ENGINEERING 6961

FLUID MECHANICS II

FINAL EXAMINATION

FALL 2012

INSTRUCTIONS

NO NOTES OR TEXTS ALLOWED

NO ELECTRONIC DEVICE ALLOWED

GIVE CONCISE ANSWERS

HAND IN ALL SHEETS

ASK NO QUESTIONS

TRUE OR FALSE QUESTIONS

Give a TRUE or FALSE response to each of the following statements. Briefly explain each answer. (1) Shock waves cannot occur in water. (2) A free falling body cannot move faster than the speed of sound. (3) An expansion shock wave where M goes suddenly from subsonic to supersonic is not possible. (4) Flow in hydrodynamic lubrication bearings is turbulent. (5) A potential vortex is a rotational flow. (6) An incompressible fluid has an infinite speed of sound. (7) Eddies in a turbulent flow make the fluid appear more viscous locally. (8) Gas moving through a normal shock wave undergoes an isentropic process. (9) Choked flow is a subsonic flow phenomenon. (10) The speed of the fluid at a singularity in a flow is zero.

[THIS QUESTION IS WORTH 10%: EACH QUESTION PART IS WORTH 1%]

LAB QUESTIONS

Describe the purpose, setup, procedure and main observations from each of the following labs: water hammer lab [3] shock tube lab [3] Joukowsky foil lab [4]. In each case, sketch the system and identify the important equations tested in the lab.

[THIS QUESTION IS WORTH 10%: PART QUESTION MARKS IN [] BRACKETS]

DERIVE QUESTIONS

Sheets for 3 derivations are provided. Title any 2 of the derivations and in the spaces provided on the sheets outline the derivation steps. Note: Sheets are double sided.

[THIS QUESTION IS WORTH 10%: EACH QUESTION PART IS WORTH 5%]

PROJECT QUESTIONS

EFLUIDS: Write a brief summary of 1 project fluids video.
CFD: Sketch and explain the main menus of the project cfd.

[THIS QUESTION IS WORTH 10%: EACH QUESTION PART IS WORTH 5%]

WORDS FORMULAS SKETCHES QUESTIONS

With words and formulas describe, in step format, how you would calculate any 3 of the following: (1) the lift and drag on a hemispherical hut (2) the lift and drag on a supersonic foil (3) high speed gas flow in a pipe (4) stagnation point conditions on a supersonic blunt object. Make extensive use of the formula sheets for this question. State all assumptions and approximations. Use sketches to illustrate your answers.

[THIS QUESTION IS WORTH 30%: EACH QUESTION PART IS WORTH 10%]

NUMBERS QUESTIONS

[THIS QUESTION IS WORTH 30%: EACH QUESTION PART IS WORTH 10%]

A certain pipe network consists of a pipe connecting two water tanks. The pipe has a valve midway along its length. A small deadend pipe is attached to the section of pipe downstream of the valve. Initially the valve is partially open and the conditions in the section of the pipe upstream of it are P_o=60BAR U_o=2m/s while the conditions downstream are P_o=40BAR U_o=2m/s. Water density ρ is 1000kg/m^3. The wave speed **a** for the main pipe is 1000m/s. The wave speed **a** for the deadend pipe is 500m/s.

Using wave propagation concepts, explain what happens in the downstream pipe following a sudden valve closure.

Using algebraic water hammer analysis, determine the pressure and velocity at the ends of the downstream pipe following a sudden valve closure for 4 steps in time.

Using wave propagation concepts, explain what happens in the deadend pipe following the sudden valve closure.

Using algebraic water hammer analysis, determine the pressure and velocity at the ends of the deadend pipe for 4 steps in time following the sudden valve closure.

[THIS QUESTION IS WORTH 10%: EACH QUESTION PART IS WORTH 2.5%]

A certain subsonic air foil is constructed from a circle with radius R=0.25m and offsets m=0m and n=0m. Its angle of attack θ is 45° and its speed S is [100√2]m/s. Air density ρ is [1/√2]kg/m^3.

Calculate the theoretical lift on the foil.

Map the front and back points on the circle to a foil plane.

Calculate the pressure at points midway between mapped points.

Calculate the lift on the foil from the calculated pressures.

[THIS QUESTION IS WORTH 10%: EACH QUESTION PART IS WORTH 2.5%]

A certain Cartesian hydrodynamic lubrication bearing has a front gap of 5mm and a back gap of 1mm. The pad is 0.2m long by 0.2m wide. The speed of the pad is 0.2m/s. The viscosity of its oil is 1/6 Ns/m^2. The sides of the bearing are open to atmosphere.

Derive a 5 point CFD template for the bearing.

Calculate the pressure at the middle of the bearing.

Calculate the load supported by the bearing.

Calculate the load for a blocked sides case.

