Theta \; w \; dx = \rho g \; K \; \Theta$$

where

$$K = \int_{-G}^{+G} x^2 \; w \; dx$$

$$= 2 * 2L * G^3/3$$

The volume of the barge is

$$V = 2L * 2G * h$$

Manipulation gives

$$S = K/V \; \Theta = R \; \Theta$$

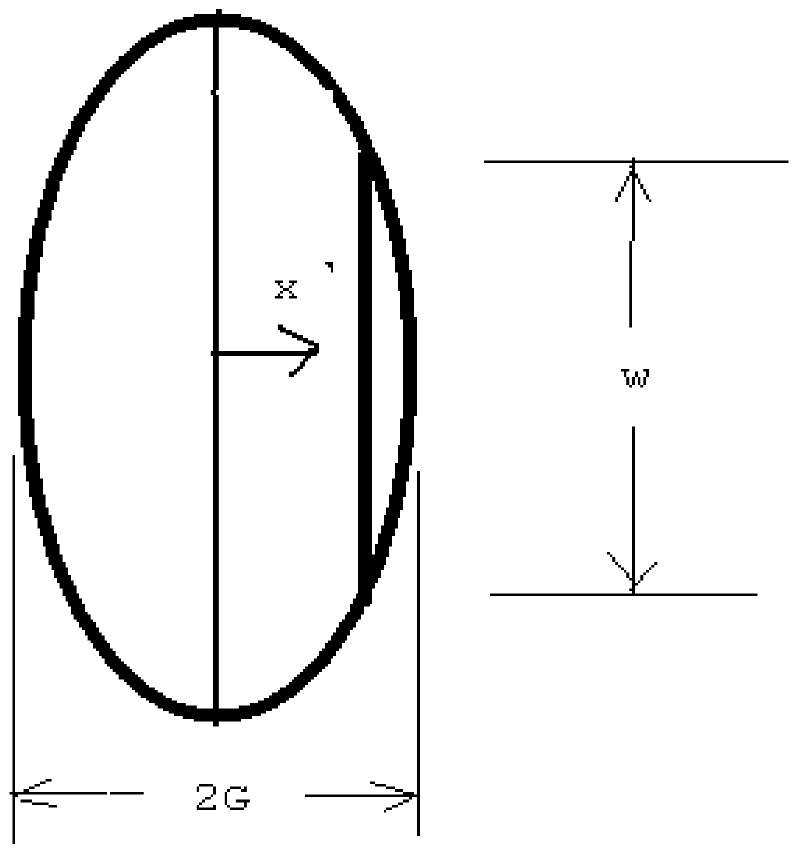
$$= G^2/[3h] \; \Theta$$

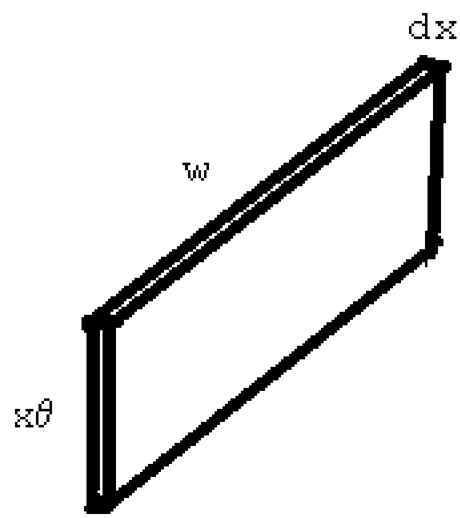
So the roll metacentric radius is

$$R = G^2/[3h]$$

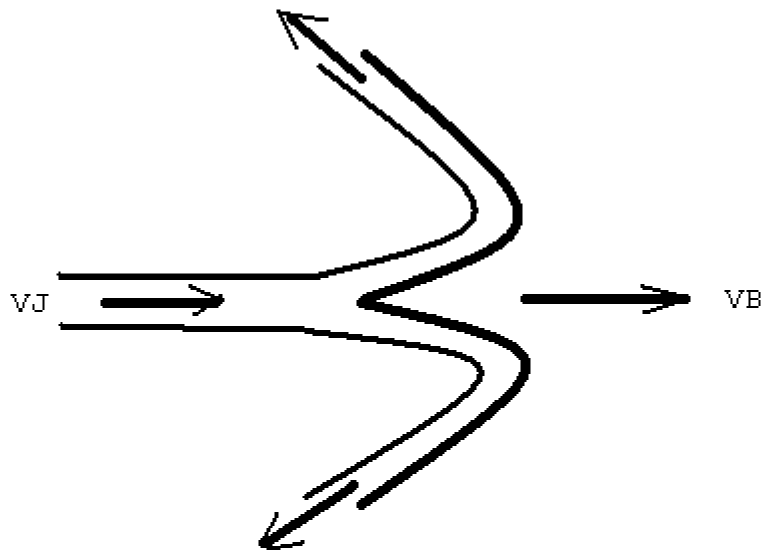
By analogy the pitch metacentric radius is

$$R = L^2/[3h]$$


x'
w
2G


dx
w
xθ

A sketch of one bucket of a Pelton Wheel Turbine is shown below. Derive an equation for the power output of this turbine in terms of its bucket speed V_B and its jet speed V_J. [15] Determine the peak power of the turbine. [10] Write down the C_P and C_S scaling laws for the turbine. [5]



The force at the inlet or outlet of a turbine associated with the tangential momentum there is:

$$\rho Q\ V_T$$

The speed of this force is

$$R\ \omega$$

Power is force times speed

$$\rho Q\ V_T \quad R\ \omega$$

The power of the turbine is

$$\mathbf{P} = \Delta[\ \rho Q\ V_T \quad R\ \omega\ ]$$

For a Pelton Wheel, Q and Rω are the same at the inlet and the outlet. So the power becomes

$$\mathbf{P} = \rho Q\ R\omega\ \Delta[V_T]$$

$$= \rho Q\ V_B\ \Delta[V_T]$$

The absolute tangential velocities are

$$V_{IN} = V_J$$

$$V_{OUT} = (V_J - V_B)\ \text{Cos}\beta + V_B$$

With this power becomes

$$\mathbf{P} = \rho Q\ V_B\ (V_J - V_B)\ (1 - \cos\beta)$$

Differentiation with respect to V_B shows that the power peaks when the bucket speed V_B is half the jet speed V_J. Substitution into power gives

$$\mathbf{P} = \rho Q\ [V_J]^2/4\ (1 - \cos\beta)$$

The scaling laws for the turbine are:

Power Coefficient $C_{\mathbf{P}}$

$$C_{\mathbf{P}} = \mathbf{P}\ /\ PQ$$

$$= \mathbf{P}\ /\ (\ \rho\ [V_J]^2\ /\ 2\quad V_J\ A\ )$$

Speed Coefficient C_S

$$C_S = V_B\ /\ V_J$$

$$= R\omega\ /\ V_J$$

Give a statement in words for each of the three conservation laws for fluid flow. [10]

Conservation of Mass: The time rate of change of the mass of an arbitrary specific group of fluid particles in a flow is zero.

Conservation of Momentum: The time rate of change of the momentum of an arbitrary specific group of fluid particles in a flow is equal to the force acting on the group. The force can be of two types: body forces and surface forces. Body forces are generally due to gravity. Surface forces are due to pressure and viscous traction.

Conservation of Mass: The time rate of change of the energy of an arbitrary specific group of fluid particles in a flow is equal to the net work done on the group by the surroundings plus the net heat flux into the group from the surroundings.

The integral form of the conservation laws is given below. Identify each law. [5] The PDE form of each law is on the next page. Identify each PDE. [5]

Conservation of Mass

$$\frac{D}{Dt} \int_{V(t)} \rho \, dV = 0$$

$$\int_{V(t)} \left[\partial\rho/\partial t + \nabla.(\rho\mathbf{v}) \right] dV = 0$$

Conservation of Momentum

$$\frac{D}{Dt} \int_{V(t)} \rho\mathbf{v} \, dV = \int_{S(t)} \boldsymbol{\sigma} \, dS + \int_{V(t)} \rho\mathbf{b} \, dV$$

$$\int_{V(t)} \left[\partial(\rho\mathbf{v})/\partial t + \nabla.(\rho\mathbf{v}\mathbf{v}) \right] dV = \int_{S(t)} \boldsymbol{\sigma} \, dS + \int_{V(t)} \rho\mathbf{b} \, dV$$

Conservation of Energy

$$\frac{D}{Dt} \int_{V(t)} \rho e \, dV = - \int_{S(t)} \mathbf{q}.\mathbf{n} \, dS + \int_{S(t)} \mathbf{v}.\boldsymbol{\sigma} \, dS$$

$$e = u + \mathbf{v}.\mathbf{v}/2 + gz$$

$$\int_{V(t)} \left[\partial(\rho e)/\partial t + \nabla.(\rho e\mathbf{v}) \right] dV = - \int_{S(t)} \mathbf{q}.\mathbf{n} \, dS + \int_{S(t)} \mathbf{v}.\boldsymbol{\sigma} \, dS$$

Conservation of Mass

$$\partial U/\partial x + \partial V/\partial y + \partial W/\partial z = 0$$

Conservation of Momentum

$$\rho \partial U/\partial t + \rho (U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z) = - \partial P/\partial x$$
$$+ \mu (\partial^2 U/\partial x^2 + \partial^2 U/\partial y^2 + \partial^2 U/\partial z^2)$$

$$\rho \partial V/\partial t + \rho (U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z) = - \partial P/\partial y$$
$$+ \mu (\partial^2 V/\partial x^2 + \partial^2 V/\partial y^2 + \partial^2 V/\partial z^2)$$

$$\rho \partial W/\partial t + \rho (U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z) = - \partial P/\partial z - \rho g$$
$$+ \mu (\partial^2 W/\partial x^2 + \partial^2 W/\partial y^2 + \partial^2 W/\partial z^2)$$

Conservation of Energy

$$\rho C \partial T/\partial t + \rho C (U\partial T/\partial x + V\partial T/\partial y + W\partial T/\partial z) = \mu \Phi$$
$$+ \partial/\partial x(k\partial T/\partial x) + \partial/\partial y(k\partial T/\partial y) + \partial/\partial z(k\partial T/\partial z)$$

The PDEs for Turbulent Wake Flows are given on the next few pages. Identify each PDE. [10] Explain briefly how CFD can be used to get flows step by step in time. [10]

Conservation of X Momentum

$$\rho \left(\partial U/\partial t + U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z \right) + A = - \partial P/\partial x$$
$$+ \left[\partial/\partial x \left(\mu\, \partial U/\partial x\right) + \partial/\partial y \left(\mu\, \partial U/\partial y\right) + \partial/\partial z \left(\mu\, \partial U/\partial z\right) \right]$$