[THIS QUESTION IS WORTH 10%: EACH QUESTION PART IS WORTH 2.5%]

NUMBERS SHEET

$\mathrm{Sin}[45] = 1/\sqrt{2}$ $\mathrm{Cos}[45] = 1/\sqrt{2}$

pi=3.14159   approximation   pi=3

$4^3 = 64$ $2^3 = 8$ $3^3 = 27$

64+8+27+27=126   1/126=0.008

64+8=72   1/72=0.014

$[10^{-3}]^3 = 10^{-9}$ $10^{-3} * 10^{+9} = 10^{+6}$

1000 * 1000 = 10 BAR

1000 * 500 = 5 BAR

TITLE ____

$$\partial U/\partial x + \partial V/\partial y + \partial W/\partial z = 0$$

$$\partial P/\partial x = \mu \partial^2 U/\partial z^2$$

$$\partial P/\partial y = \mu \partial^2 V/\partial z^2$$

$$0 = \mu \partial^2 W/\partial z^2$$

$$\int [\partial U/\partial x + \partial V/\partial y + \partial W/\partial z]\, dz = 0$$

$$I = \int U dz \qquad J = \int V dz \qquad K = \int W dz$$

$$\partial I/\partial x + \partial J/\partial y + \partial K/\partial z = 0$$

$$U = \partial P/\partial x \ (z^2 - zh)/2\mu + (U_T - U_B) z/h + U_B$$

$$V = \partial P/\partial y \ (z^2 - zh)/2\mu + (V_T - V_B) z/h + V_B$$

$$W = (W_T - W_B) z/h + W_B$$

$$I = \partial P/\partial x \ (-h^3/12\mu) + (U_T - U_B) h/2 + U_B h$$

$$J = \partial P/\partial y \ (-h^3/12\mu) + (V_T - V_B) h/2 + V_B h$$

$$K = (W_T - W_B) z^2/2h + W_B z$$

$$\partial/\partial x \ (h^3/12\mu \ \partial P/\partial x) + \partial/\partial y \ (h^3/12\mu \ \partial P/\partial y)$$

$$= \partial[h(U_T + U_B)/2]/\partial x + \partial[h(V_T + V_B)/2]/\partial y + (W_T - W_B)$$

TITLE: ____

$$\dot{M} = \rho_1 U_1 \Delta A = \rho_2 U_2 \Delta A$$

$$\dot{M} (U_2 - U_1) = (P_1 - P_2) \Delta A$$

$$\rho_2 U_2 \Delta A U_2 - \rho_1 U_1 \Delta A U_1 = (P_1 - P_2) \Delta A$$

$$P_1 + \rho_1 U_1 U_1 = P_2 + \rho_2 U_2 U_2$$

$$P_1 (1 + k M_1 M_1) = P_2 (1 + k M_2 M_2)$$

$$h_1 + [U_1 U_1]/2 = h_2 + [U_2 U_2]/2$$

$$T_1 (1 + (k-1)/2 M_1 M_1) = T_2 (1 + (k-1)/2 M_2 M_2)$$

$$\rho_1 \; U_1 \; \Delta A \; = \; \rho_2 \; U_2 \; \Delta A$$

$$\rho_2 \;/\; \rho_1 = \; U_1 \;/\; U_2$$

$$= [M_1 \; a_1] \;/\; [M_2 \; a_2] = [M_1 \; \sqrt{T_1}] \;/\; [M_2 \; \sqrt{T_2}]$$

$$= \sqrt{\{[1 + (k-1)/2 \; M_2 M_2] \;/\; [1 + (k-1)/2 \; M_1 M_1]\}} \;\; M_1/M_2$$

$$P_1 \;/\; [\rho_1 \; R \; T_1] \; = \; P_2 \;/\; [\rho_2 \; R \; T_2]$$

$$\rho_2/\rho_1 \; T_2/T_1 \; = \; P_2/P_1$$

$$M_2 M_2 \; = [(k-1) \; M_1 M_1 + 2] \;/\; [2k \; M_1 M_1 - (k-1)]$$

$$P_2/P_1 = 1 + 2k/(k+1) \; (M_1 M_1 - 1)$$

$$T_2/T_1 = ([1+(k-1)/2 \; M_1 M_1][2k \; M_1 M_1 - (k-1)])/[(k+1)^2/2 \; M_1 M_1]$$

$$\rho_2/\rho_1 = [(k+1) \; M_1 M_1] \;/\; [2 + (k-1) \; M_1 M_1]$$

TITLE: ______________________________

$$\rho\ \partial U/\partial t + \rho U\ \partial U/\partial x + \partial P/\partial x + \rho C = 0$$

$$C = f/D\ U|U|/2 - g\ \sin\alpha$$

$$\partial P/\partial t + U\ \partial P/\partial x + \rho a^2\ \partial U/\partial x = 0$$