Conservation of Y Momentum

$$\rho \left(\partial V/\partial t + U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z \right) + B = - \partial P/\partial y$$
$$+ \left[\partial/\partial x \left(\mu\, \partial V/\partial x\right) + \partial/\partial y \left(\mu\, \partial V/\partial y\right) + \partial/\partial z \left(\mu\, \partial V/\partial z\right) \right]$$

Conservation of Z Momentum

$$\rho \left(\partial W/\partial t + U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z \right) + C = - \partial P/\partial z - \rho g$$
$$+ \left[\partial/\partial x \left(\mu\, \partial W/\partial x\right) + \partial/\partial y \left(\mu\, \partial W/\partial y\right) + \partial/\partial z \left(\mu\, \partial W/\partial z\right) \right]$$

Conservation of Mass

$$\partial P/\partial t + \rho\, c^2 \left(\partial U/\partial x + \partial V/\partial y + \partial W/\partial z \right) = 0$$

Water Surface Tracker

$$\partial F/\partial t + U\partial F/\partial x + V\partial F/\partial y + W\partial F/\partial z = 0$$

Kinetic Energy of Turbulence

$$\partial k/\partial t + U\partial k/\partial x + V\partial k/\partial y + W\partial k/\partial z = T_P - T_D$$

$$+ \left[\partial/\partial x \, (\mu/a \, \partial k/\partial x) + \partial/\partial y \, (\mu/a \, \partial k/\partial y) + \partial/\partial z \, (\mu/a \, \partial k/\partial z) \right]$$

Dissipation Rate of Turbulence

$$\partial \varepsilon/\partial t + U\partial \varepsilon/\partial x + V\partial \varepsilon/\partial y + W\partial \varepsilon/\partial z = D_P - D_D$$

$$+ \left[\partial/\partial x \, (\mu/b \, \partial \varepsilon/\partial x) + \partial/\partial y \, (\mu/b \, \partial \varepsilon/\partial y) + \partial/\partial z \, (\mu/b \, \partial \varepsilon/\partial z) \right]$$

Time Stepping Template

$$\partial M/\partial t = N \qquad M_{NEW} = M_{OLD} + \Delta t \, N_{OLD}$$

Production and Dissipation Functions

$$T_P = G \, \mu_t / \rho \qquad D_P = T_P \, C_1 \, \varepsilon / k$$

$$T_D = C_D \, \varepsilon \qquad D_D = C_2 \, \varepsilon^2 / k$$

Eddy and Effective Viscosity

$$\mu_t = C_3 \, k^2 / \varepsilon \qquad \mu = \mu_t + \mu_1$$

The region of interest is divided by a CFD grid. CFD cells surround each point where grid lines cross. Each PDE is put into the form:

$$\partial M/\partial t = N$$

Application of simple time stepping gives:

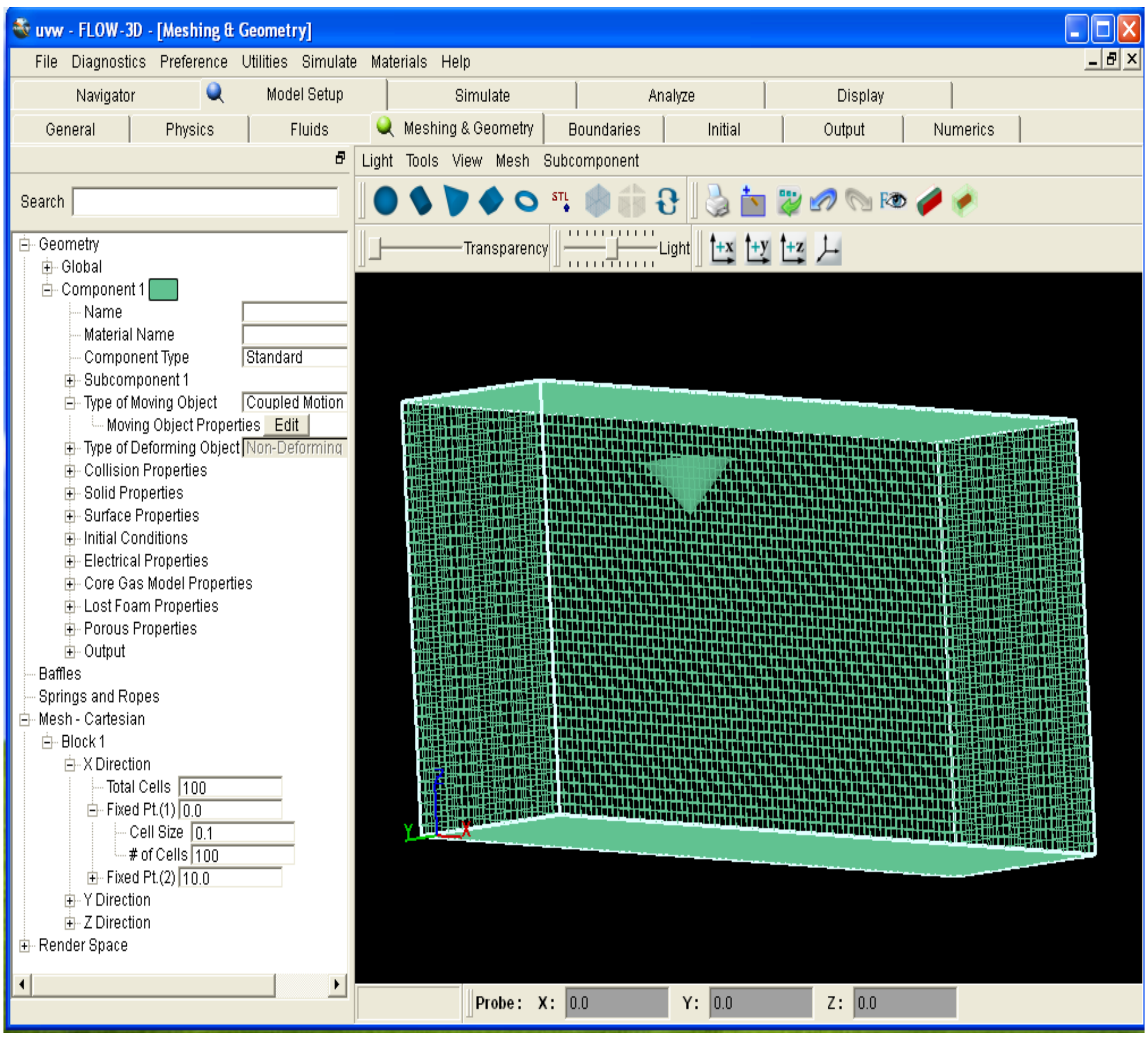
$$M_{NEW} = M_{OLD} + \Delta t\ N_{OLD}$$

This template is applied to each PDE at each point in the grid. Finite differences are used to approximate the various derivatives in N. Central differences are used to approximate the diffusion terms. Upwind differences are used to approximate the convective terms. The eddy viscosity concept is used to model turbulence. The volume of fluid concept is used to track the water surface. The function F is 1 inside water and 0 outside it: cells with F between 1 and 0 contain the water surface. The Semi Implicit Method for Pressure Linked Equations or SIMPLE procedure is used to update pressure and correct velocities so that they satisfy mass and momentum.

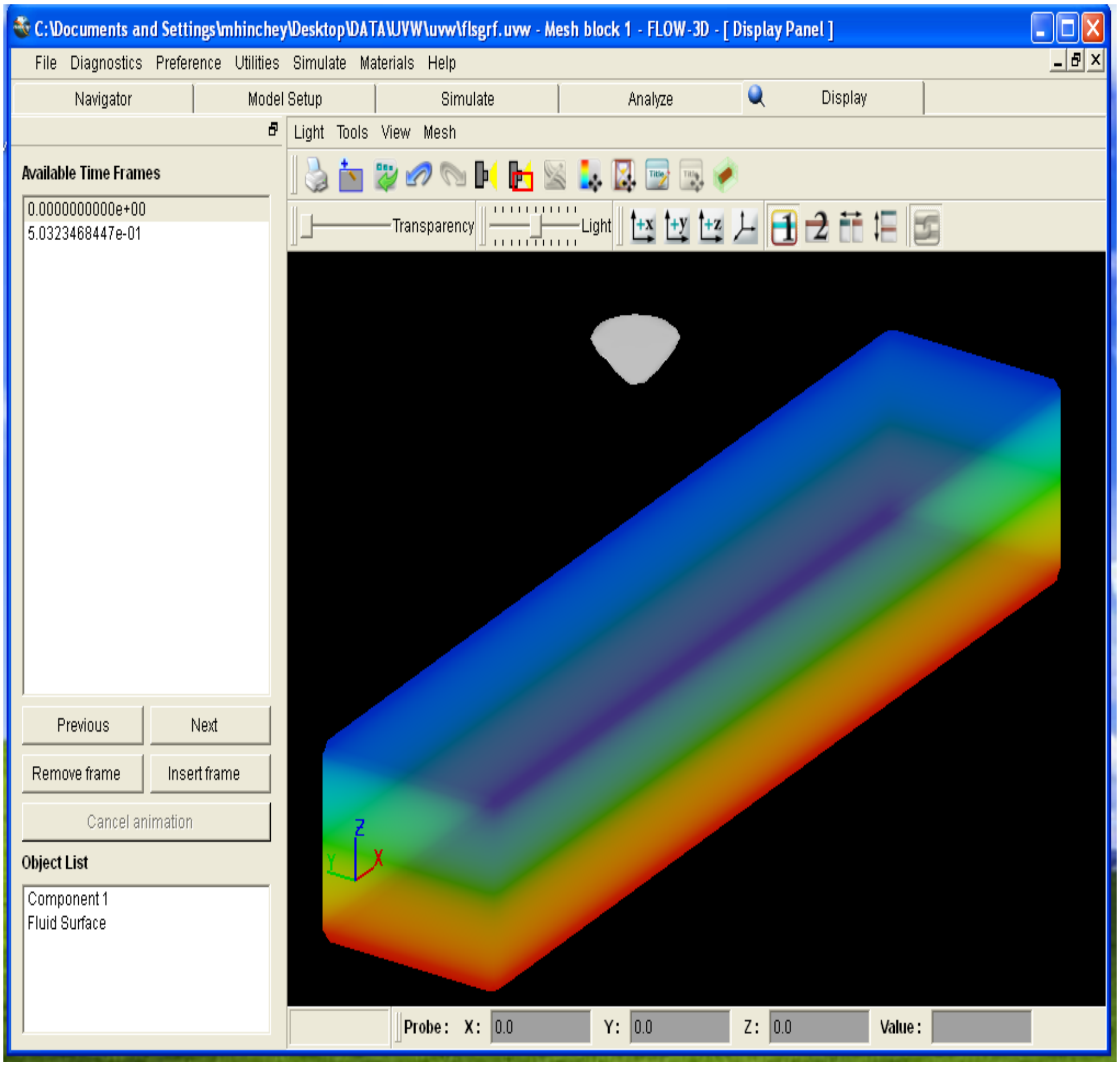
BONUS [5]

Two menus from FLOW 3D are given below. Make notes on these menus to indicate the purpose of each submenu.

This shows the Meshing and Geometry Submenu.



This shows the 3D output display submenu.



Navigator: Sets up workspace.

Model Setup: Series of menus for a simulation.

General: Sets things like simulation duration, number of fluids and type of fluid interface.

Physics: Sets things like gravity, GMO and turbulence.

Fluids: Imports the fluid properties from a data base.

Mesh & Geometry: Sets the xyz mesh and GMO properties.

Boundaries: Sets the boundary conditions on the mesh.

Initial: Sets the initial state of the fluid.

Output: Sets the output time step.

Numerics: Sets details of CFD.

Simulate: Runs the simulation.

Analyze: Picks what data to look at.

Display: Displays the data in various formats.

FORMULA SHEET

$$S \; \rho g \; V = \int_{-G}^{+G} x \; \rho g \; x\Theta \; w \; dx$$

$$K = \int_{-G}^{+G} x^2 \; w \; dx \qquad V = A \; h$$

$$S = K/V \; \Theta = R \; \Theta$$

$$T = \Delta[\; \rho Q \; V_T \; R \;]$$

$$\mathbf{P} = T \; \omega = \Delta[\; \rho Q \; V_T \; R \; \omega \;]$$

$$P \; Q \qquad V_B = R \; \omega$$

$$P = \rho \; [V_J]^2 \; / \; 2 \qquad Q = V_J \; A$$

NAME: ____

ENGINEERING 6961

FLUID MECHANICS II

MECHANICAL CLASS

QUIZ #2

NO NOTES OR TEXTS ALLOWED

NO CALCULATORS ALLOWED

GIVE ANSWERS IN POINT FORMAT

GIVE CONCISE ANSWERS

ASK NO QUESTIONS

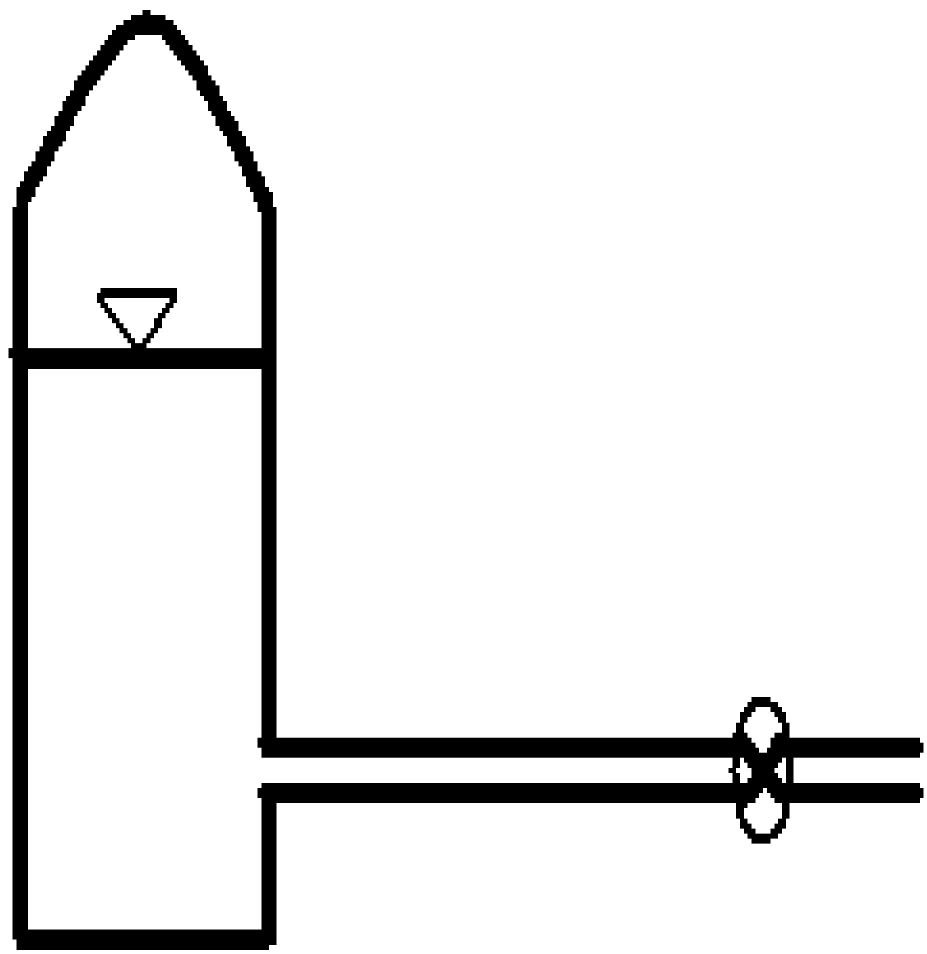
Consider a pipe with a valve at its downstream end and a large pressurized tank at its upstream end. Using wave reflection concepts, explain what happens inside the pipe when there is a sudden valve closure. [10]

When the valve is suddenly closed, it creates a flow imbalance. A high pressure or surge wave propagates up the pipe. As it does so, it brings the fluid to rest. The pipe has high pressure all along its length.

When the surge wave reaches the tank, it creates a pressure imbalance. A backflow wave is created. The backflow wave propagates down the pipe restoring pressure everywhere to its original level.

When the backflow wave reaches the valve, it creates a flow imbalance. This causes a low pressure or suction wave to propagate up the pipe. As it does so, it brings the fluid to rest. The pipe has low pressure all along its length.

When the suction wave reaches the tank, it creates a pressure imbalance. An inflow wave is created. The inflow wave travels down the pipe restoring pressure to its original level. Conditions in the pipe become what they were just before the valve was closed.



Consider a small deadend pipe attached to a large pipe with a valve at its downstream end and a large pressurized tank at its upstream end. Using algerbraic waterhammer concepts, explain how you would calculate pressure and flow velocity in the deadend pipe following a sudden valve closure in the large pipe. Sketch the PU plot for the deadend pipe. [15] Identify the important equations.

The small deadend pipe does not influence waves in the large pipe. The surge wave in the large pipe caused by a sudden valve closure causes a sudden pressure rise at the entrance to the deadend pipe. It fixes the pressure there. Let the entrance be A and the deadend be B.

Move along an F wave from B to A. For an F wave:

$$\Delta P = + \rho a \ \Delta U$$

$$[P_A - P_B] = + \rho a \ [U_A - U_B]$$

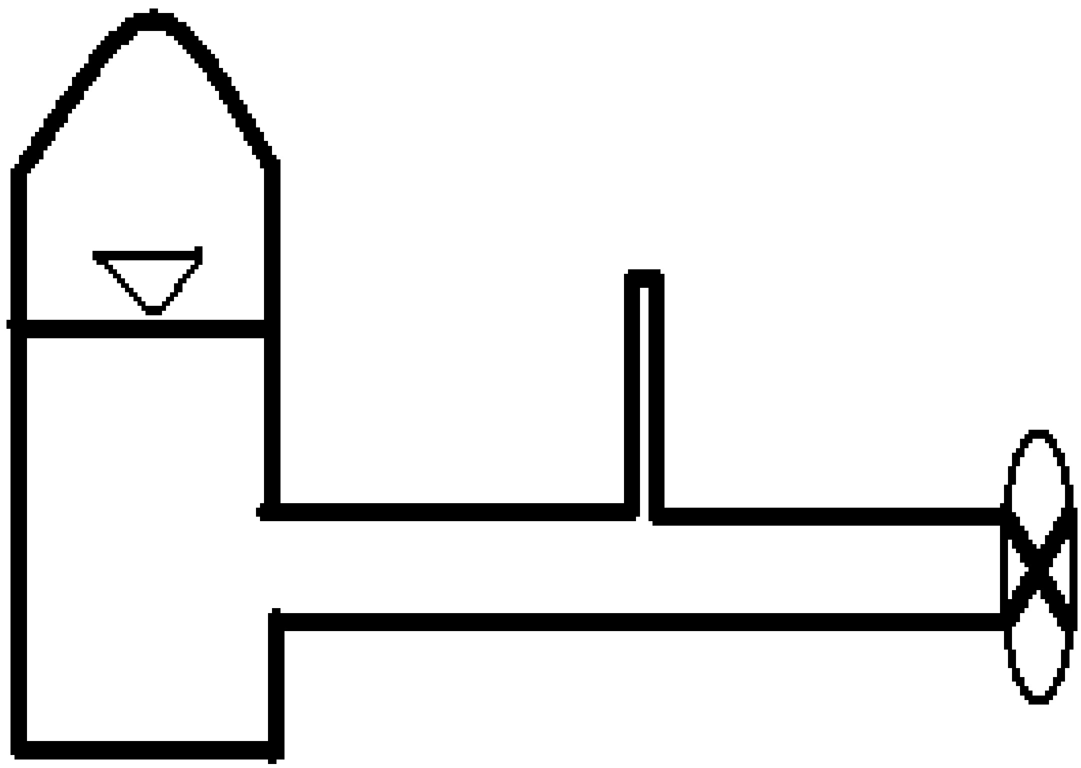
In this equation, $U_B=0$; $P_B=P_O$; $P_A=P_S$. It gives U_A.

Move along an f wave from A to B. For an f wave:

$$\Delta P = - \rho a \ \Delta U$$

$$[P_B - P_A] = - \rho a \ [U_B - U_A]$$

In this equation, $U_B=0$; $P_A=P_S$; U_A known. It gives P_B.



Explain how you would calculate the drift speed generated by an explosion. [15] Identify the important equations.