$$\rho\ \partial U/\partial t + \partial P/\partial x = 0$$

$$\partial P/\partial t + \rho a^2\ \partial U/\partial x = 0$$

$$\partial^2 P/\partial t^2 = a^2\ \partial^2 P/\partial x^2$$

$$\partial^2 U/\partial t^2 = a^2\ \partial^2 U/\partial x^2$$

$$P - P_o = f(N) + F(M)$$

$$U - U_o = [f(N) - F(M)] / [\rho a]$$

$$N = x - a t \qquad M = x + a t$$

$$[P-P_o] - \rho a[U-U_o] = 2F(M)$$

$$\Delta P = + \rho a \ \Delta U$$

$$[P-P_o] + \rho a[U-U_o] = 2f(N)$$

$$\Delta P = - \rho a \ \Delta U$$

NAME ______________________________

$$\Delta M^2/M^2 = + kM^2[1+[(k-1)/2]M^2] / [1-M^2] f\Delta x/D$$

$$\Delta P/P = - kM^2[1+(k-1) M^2]/[2(1-M^2)] f\Delta x/D$$

$$\Delta T/T = - k(k-1)M^4/[2(1-M^2)] f\Delta x/D$$

$$\Delta\rho/\rho = - kM^2/[2(1-M^2)] f\Delta x/D$$

$$\Delta G = H \Delta x \qquad G_{NEW} = G_{OLD} + H_{OLD} [x_{NEW} - x_{OLD}]$$

$$fL^*/D = (1-M^2)/(kM^2) + [(k+1)/(2k)] \ln[(k+1)M^2/(2+(k-1)M^2)]$$

$$fL^*/D = (1-kM^2)/(kM^2) + \ln[kM^2]$$

$$T_D/T_U = [(1 + [(k-1)/2] M_U M_U) / (1 + [(k-1)/2] M_D M_D)]$$

$$P_D/P_U = [T_D/T_U]^x \qquad x = k/(k-1)$$

$$\dot{M} = \rho AU \qquad M = U/a \qquad a = \sqrt{kRT}$$

$$\rho = P/[RT] \qquad \dot{M} U + \Delta P A$$

$$d\rho/\rho + dA/A + dU/U = 0 \qquad UdU + a^2 d\rho/\rho = 0$$

$$dU = UdA / [A(M^2-1)]$$

$$P_D/P_U = [\ 1 + k\ M_U M_U\ ]\ /\ [\ 1 + k\ M_D M_D\ ]$$

$$T_D/T_U = [\ (1 + [(k-1)/2]\ M_U M_U)\ /\ (1 + [(k-1)/2]\ M_D M_D)\ ]$$

$$M_D M_D = [(k-1)\ M_U M_U + 2]\ /\ [2k\ M_U M_U - (k-1)]$$

$$P_D/P_U = 1 + [2k/(k+1)]\ (M_U M_U - 1)$$

$$P_D/P_U = 1 + [2k/(k+1)]\ (N_U N_U - 1)$$

$$N_D N_D = [(k-1)\ N_U N_U + 2]\ /\ [2k\ N_U N_U - (k-1)]$$

$$N_U = M_U \text{ Sin}\beta \qquad N_D = M_D \text{ Sin}\kappa \qquad \kappa=\beta-\Theta$$

$$\tan(\beta)/\tan(\kappa) = [(k+1)\ N_U\ N_U\ ]\ /\ [\ (k-1)\ N_U\ N_U + 2\ ]$$

$$\upsilon = \sqrt{[K]}\ \tan^{-1}\sqrt{[(M^2-1)/K]} - \tan^{-1}\sqrt{[M^2-1]}$$

$$K = (k+1)/(k-1) \qquad \upsilon_D = \upsilon_U + \Theta$$

$$P\,A\ \text{Sin}(\boldsymbol{\theta}-\Theta) \qquad P\,A\ \text{Cos}(\boldsymbol{\theta}-\Theta)$$

$$\partial/\partial x \ (h^3/12\mu \ \partial P/\partial x) \ + \ \partial/\partial y \ (h^3/12\mu \ \partial P/\partial y)$$

$$= \ \partial[h(U_T+U_B)/2]/\partial x \ + \ \partial[h(V_T+V_B)/2]/\partial y \ + \ (W_T-W_B)$$

$$A = [(h_E+h_P)/2]^3 \ / \ [\Delta x^2]$$

$$B = [(h_W+h_P)/2]^3 \ / \ [\Delta x^2]$$

$$C = [(h_N+h_P)/2]^3 \ / \ [\Delta y^2]$$

$$D = [(h_S+h_P)/2]^3 \ / \ [\Delta y^2]$$

$$H = - \ 6\mu \ S \ (h_E-h_W)/[2\Delta x]$$

$$\partial/\partial r \ (rh^3/12\mu \ \partial P/\partial r) \ + \ r \ \partial/\partial c \ (h^3/12\mu \ \partial P/\partial c)$$

$$= \ \partial[rh(U_T+U_B)/2]/\partial r \ + \ \partial[h(V_T+V_B)/2]/\partial\Theta \ + \ r(W_T-W_B)$$

$$A = [(h_E+h_P)/2]^3 \ r_P \ / \ [\Delta c^2]$$

$$B = [(h_W+h_P)/2]^3 \ r_P \ / \ [\Delta c^2]$$

$$C = [(h_N+h_P)/2]^3 \ [(r_N+r_P)/2] \ / \ [\Delta r^2]$$

$$D = [(h_S+h_P)/2]^3 \ [(r_S+r_P)/2] \ / \ [\Delta r^2]$$

$$H = - \ 6\mu \ r_P\omega \ (h_E-h_W)/[2\Delta\Theta]$$

$$P_P = \frac{(A \ P_E + B \ P_W + C \ P_N + D \ P_S + H)}{(A + B + C + D)}$$

$$\frac{d}{dx}\left(h^3 \frac{dP}{dx}\right) = 6\mu S \frac{dh}{dx} = H \frac{dh}{dx}$$

$$h^3 \frac{dP}{dx} = H h + A \qquad \frac{dP}{dx} = H/h^2 + A/h^3$$

$$\frac{dP}{dx} = H/(sx+b)^2 + A/(sx+b)^3 \qquad s=[a-b]/d$$

$$P = -H/[s(sx+b)] - A/[2s(sx+b)^2] + B$$

$$A = \mathbf{[P_I-P_O}][2sa^2b^2]/[b^2-a^2] - 2Hba/[b+a]$$

$$B = \mathbf{[P_I}b^2\mathbf{-P_O}a^2]/[b^2-a^2] + H/[s(b+a)]$$

$$\Delta[h^3 \, dP/dx] = H \, \Delta h$$

$$a^3 [P_O-\mathbf{P}]/v - b^3 [\mathbf{P}-P_I]/w = H [a-b]$$

$$\mathbf{P} = [a^3/v \, P_O + b^3/w \, P_I + H [b-a]] / [a^3/v + b^3/w]$$

$$\frac{d}{dy}\left(h^3 \frac{dP}{dy}\right) = 6\mu S \frac{dh}{dx} = H \frac{dh}{dx}$$

$$\frac{d}{dy}\left(\frac{dP}{dy}\right) = H/h^3 \, dh/dx = G$$

$$P = G/2 \, y^2 + Ay + B$$

$$\varphi = S\ [\ X + XR^2/(X^2+Y^2)\ ]\ +\ \Gamma/[2\pi]\ \sigma$$

$$\varphi = 2\ S\ X\ +\ \Gamma/[2\pi]\ \sigma$$

$$\rho/2\ [\ S^2 - (\partial\varphi/\partial c)^2\ ]$$

$$\rho/2\ [\ S^2 - (\Delta\varphi/\Delta c)^2\ ]$$

$$\alpha = x + xa^2/(x^2+y^2)$$

$$\beta = y - ya^2/(x^2+y^2)$$

$$m^2 + (a+n)^2 = R^2$$

$$\Delta\varphi = 2\ S\ \Delta X\ +\ \Gamma/[2\pi]\ \Delta\sigma$$

$$\Delta c = \sqrt{[\Delta\alpha^2+\Delta\beta^2]}$$

$$X = \mathbf{X}\ \mathrm{Cos}\Theta + \mathbf{Y}\ \mathrm{Sin}\Theta$$

$$Y = \mathbf{Y}\ \mathrm{Cos}\Theta - \mathbf{X}\ \mathrm{Sin}\Theta$$

$$\mathbf{X} = x + n \qquad \mathbf{Y} = y - m$$

$$\rho S\Gamma$$

$$\Gamma = 4\pi SR\ \mathrm{Sin}\kappa$$

$$\kappa = \Theta + \varepsilon$$

$$\varepsilon = \tan^{-1}\ [m/(n+a)]$$

$$P\Delta c \; \sin(\boldsymbol{\theta}-\Theta) \qquad P\Delta c \; \cos(\boldsymbol{\theta}-\Theta)$$

$$\boldsymbol{\theta} = \tan^{-1}[-\Delta\alpha/+\Delta\beta]$$

$$L=\Sigma\Delta L \qquad D=\Sigma\Delta D$$

$$\varphi = - S \; r \; \cos[\sigma\,] - S/2 \; R^3/r^2 \; \cos[\sigma]$$

$$\varphi = - S \; R \; \cos[c/R] - S/2 \; R \; \cos[c/R]$$

$$\partial\varphi/\partial c = 3/2 \; S \; \sin\sigma$$

$$P = \rho/2 \; [\; S^2 - (\partial\varphi/\partial c)^2 \;]$$

$$dA = R \; \sin\sigma \; d\Theta \quad R \; d\sigma$$

$$dF = P \; dA = P \; R^2 \; \sin\sigma \; d\sigma \; d\Theta$$

$$dD = + \; dF \; \cos\sigma$$

$$dL = - \; dF \; \sin\sigma \; \cos\Theta$$

$$\int\int + P \; R^2 \; \sin\sigma \; \cos\sigma \; d\sigma \; d\Theta$$

$$\int\int - P \; R^2 \; \sin\sigma \; \sin\sigma \; \cos\Theta \; d\sigma \; d\Theta$$