Assume that the pressure ratio across the shock wave generated by the explosion is known. In the shock frame, air upstream moves towards the shock at supersonic speed and air downstream moves away from it at subsonic speed. The pressure ratio equation gives M_U:

$$P_D/P_U = 1 + [2k/(k+1)]\ (M_U M_U - 1)$$

The wave speed equation gives a_U:

$$a_U = \sqrt{[kRT_U]}$$

The Mach Number equation gives U_U:

$$M_U = U_U/a_U$$

The Mach Number connection gives M_D:

$$M_D M_D = [(k-1)\ M_U M_U + 2]\ /\ [2k\ M_U M_U - (k-1)]$$

The temperature ratio equation gives T_D:

$$T_D/T_U = [\ (1 + [(k-1)/2]\ M_U M_U)\ /\ (1 + [(k-1)/2]\ M_D M_D)\ ]$$

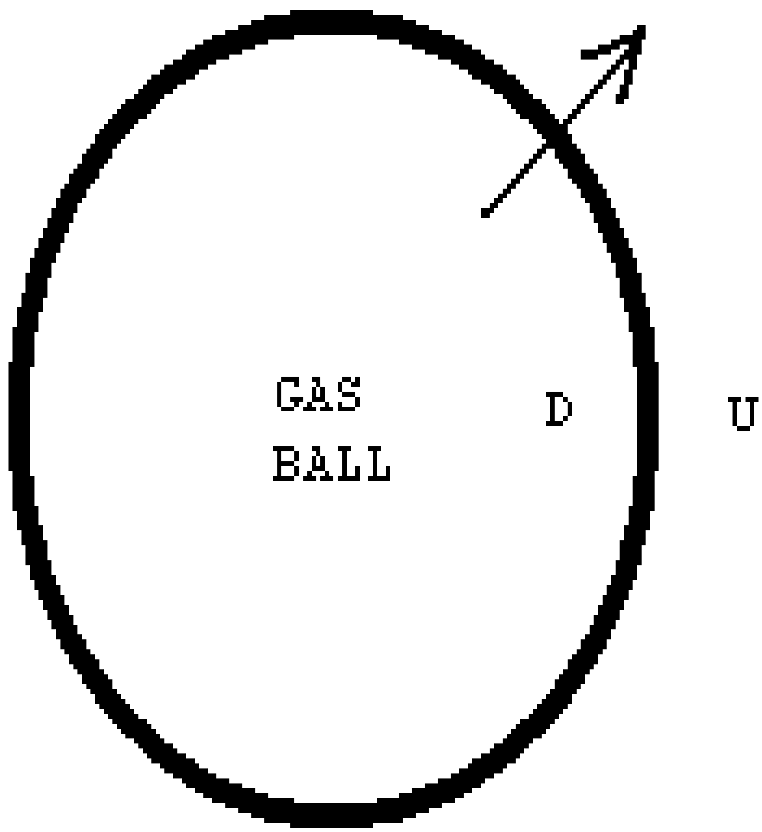
The wave speed equation gives a_D:

$$a_D = \sqrt{[kRT_D]}$$

The Mach Number equation gives U_D:

$$M_D = U_D/a_D$$

The drift speed is U_U minus U_D.


GAS
BALL
D
U

Explain how you would calculate the thrust of an ideal rocket nozzle. [15] Identify the important equations.

Let the combustion chamber be U and the nozzle exit be D and the nozzle throat be T. For an ideal rocket nozzle, M_U is zero and M_T is unity. Flow is isentropic throughout the nozzle. Pressure and temperature in the combustion chamber are known. The nozzle throat diameter is known. The nozzle exit pressure is atmospheric.

The thrust is $\dot{M}\,U_D$. The mass flow rate is:

$$\dot{M} = \rho_T A_T U_T = P_T/[RT_T]\ A_T \sqrt{[kRT_T]}$$

The isentropic temperature ratio equation gives T_T:

$$T_T/T_U = (1 + [(k-1)/2]\ M_U M_U) \ / \ (1 + [(k-1)/2]\ M_T M_T)$$

The isentropic pressure ratio equation gives P_T:

$$P_T/P_U = [T_T/T_U]^x \qquad x = k/(k-1)$$

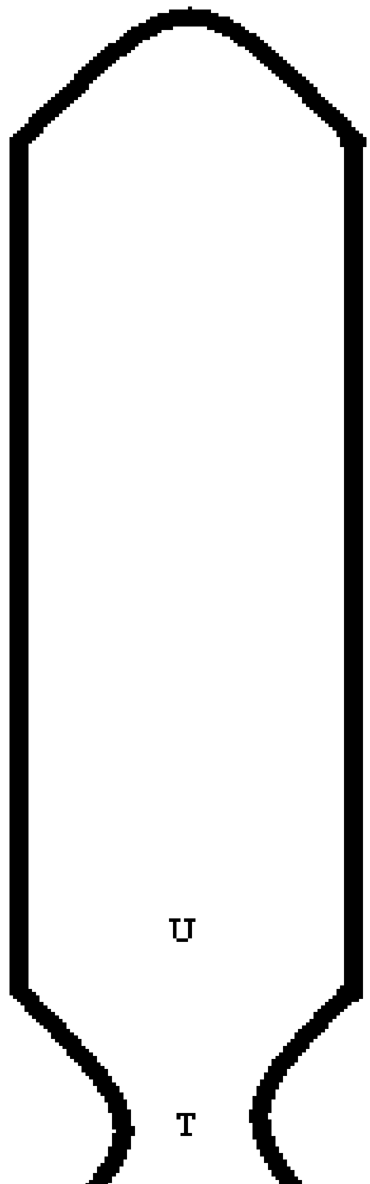
The isentropic pressure ratio equation gives M_D:

$$P_D/P_U = [\ (1 + [(k-1)/2]\ M_U M_U) \ / \ (1 + [(k-1)/2]\ M_D M_D)\ ]^x$$

The isentropic temperature ratio equation give T_D:

$$T_D/T_U = (1 + [(k-1)/2]\ M_U M_U) \ / \ (1 + [(k-1)/2]\ M_D M_D)$$

The wave speed equation $a_D = \sqrt{[kRT_D]}$ gives a_D. The Mach Number equation $M_D = U_D/a_D$ gives U_D.


U
T
D

Explain how you would calculate the lift on a supersonic flat plate foil. Identify the important equations. [15]

A flat plate supersonic foil has an expansion wave at the leading edge at the top and an oblique shock wave at the leading edge on the bottom.

With known upstream Mach Number M_U and attack angle Θ, the oblique shock plot gives the shock angle β. Substitution into the normal Mach Number equation gives N_U:

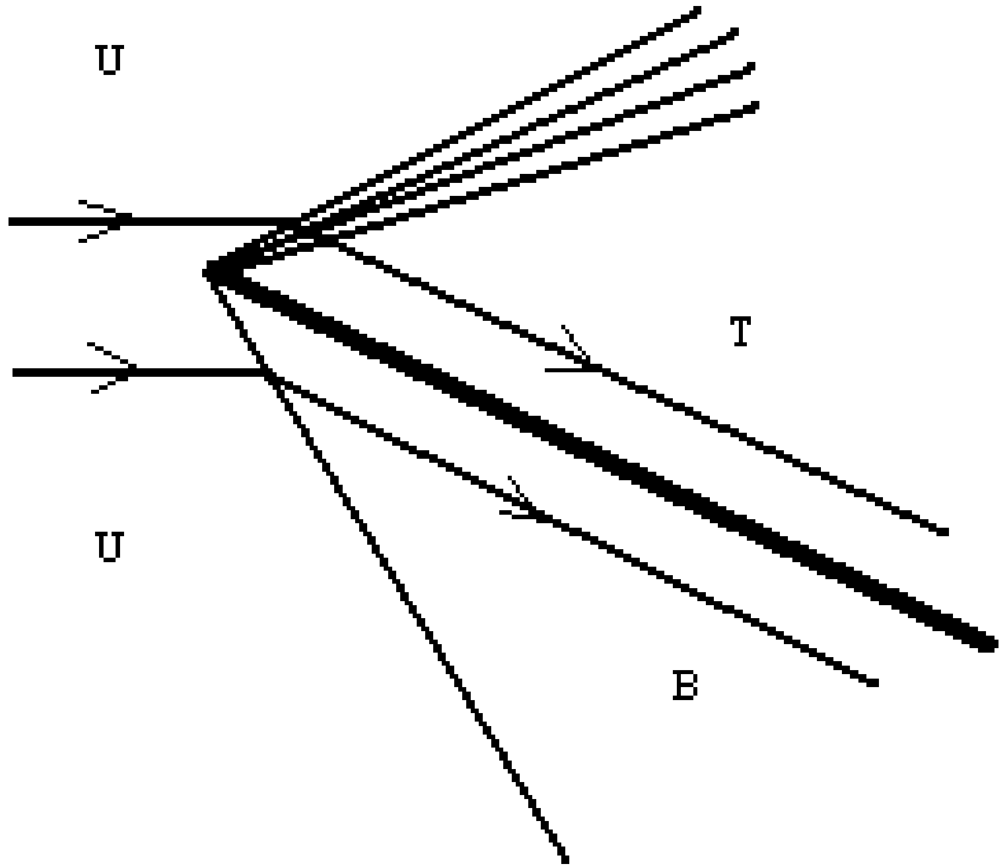
$$N_U = M_U \operatorname{Sin}\beta$$

Substitution into the pressure ratio equation gives P_B:

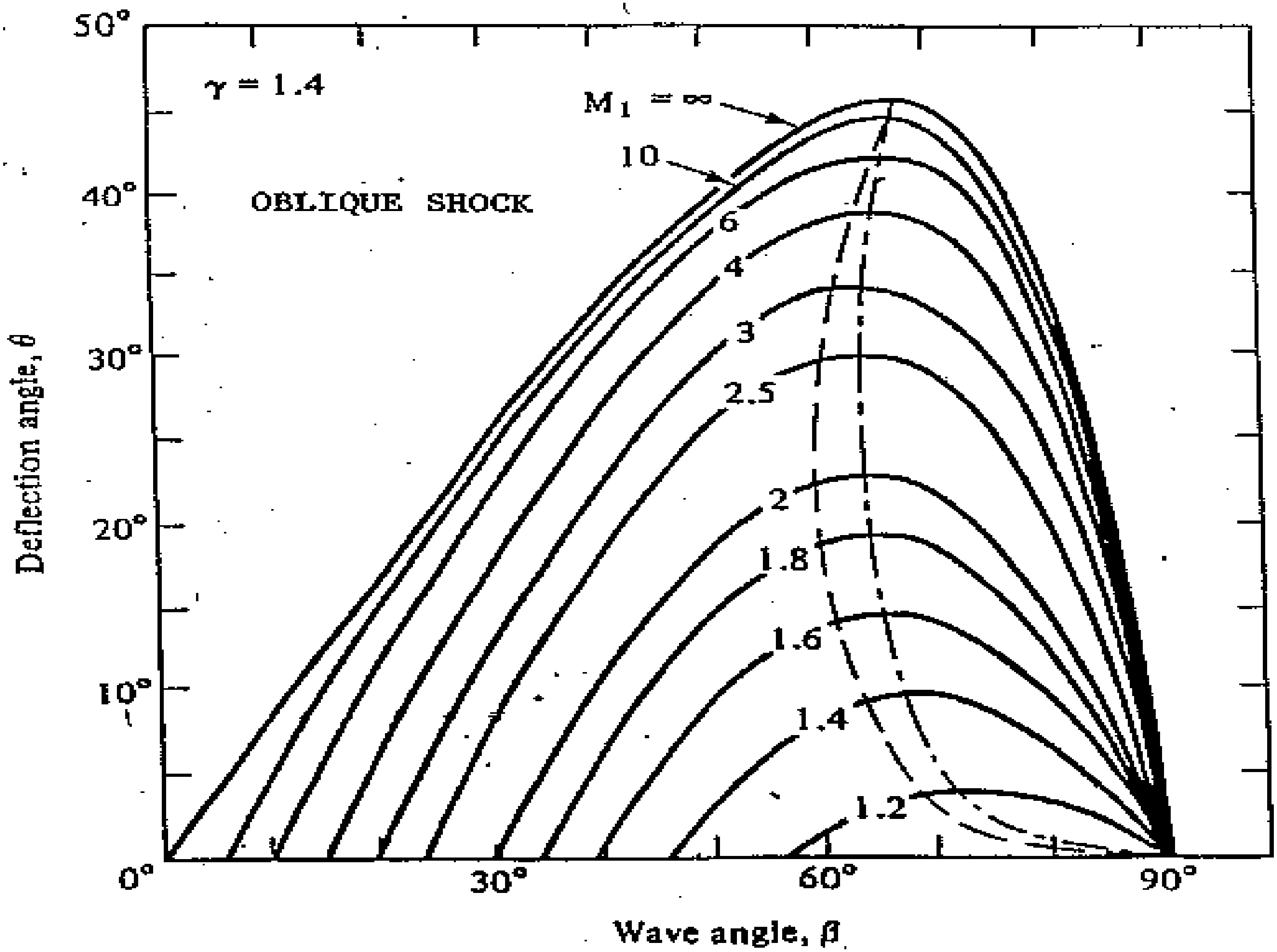
$$P_B/P_U = 1 + [2k/(k+1)] (N_U N_U - 1)$$

With known upstream Mach Number M_U and attack angle Θ, the expansion wave plot gives M_T. The isentropic pressure ratio equation then gives P_T. The lift is:

$$P_B \; C \; \operatorname{Cos}\Theta \; - P_T \; C \; \operatorname{Cos}\Theta$$

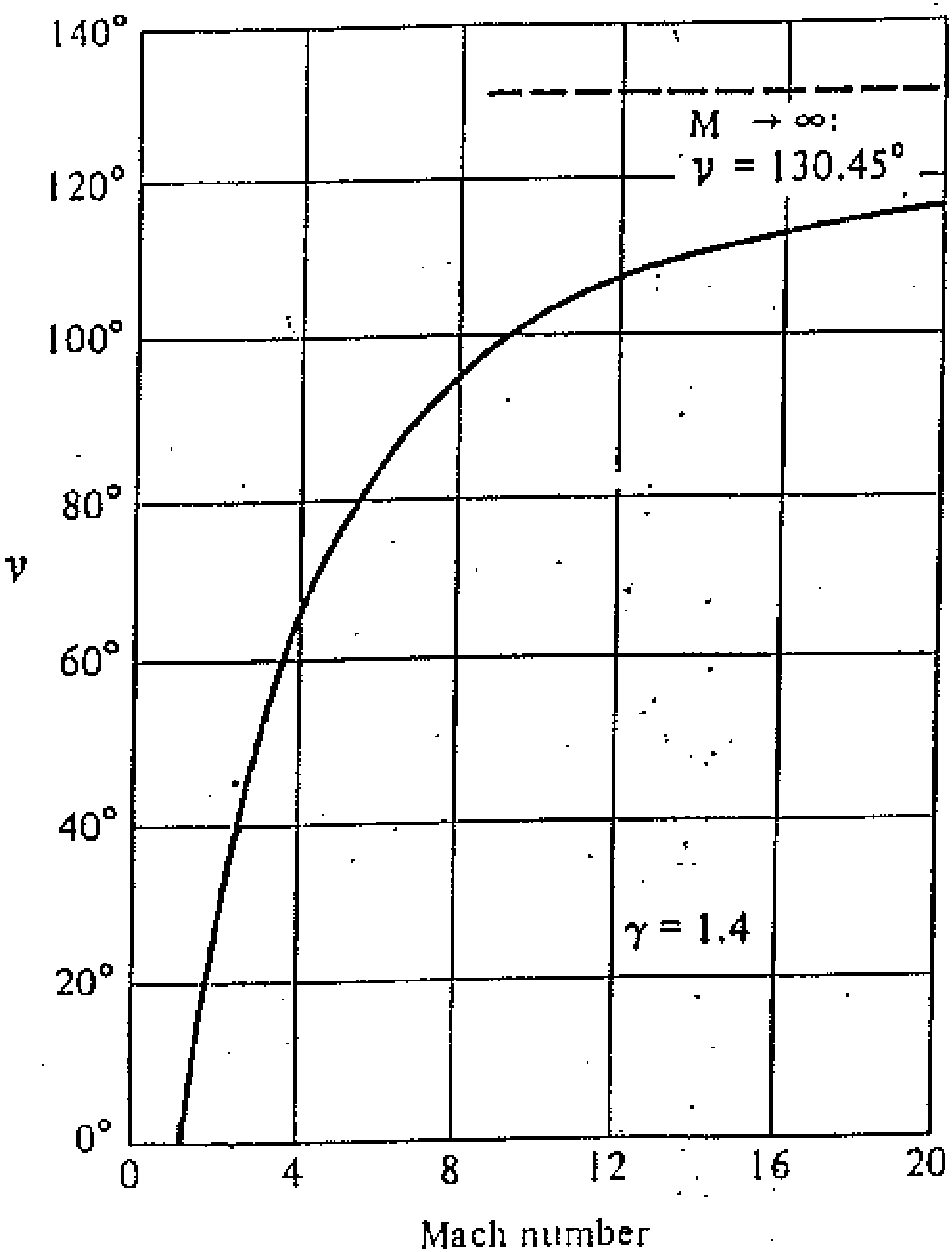


U
T
U
B



γ = 1.4
OBLIQUE SHOCK
M1 = ∞
10
6
4
3
2.5
2
1.8
1.6
1.4
1.2
Deflection angle, θ
50°
40°
30°
20°
10°
0°
30°
60°
90°
Wave angle, β

EXPANSION WAVE



Explain how you would calculate the pressure at points on a Joukowsky foil. [10] Identify the important equations.

An application of the Bernoulli equation from a point far upstream of the foil to a point on the foil gives:

$$P = \rho/2\ [\ S^2 - (\partial\varphi/\partial c)^2\ ]$$

An approximation to this is:

$$\rho/2\ [\ S^2 - (\Delta\varphi/\Delta c)^2\ ]$$

Geometry gives:

$$\Delta c = \sqrt{[\Delta\alpha^2 + \Delta\beta^2]}$$

$$\alpha = x + xa^2/(x^2+y^2) \qquad \beta = y - ya^2/(x^2+y^2)$$

$$x = \mathbf{X} - n \qquad y = \mathbf{Y} + m$$

$$\mathbf{X} = -\ R\ \mathrm{Cos}\Upsilon \qquad \mathbf{Y} = +\ R\ \mathrm{Sin}\Upsilon$$

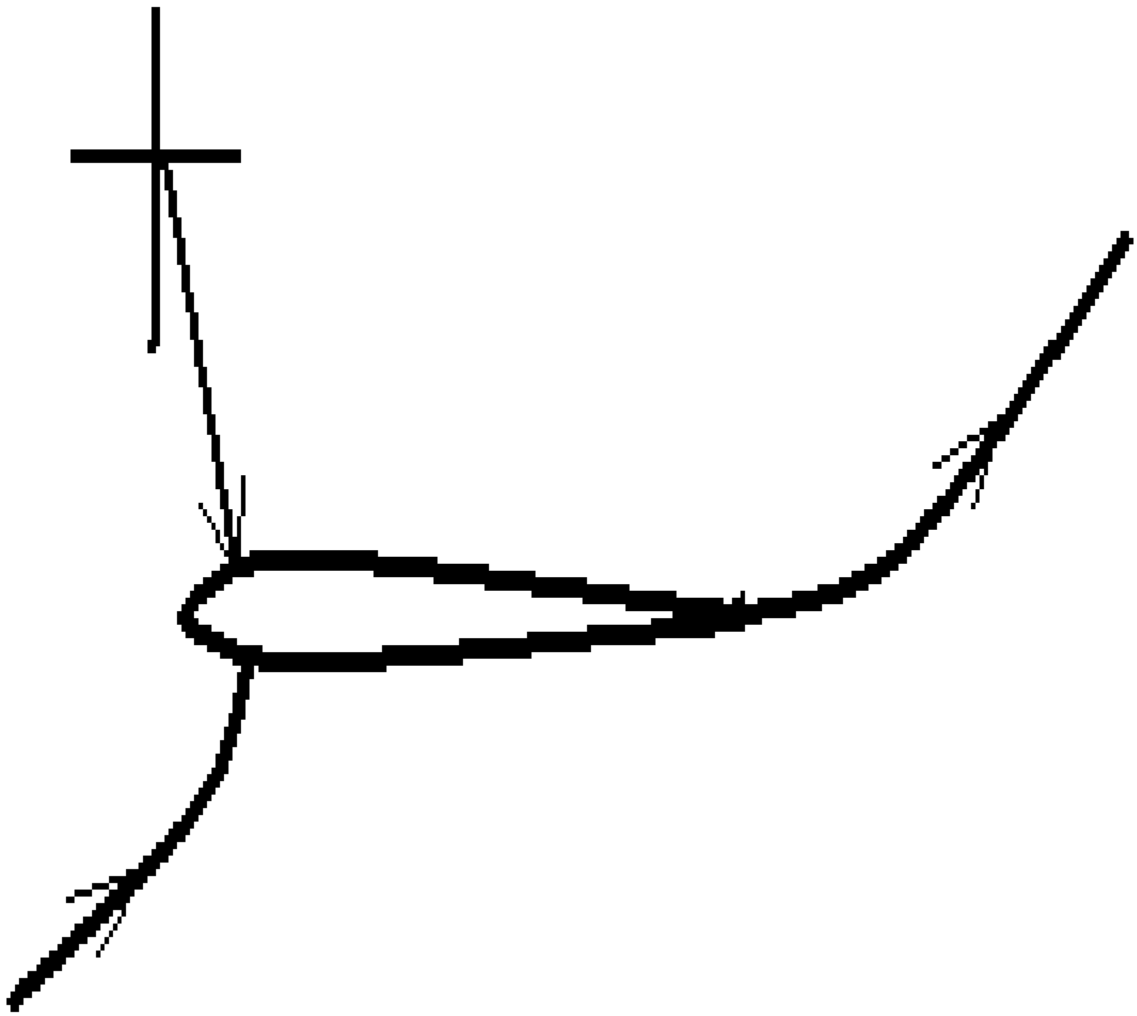
Potential flow theory gives:

$$\Gamma = 4\pi SR\ \mathrm{Sin}\kappa$$

$$\kappa = \Theta + \varepsilon \qquad \varepsilon = \tan^{-1}\ [m/(n+a)]$$

$$\Delta\varphi = 2S\ \Delta X + \ \Gamma/[2\pi]\ \Delta\sigma$$

$$X = \mathbf{X}\ \mathrm{Cos}\Theta + \mathbf{Y}\ \mathrm{Sin}\Theta$$



Explain how you would calculate lift on a Joukowsky foil from calculated pressure. What is the theoretical lift on the foil? [10] Identify the important equations.