$$\Sigma\Sigma \; [\; + P \; R^2 \; \sin\sigma \; \cos\sigma \;] \; \Delta\sigma \; \Delta\Theta$$

$$\Sigma\Sigma \; [\; - P \; R^2 \; \sin\sigma \; \sin\sigma \; \cos\Theta \,] \; \Delta\sigma \; \Delta\Theta$$

$$a = \sqrt{[\mathbf{K}/\rho]}$$

$$\mathbf{K} = K / [1 + [DK]/[Ee]]$$

$$a = \sqrt{[k\ R\ T]} = \sqrt{[K/\rho]}$$

$$K/\rho = k\ R\ T \qquad K = k\ \rho\ R\ T$$

$$K = k\ P$$

$$a_M = \sqrt{[K_M/\rho_M]}$$

$$\rho_M = \Sigma[\rho_S V_S]/V_M \qquad K_M = V_M/\Sigma[V_S/K_S]$$

$$\leftarrow F : \Delta P = + \rho a\ \Delta U$$

$$\rightarrow f : \Delta P = - \rho a\ \Delta U$$

$$P_J = [G - H] / M$$

$$G = [A_A/a_A\ P_X + A_B/a_B\ P_Y + A_C/a_C\ P_Z]$$

$$H = \rho [A_A U_X + A_B U_Y + A_C U_Z]$$

$$M = [A_A/a_A + A_B/a_B + A_C/a_C]$$

$$P_M = (P_Z+P_X)/2 - [\rho a][U_Z-U_X]/2$$

$$U_M = (U_Z+U_X)/2 - [P_Z-P_X]/[2\rho a]$$

$$U = U_\circ \, M/M_\circ \, \zeta \, a$$

$$U_\circ = D/\mathbf{T} \qquad M_\circ = \rho D^2$$

$$U = \beta/\mathbf{T} \, \sqrt{[M\delta/\rho]}$$

$$U = \beta U_\circ \, \sqrt{[\delta M/M_\circ]}$$

$$U = D/[S\mathrm{T}]$$

$$\mathrm{T} = \mathbf{T}$$

$$U^2 = [\ EI/[\rho A]\ \pi^2/L^2 + T/[\rho A] - P/\rho\ ]$$

$$U = [4 + 14\ M_\circ/M]\ U_\circ$$

$$U_\circ = \sqrt{[EI]/[M_\circ L^2]} \qquad M_\circ = \rho A$$

$$\mathbf{T}_n = [2L/n]\ \sqrt{[m/T]}$$

$$\mathbf{T}_n = [L/n]^2\ [2/\pi]\ \sqrt{[m/EI]}$$

$$\mathbf{T}_n = 2\pi L^2/K_n\ \sqrt{[m/EI]}$$

$$\frac{D}{Dt} \int_{V(t)} \rho \, dV = 0$$

$$\partial U/\partial x + \partial V/\partial y + \partial W/\partial z = 0$$

$$\frac{D}{Dt} \int_{V(t)} \rho \mathbf{v} \, dV = \int_{S(t)} \boldsymbol{\sigma} \, dS + \int_{V(t)} \rho \mathbf{b} \, dV$$

$$\rho \partial U/\partial t + \rho \, (U \partial U/\partial x + V \partial U/\partial y + W \partial U/\partial z) = - \partial P/\partial x$$

$$+ \mu \, (\partial^2 U/\partial x^2 + \partial^2 U/\partial y^2 + \partial^2 U/\partial z^2)$$

$$\rho \partial V/\partial t + \rho \, (U \partial V/\partial x + V \partial V/\partial y + W \partial V/\partial z) = - \partial P/\partial y$$

$$+ \mu \, (\partial^2 V/\partial x^2 + \partial^2 V/\partial y^2 + \partial^2 V/\partial z^2)$$

$$\rho \partial W/\partial t + \rho \, (U \partial W/\partial x + V \partial W/\partial y + W \partial W/\partial z) = - \partial P/\partial z - \rho g$$

$$+ \mu \, (\partial^2 W/\partial x^2 + \partial^2 W/\partial y^2 + \partial^2 W/\partial z^2)$$

$$\frac{D}{Dt} \int_{V(t)} \rho e \, dV = - \int_{S(t)} \mathbf{q.n} \, dS + \int_{S(t)} \mathbf{v}.\boldsymbol{\sigma} \, dS$$

$$\rho C \, \partial T/\partial t + \rho C \, (U \partial T/\partial x + V \partial T/\partial y + W \partial T/\partial z) = \mu \, \Phi$$

$$+ \partial/\partial x(k \partial T/\partial x) + \partial/\partial y(k \partial T/\partial y) + \partial/\partial z(k \partial T/\partial z)$$