The lift on the foil is:

$$\Delta L = P\Delta c \, \mathrm{Sin}(\boldsymbol{\theta}-\Theta)$$

The foil normal $\boldsymbol{\theta}$ is:

$$\boldsymbol{\theta} = \tan^{-1}[-\Delta\alpha/+\Delta\beta]$$

The total lift L is:

$$L=\Sigma\Delta L$$

The theoretical lift is:

$$\rho S\Gamma$$

The circulation is:

$$\Gamma = 4\pi SR \, \mathrm{Sin}\kappa$$

$$\kappa = \Theta + \varepsilon \qquad \varepsilon = \tan^{-1}[m/(n+a)]$$

Answer TRUE or FALSE and briefly explain each answer [10]:

(1) Shock waves cannot occur in water. False
Water has a finite speed of sound.

(2) Waterhammer waves travel at supersonic speeds. False
Waterhammer waves travel at sonic or subsonic speeds.

(3) A single Mach wave cannot be heard. True
It is generated by an infinitesimal disturbance.

(4) A free falling body cannot have M>1. False
For a spear like body drag will be less than weight.

(5) At singularities, flow speed is zero. False
At singularities flow speed is infinite.

(6) An expansion shock wave is not possible. True
The Second Law shows $\Delta S<0$ when M<1 goes M>1.

(7) A doublet is the source of foil lift. False
A vortex is the source of foil lift.

(8) Viscous flows are irrotational flows. False
Fluid spin is not zero in viscous flows.

(9) Flow is isentropic in ideal rocket engines. True
Friction and heat transfer are insignificant.

(10) Joukowsky foils are subsonic foils. True
Theory assumes that the fluid is incompressible.

BONUS [5]

Explain how you would calculate the pressure and the temperature at the stagnation point on a blunt object in a supersonic flow. Identify the important equations.

A bow shock wave forms directly in front of a blunt object in a supersonic flow. Let A be just upstream of the shock and B be just downstream. At the stagnation point on the object, M_S is zero. Conditions at A are known.

The temperature ratio equation based on energy gives the stagnation point temperature T_S:

$$T_S/T_A = [\ (1 + [(k-1)/2]\ M_A M_A)\ /\ (1 + [(k-1)/2]\ M_S M_S)\ ]$$

The pressure ratio equation gives the pressure downstream of the bow shock wave P_B:

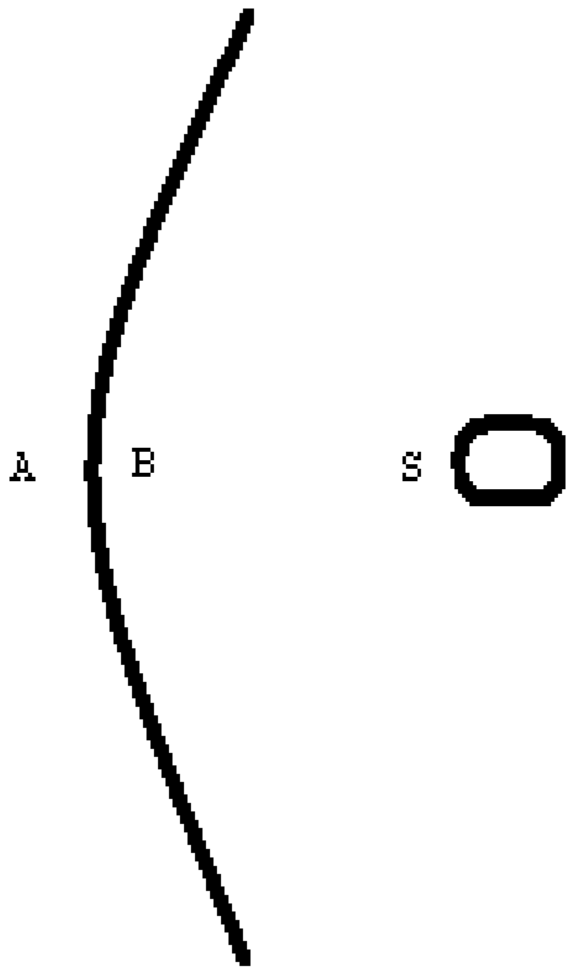
$$P_B/P_A = 1 + [2k/(k+1)]\ (M_A M_A - 1)$$

The Mach Number connection gives M_B:

$$M_B M_B = [(k-1)\ M_A M_A + 2]\ /\ [2k\ M_A M_A - (k-1)]$$

The isentropic pressure ratio equation gives P_S:

$$P_S/P_B = [\ (1 + [(k-1)/2]\ M_B M_B)\ /\ (1 + [(k-1)/2]\ M_S M_S)\ ]^x$$


A
B
S

NAME: ____

ENGINEERING 6961

FLUID MECHANICS II

PROCESS CLASS

QUIZ #2

NO NOTES OR TEXTS ALLOWED

NO CALCULATORS ALLOWED

GIVE ANSWERS IN POINT FORMAT

GIVE CONCISE ANSWERS

ASK NO QUESTIONS

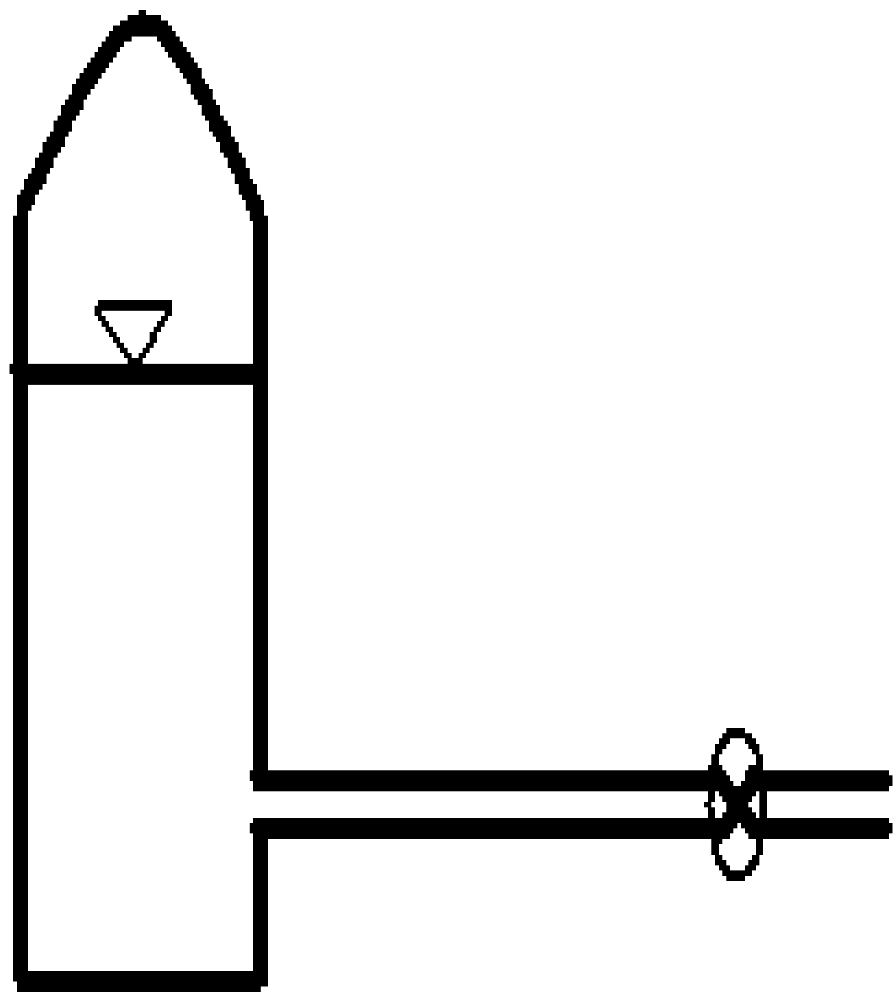
Consider a pipe with a valve at its downstream end and a large pressurized tank at its upstream end. Using wave reflection concepts, explain what happens inside the pipe when there is a sudden valve closure. [10]

When the valve is suddenly closed, it creates a flow imbalance. A high pressure or surge wave propagates up the pipe. As it does so, it brings the fluid to rest. The pipe has high pressure all along its length.

When the surge wave reaches the tank, it creates a pressure imbalance. A backflow wave is created. The backflow wave propagates down the pipe restoring pressure everywhere to its original level.

When the backflow wave reaches the valve, it creates a flow imbalance. This causes a low pressure or suction wave to propagate up the pipe. As it does so, it brings the fluid to rest. The pipe has low pressure all along its length.

When the suction wave reaches the tank, it creates a pressure imbalance. An inflow wave is created. The inflow wave travels down the pipe restoring pressure to its original level. Conditions in the pipe become what they were just before the valve was closed.



Consider a small deadend pipe attached to a large pipe with a valve at its downstream end and a large pressurized tank at its upstream end. Using the method of characteristics, explain how you would calculate the pressure and flow velocity at 3 points in the deadend pipe following a sudden valve closure in the large pipe. Do surge wave case only. Use waterhammer analysis if you cannot remember method of characteristics. Identify the important equations. [15]

The deadend pipe does not influence the large pipe. The surge wave in the large pipe caused by a sudden valve closure causes a sudden pressure rise at the entrance to the deadend pipe. It fixes the pressure there. Let the entrance be A and the deadend be B.

Move along an F wave from B to A. For an F wave:

$$\Delta P = + \rho a \ \Delta U$$

$$[P_A - P_B] = + \rho a \ [U_A - U_B]$$

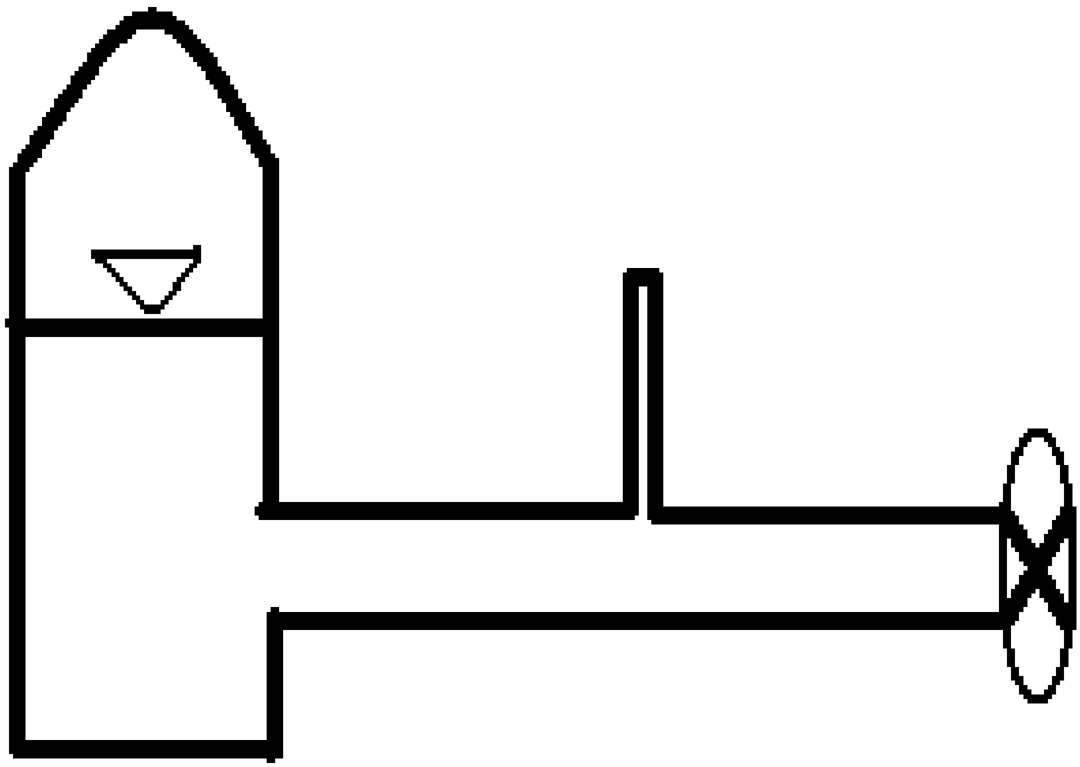
In this equation, $U_B=0$; $P_B=P_O$; $P_A=P_S$. It gives U_A.

Move along an f wave from A to B. For an f wave:

$$\Delta P = - \rho a \ \Delta U$$

$$[P_B - P_A] = - \rho a \ [U_B - U_A]$$

In this equation, $U_B=0$; $P_A=P_S$; U_A known. It gives P_B.



Let A B C indicate present pressure and flow velocity at 3 points along the deadend pipe: A is at the entrance to the deadend pipe, B is in the middle of the deadend pipe and C is at the deadend. Let I J K indicate pressure and flow velocity a step forward in time at the corresponding points. At the entrance the pressure is the surge pressure P_S. At the deadend the flow velocity is zero.

At the deadend, the C^+ characteristic gives

$$U_K - U_B + (P_K-P_B)/[\rho a] = 0$$

$$P_K = + [\rho a]\ U_B + P_B$$

At the entrance, the C^- characteristic gives

$$U_I - U_B - (P_I-P_B)/[\rho a] = 0$$

$$U_I = U_B + (P_I-P_B)/[\rho a]$$

At the middle, the C^+ and C^- characteristics give

$$U_J = 0.5\ [\ U_A + U_C + [P_A-P_C]/[\rho a]\ ]$$

$$P_J = 0.5\ [P_A + P_B + [\rho a][U_A-U_C]\ ]$$

Explain how you would calculate the drift speed generated by an explosion. [15] Identify the important equations.

Assume that the pressure ratio across the shock wave generated by the explosion is known. In the shock frame, air upstream moves towards the shock at supersonic speed and air downstream moves away from it at subsonic speed. The pressure ratio equation gives M_U:

$$P_D/P_U = 1 + [2k/(k+1)]\ (M_U M_U - 1)$$

The wave speed equation gives a_U:

$$a_U = \sqrt{[kRT_U]}$$

The Mach Number equation gives U_U:

$$M_U = U_U/a_U$$

The Mach Number connection gives M_D:

$$M_D M_D = [(k-1)\ M_U M_U + 2]\ /\ [2k\ M_U M_U - (k-1)]$$

The temperature ratio equation gives T_D:

$$T_D/T_U = [\ (1 + [(k-1)/2]\ M_U M_U)\ /\ (1 + [(k-1)/2]\ M_D M_D)\ ]$$

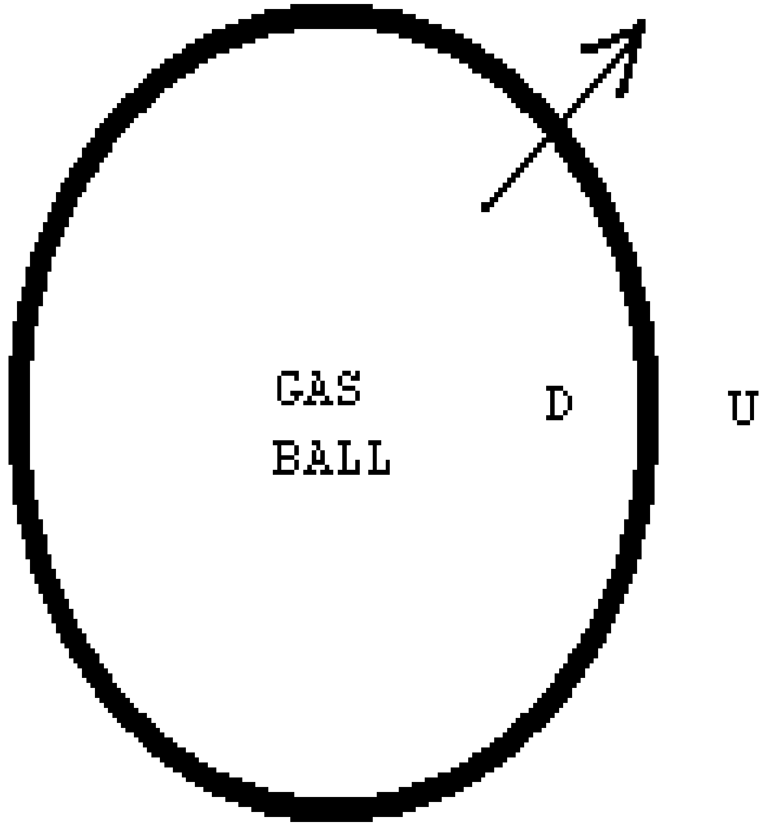
The wave speed equation gives a_D:

$$a_D = \sqrt{[kRT_D]}$$

The Mach Number equation gives U_D:

$$M_D = U_D/a_D$$

The drift speed is U_U minus U_D.


GAS
BALL
D
U

Explain how you would calculate the thrust of an ideal rocket nozzle. [15] Identify the important equations.

Let the combustion chamber be U and the nozzle exit be D and the nozzle throat be T. For an ideal rocket nozzle, M_U is zero and M_T is unity. Flow is isentropic throughout the nozzle. Pressure and temperature in the combustion chamber are known. The nozzle throat diameter is known. The nozzle exit pressure is atmospheric.

The thrust is $\dot{M} U_D$. The mass flow rate is:

$$\dot{M} = \rho_T A_T U_T = P_T/[RT_T] \; A_T \sqrt{[kRT_T]}$$

The isentropic temperature ratio equation gives T_T:

$$T_T/T_U = (1 + [(k-1)/2] M_U M_U) / (1 + [(k-1)/2] M_T M_T)$$

The isentropic pressure ratio equation gives P_T:

$$P_T/P_U = [T_T/T_U]^x \qquad x = k/(k-1)$$

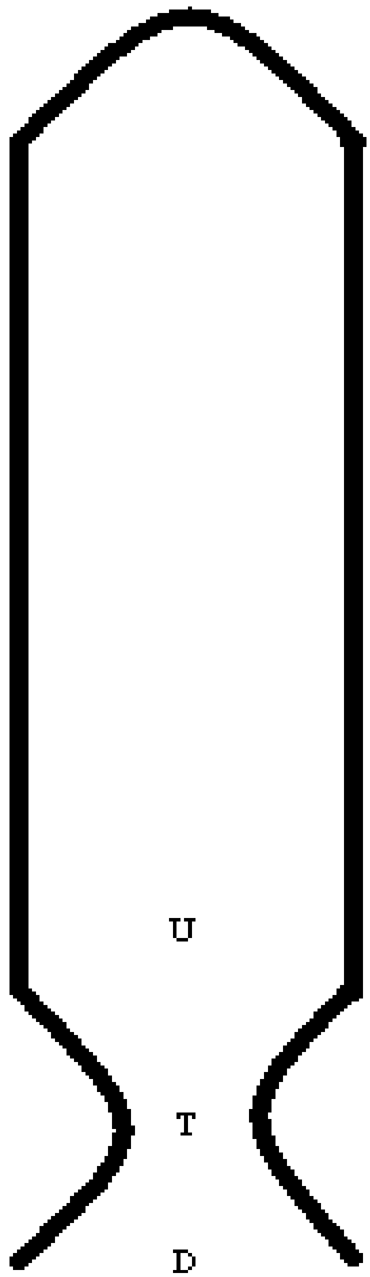
The isentropic pressure ratio equation gives M_D:

$$P_D/P_U = [\; (1 + [(k-1)/2] M_U M_U) / (1 + [(k-1)/2] M_D M_D) \;]^x$$

The isentropic temperature ratio equation give T_D:

$$T_D/T_U = (1 + [(k-1)/2] M_U M_U) / (1 + [(k-1)/2] M_D M_D)$$

The wave speed equation $a_D = \sqrt{[kRT_D]}$ gives a_D. The Mach Number equation $M_D = U_D/a_D$ gives U_D.


U
T
D

Explain how you would calculate the pressure and flow velocity changes which occur when a high speed flow moves down a pipe. Identify the important equations. [15]

Gas dynamics theory gives

$$\Delta M^2/M^2 = kM^2[1+[(k-1)/2]M^2] /[1-M^2] \; f\Delta x/D$$

$$\Delta P/P = -kM^2[1+(k-1) M^2]/[2(1-M^2)] \; f\Delta x/D$$

$$\Delta T/T = -k(k-1)M^4/[2(1-M^2)] \; f\Delta x/D$$

These can be put into the form:

$$\Delta M^2 = A \; \Delta x$$

$$\Delta P = B \; \Delta x$$

$$\Delta T = C \; \Delta x$$

For moving a step down a pipe, these equations give:

$$[M^2]_{NEW} = [M^2]_{OLD} + A_{OLD} \; \Delta x$$

$$P_{NEW} = P_{OLD} + B_{OLD} \; \Delta x$$

$$T_{NEW} = T_{OLD} + C_{OLD} \; \Delta x$$

Note that A B C is each a function of M^2 so they must be updated after each step down the pipe.

Explain how you would calculate the pressure at points on a Joukowsky foil. [10] Identify the important equations.