$$\rho \left(\partial U/\partial t + U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z \right) + A = - \partial P/\partial x$$
$$+ \left[\partial/\partial x \left(\mu\ \partial U/\partial x\right) + \partial/\partial y \left(\mu\ \partial U/\partial y\right) + \partial/\partial z \left(\mu\ \partial U/\partial z\right) \right]$$

$$\rho \left(\partial V/\partial t + U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z \right) + B = - \partial P/\partial y$$
$$+ \left[\partial/\partial x \left(\mu\ \partial V/\partial x\right) + \partial/\partial y \left(\mu\ \partial V/\partial y\right) + \partial/\partial z \left(\mu\ \partial V/\partial z\right) \right]$$

$$\rho \left(\partial W/\partial t + U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z \right) + C = - \partial P/\partial z - \rho g$$
$$+ \left[\partial/\partial x \left(\mu\ \partial W/\partial x\right) + \partial/\partial y \left(\mu\ \partial W/\partial y\right) + \partial/\partial z \left(\mu\ \partial W/\partial z\right) \right]$$

$$\partial P/\partial t + \rho\ c^2 \left(\partial U/\partial x + \partial V/\partial y + \partial W/\partial z \right) = 0$$

$$\partial F/\partial t + U\partial F/\partial x + V\partial F/\partial y + W\partial F/\partial z = 0$$

$$\partial k/\partial t + U\partial k/\partial x + V\partial k/\partial y + W\partial k/\partial z = T_P - T_D$$
$$+ \left[\partial/\partial x \left(\mu/a\ \partial k/\partial x\right) + \partial/\partial y \left(\mu/a\ \partial k/\partial y\right) + \partial/\partial z \left(\mu/a\ \partial k/\partial z\right) \right]$$

$$\partial \varepsilon/\partial t + U\partial \varepsilon/\partial x + V\partial \varepsilon/\partial y + W\partial \varepsilon/\partial z = D_P - D_D$$
$$+ \left[\partial/\partial x \left(\mu/b\ \partial \varepsilon/\partial x\right) + \partial/\partial y \left(\mu/b\ \partial \varepsilon/\partial y\right) + \partial/\partial z \left(\mu/b\ \partial \varepsilon/\partial z\right) \right]$$

$$\partial M/\partial t = N \qquad M_{NEW} = M_{OLD} + \Delta t\ N_{OLD}$$

$$\mathbf{v} = - K \nabla P \qquad \nabla . \mathbf{v} = 0$$

$$\nabla . [K \nabla P] = 0$$

$$\partial/\partial x [K \partial P/\partial x] + \partial/\partial y [K \partial P/\partial y] + \partial/\partial z [K \partial P/\partial z] = 0$$

$$[(K_E+K_P)/2 (P_E-P_P)/\Delta x - (K_W+K_P)/2 (P_P-P_W)/\Delta x] / \Delta x$$
$$+$$
$$[(K_N+K_P)/2 (P_N-P_P)/\Delta y - (K_S+K_P)/2 (P_P-P_S)/\Delta y] / \Delta y$$
$$+$$
$$[(K_J+K_P)/2 (P_J-P_P)/\Delta z - (K_I+K_P)/2 (P_P-P_I)/\Delta z] / \Delta z$$
$$= 0$$

$$P_P = \frac{(A P_E + B P_W + C P_N + D P_S + V P_J + W P_I)}{(A + B + C + D + V + W)}$$

$$d/dx [K dP/dx] = 0$$

$$K dP/dx = G \qquad dP/dx = G/K$$

$$K = ax + b \qquad dP/dx = G / [ax + b]$$

$$P = G/a \ln[ax+b] + H$$

$$S_\eta = A/\omega^5 \, e^{-B/\omega 4} \qquad A=346H^2/T^4 \qquad B=691/T^4$$

$$S_R = RAO^2 \, S_\eta \qquad M_n = 1/2 \int S_R(\omega) \, \omega^n \, d\omega$$

$$2 \, R_S = 4 \sqrt{M_0} \qquad T_S = 2\pi \, M_0/M_1$$

$$P(R_o > R_\bullet) = e^{-X} \qquad X = R_\bullet R_\bullet / [2M_0]$$

$$\nabla . \mathbf{v} = 0$$

$$\rho \partial \mathbf{v}/\partial t + \rho \mathbf{v} . \nabla \mathbf{v} + \nabla P + \nabla \rho g z - \mu \nabla^2 \mathbf{v} = 0$$

$$\rho \partial \mathbf{v}/\partial t + \rho \mathbf{v} . \nabla \mathbf{v} + \nabla P + \nabla \rho g z = 0$$

$$\boldsymbol{\omega} = 2\boldsymbol{\Omega} = \nabla x \mathbf{v} = 0 \qquad \nabla x \nabla \varphi = 0 \qquad \mathbf{v} = \nabla \varphi$$

$$\mathbf{v} . \nabla \mathbf{v} = \nabla [\mathbf{v} . \mathbf{v}]/2 - \mathbf{v} x \, \boldsymbol{\omega} \qquad \mathbf{v} . \nabla \mathbf{v} = \nabla [\mathbf{v} . \mathbf{v}]/2$$

$$\nabla^2 \varphi = \nabla^2 \varphi = \partial^2 \varphi/\partial X^2 + \partial^2 \varphi/\partial Y^2 + \partial^2 \varphi/\partial Z^2 = 0$$

$$\rho \partial \mathbf{v}/\partial t + \rho \nabla [\mathbf{v} . \mathbf{v}]/2 + \nabla P + \nabla \rho g z = 0$$

$$\partial \varphi/\partial t + (\nabla \varphi . \nabla \varphi)/2 + P/\rho + g z = C$$

FLUID MECHANICS II

QUIZ #1

NOTE: MARKS IN SQUARE [] BRACKETS

A mixed flow pump has an axial flow type of inlet and a radial flow type of outlet. In other words, it looks like a propeller pump at the inlet and it looks like a centrifugal pump at the outlet. Derive an equation for the pressure versus flow characteristic of the pump. Assume that the pump does not have any guide vanes. [40]

A small deadend pipe is attached to a large pipe. The wave speed for each pipe is 1000m/s and the density of the fluid in the pipes is 1000 kg/m^3. The initial pressure P_O is 40bar everywhere and the initial flow speed U_O in the large pipe is 2m/s. The small pipe is so small that it does not influence what happens in the large pipe. The large pipe looks like a tank to the small pipe. When a valve in the large pipe is suddenly shut, it sends a surge wave up the pipe. Use algebraic waterhammer analysis to predict the pressure of this surge wave. [10] Use algebraic waterhammer analysis to predict the pressure and flow velocity at the ends of the small pipe for 4 small pipe transit times just after the surge wave in the large pipe suddenly passes its entrance. [40] Use the method of reaches to predict what happens in the small pipe for 1 reach transit time. [10]

PUMP QUESTION

The pressure flow characteristic of a pump is

$$P = \rho \, \Delta(V_T V_B)$$

$$= \rho \, [\, (V_T V_B)_{OUT} - (V_T V_B)_{IN} \,]$$

Here there are no guide vanes so we use blade angles at the inlet and the outlet. In this case, the tangential speed is

$$V_T = V_B + V_N \, \mathrm{Cot}[\beta]$$

The blade speed is $V_B = R\omega$. The normal speed is $V_N = Q/A$. For the propeller pump inlet

$$R = (R_a+R_b)/2 \qquad A = \pi \, (R_a R_a - R_b R_b)$$

For the centrifugal pump outlet

$$R = R_o \qquad A = 2\pi R_o \, h$$

In these equations, R_a is the tip radius at the inlet, R_b is the hub radius at the inlet, R_o is the tip radius at the outlet and h is the rotor depth at the outlet.

WATERHAMMER QUESTION

Surge Pressure in Large Pipe

To get the surge pressure, we move along an f wave from the upstream tank to the valve:

$$\Delta P = - \rho a \, \Delta U$$

$$P_V - P_T = - \rho a \, [\, U_V - U_T \,]$$

$$P_V - 40 = - 10 \, [\, 0 - 2 \,] \qquad P_V = 60$$

Waves in Small Pipe

To get the velocity at the entrance to the small pipe, we move along an F wave from the deadend to the entrance:

$$\Delta P = + \rho a \, \Delta U$$

$$P_E - P_D = + \rho a \, [\, U_E - U_D \,]$$

$$U_E = U_D + [\, P_E - P_D \,] \, / \, [\rho a]$$

To get the pressure at the deadend of the small pipe, we move along an f wave from the entrance to the deadend:

$$\Delta P = - \rho a \, \Delta U$$

$$P_D - P_E = - \rho a \, [\, U_D - U_E \,]$$

$$P_D = P_E - \rho a \, [\, U_D - U_E \,]$$

Step #1

$$U_E = U_D + [P_E - P_D] / [\rho a]$$

$$= 0 + (60-40)/10 = +2$$

Step #2

$$P_D = P_E - \rho a [U_D - U_E]$$

$$= 60 - 10 (0 - +2) = 80$$

Step #3

$$U_E = U_D + [P_E - P_D] / [\rho a]$$

$$= 0 + (60-80)/10 = -2$$

Step #4

$$P_D = P_E - \rho a [U_D - U_E]$$

$$= 60 - 10 (0 - -2) = 40$$

Method of Reaches

To get the pressure and velocity at the middle of the deadend pipe, we follow an f wave from the entrance and an F wave from the deadend to get 2 equations in 2 unknowns:

$$P_M = [P_D+P_E]/2 - [\rho a][U_D-U_E]/2$$

$$U_M = [U_D+U_E]/2 - [P_D-P_E]/[\rho a]/2$$

To get the velocity at the entrance to the small pipe, we move along an F wave from the midpoint to the entrance:

$$U_E = U_M + [P_E - P_M] / [\rho a]$$

To get the pressure at the deadend of the small pipe, we move along an f wave from the midpoint to the deadend:

$$P_D = P_M - \rho a [U_D - U_M]$$

Before the surge wave hits the entrance, all pressures in the deadend pipe are 40 and all velocities are 0. When it hits, the pressure at the entrance suddenly jumps to 60.