An application of the Bernoulli equation from a point far upstream of the foil to a point on the foil gives:

$$P = \rho/2\ [\ S^2 - (\partial\varphi/\partial c)^2\ ]$$

An approximation to this is:

$$\rho/2\ [\ S^2 - (\Delta\varphi/\Delta c)^2\ ]$$

Geometry gives:

$$\Delta c = \sqrt{[\Delta\alpha^2+\Delta\beta^2]}$$

$$\alpha = x + xa^2/(x^2+y^2) \qquad \beta = y - ya^2/(x^2+y^2)$$

$$x = \mathbf{X} - n \qquad y = \mathbf{Y} + m$$

$$\mathbf{X} = -\ R\ \mathrm{Cos}\Upsilon \qquad \mathbf{Y} = +\ R\ \mathrm{Sin}\Upsilon$$

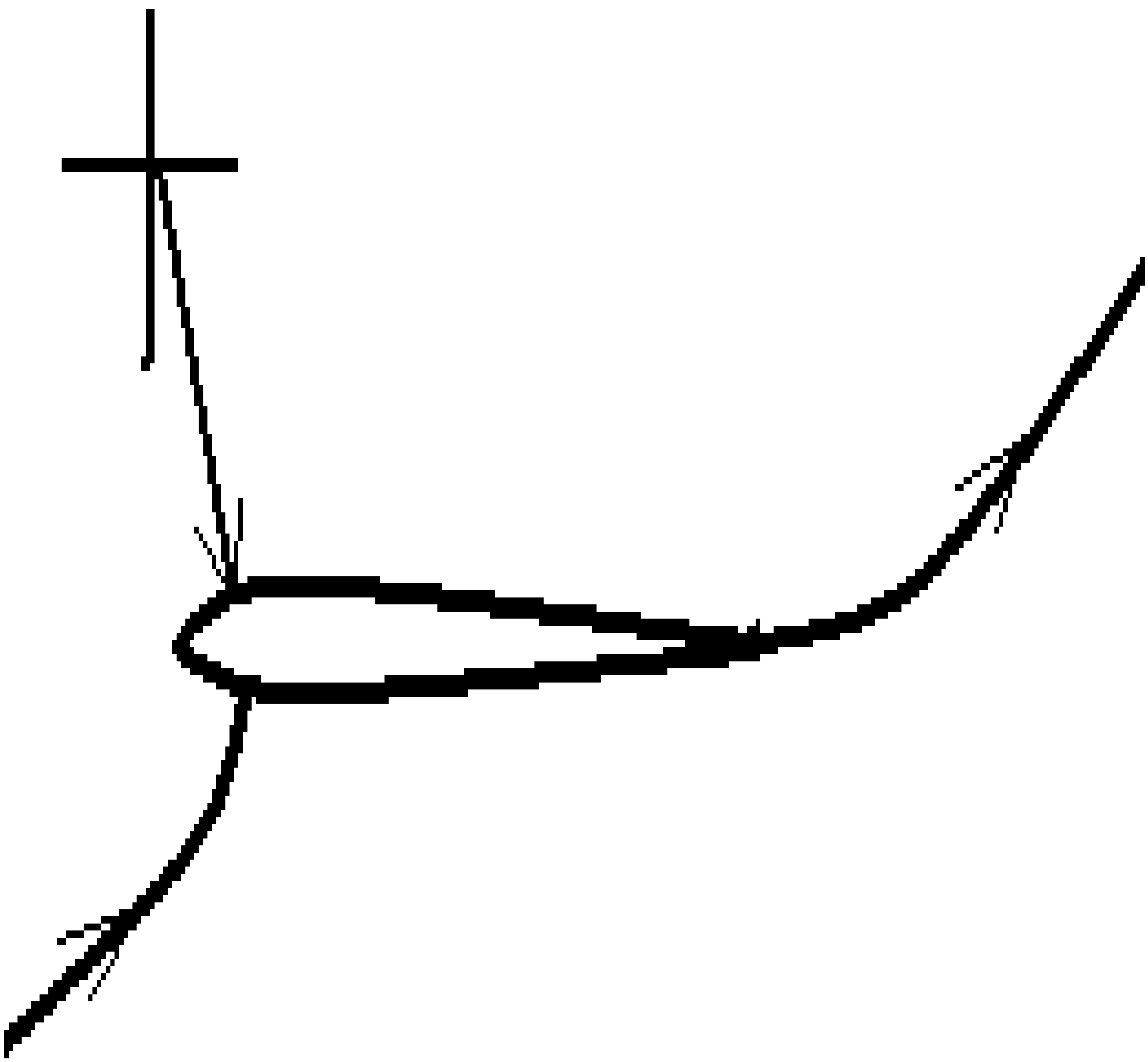
Potential flow theory gives:

$$\Gamma = 4\pi SR\ \mathrm{Sin}\kappa$$

$$\kappa = \Theta + \varepsilon \qquad \varepsilon = \tan^{-1}\ [m/(n+a)]$$

$$\Delta\varphi = 2S\ \Delta X + \ \Gamma/[2\pi]\ \Delta\sigma$$

$$X = \mathbf{X}\ \mathrm{Cos}\Theta + \mathbf{Y}\ \mathrm{Sin}\Theta$$



Explain how you would calculate lift on a Joukowsky foil from calculated pressure. What is the theoretical lift on the foil? [10] Identify the important equations.

The lift on the foil is:

$$\Delta L = P\Delta c \ \mathrm{Sin}(\boldsymbol{\theta}-\Theta)$$

The foil normal $\boldsymbol{\theta}$ is:

$$\boldsymbol{\theta} = \tan^{-1}[-\Delta\alpha/+\Delta\beta]$$

The total lift L is:

$$L=\Sigma\Delta L$$

The theoretical lift is:

$$\rho S\Gamma$$

The circulation is:

$$\Gamma = 4\pi SR \ \mathrm{Sin}\kappa$$

$$\kappa = \Theta + \varepsilon \qquad \varepsilon = \tan^{-1}\ [m/(n+a)]$$

Answer TRUE or FALSE and briefly explain each answer [10]:

(1) Shock waves cannot occur in water. False
Water has a finite speed of sound.

(2) Waterhammer waves travel at supersonic speeds. False
Waterhammer waves travel at sonic or subsonic speeds.

(3) A single Mach wave cannot be heard. True
It is generated by an infinitesimal disturbance.

(4) A free falling body cannot have $M>1$. False
For a spear like body drag will be less than weight.

(5) At singularities, flow speed is zero. False
At singularities flow speed is infinite.

(6) An expansion shock wave is not possible. True
The Second Law shows $\Delta S<0$ when $M<1$ goes $M>1$.

(7) A doublet is the source of foil lift. False
A vortex is the source of foil lift.

(8) Viscous flows are irrotational flows. False
Fluid spin is not zero in viscous flows.

(9) Flow is isentropic in ideal rocket engines. True
Friction and heat transfer are insignificant.

(10) Joukowsky foils are subsonic foils. True
Theory assumes that the fluid is incompressible.

BONUS [5]

Explain how you would calculate the pressure and the temperature at the stagnation point on a blunt object in a supersonic flow. Identify the important equations.

A bow shock wave forms directly in front of a blunt object in a supersonic flow. Let A be just upstream of the shock and B be just downstream. At the stagnation point on the object, M_S is zero. Conditions at A are known.

The temperature ratio equation based on energy gives the stagnation point temperature T_S:

$$T_S/T_A = [\ (1 + [(k-1)/2]\ M_A M_A)\ /\ (1 + [(k-1)/2]\ M_S M_S)\ ]$$

The pressure ratio equation gives the pressure downstream of the bow shock wave P_B:

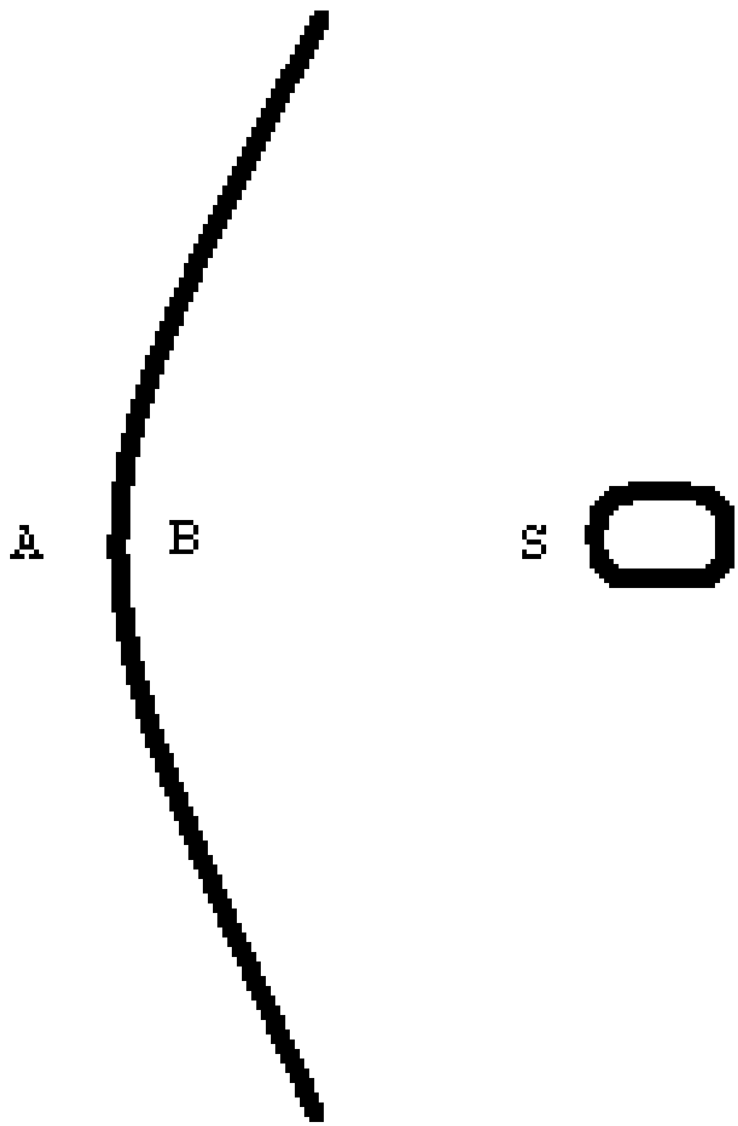
$$P_B/P_A = 1 + [2k/(k+1)]\ (M_A M_A - 1)$$

The Mach Number connection gives M_B:

$$M_B M_B = [(k-1)\ M_A M_A + 2]\ /\ [2k\ M_A M_A - (k-1)]$$

The isentropic pressure ratio equation gives P_S:

$$P_S/P_B = [\ (1 + [(k-1)/2]\ M_B M_B)\ /\ (1 + [(k-1)/2]\ M_S M_S)\ ]^x$$


A
B
S