For the midpoint P_E=40 P_D=40 U_E=0 U_D=0. Substitution into the midpoint equations gives

$$P_M = [P_D+P_E]/2 - [\rho a][U_D-U_E]/2$$

$$= [40+40]/2 - [10]\ [0-0]/2 = 40$$

$$U_M = [U_D+U_E]/2 - [P_D-P_E]/[\rho a]/2$$

$$= [0+0]/2 - [40-40]/10/2 = 0$$

For the entrance, P_E=60 P_M=40 U_M=0. Substitution gives

$$U_E = U_M + [\ P_E - P_M\ ] / [\rho a]$$

$$= 0 + [60-40]/10 = 2$$

For the deadend, P_M=40 U_M=0 U_D=0. Substitution gives

$$P_D = P_M - \rho a\ [\ U_D - U_M\ ]$$

$$= 40 - 10\ [0-0] = 40$$

FLUID MECHANICS II

QUIZ #2

EACH QUESTION IS WORTH 25 MARKS

ANSWER ANY 3 OF THE FOLLOWING 4 QUESTIONS

With words, formulas and sketches, describe how you would calculate the drift speed for an explosion in air.

With words, formulas and sketches, describe how you would calculate the lift and drag on a flat plate supersonic foil.

With words, formulas and sketches, describe how you would calculate the properties of a high speed gas flow down a pipe.

With words, formulas and sketches, describe how you would calculate the lift and the drag on an igloo.

ANSWER THE FOLLOWING QUESTION

Give a title to the derivation sheet and add statements to the sheet which explain the main steps of the derivation.

With words, formulas and sketches, describe how you would calculate the drift speed for an explosion in air.

Assume that the pressure ratio across the shock wave generated by the explosion is known. In the shock frame, air upstream moves towards the shock at supersonic speed and air downstream moves away from it at subsonic speed. The pressure ratio equation gives M_U:

$$P_D/P_U = 1 + [2k/(k+1)]\ (M_U M_U - 1)$$

The wave speed equation gives a_U:

$$a_U = \sqrt{[kRT_U]}$$

The Mach Number equation gives U_U:

$$M_U = U_U/a_U$$

The Mach Number connection gives M_D:

$$M_D M_D = [(k-1)\ M_U M_U + 2]\ /\ [2k\ M_U M_U - (k-1)]$$

The temperature ratio equation gives T_D:

$$T_D/T_U = [\ (1 + [(k-1)/2]\ M_U M_U)\ /\ (1 + [(k-1)/2]\ M_D M_D)\ ]$$

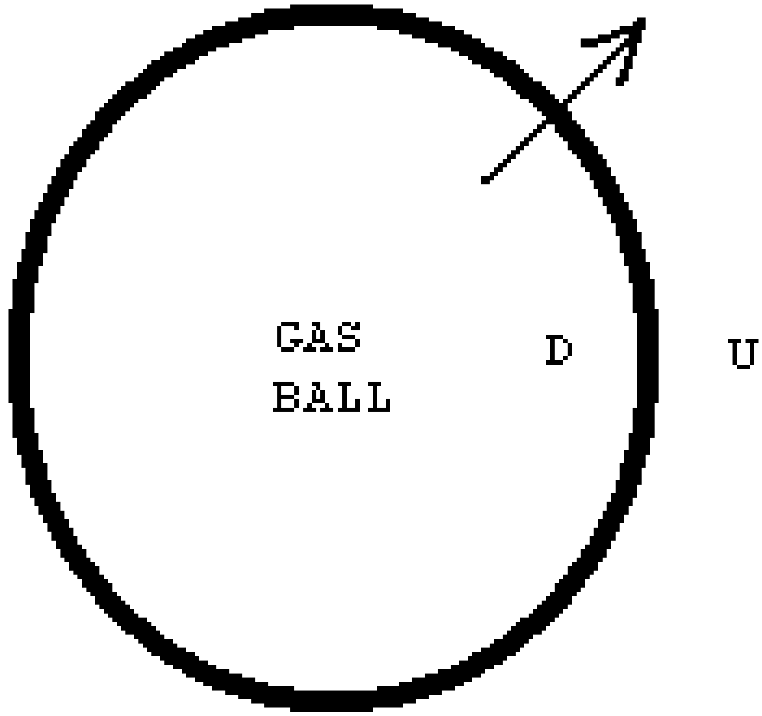
The wave speed equation gives a_D:

$$a_D = \sqrt{[kRT_D]}$$

The Mach Number equation gives U_D:

$$M_D = U_D/a_D$$

In absolute terms, the shock is moving outward at speed U_U. In relative terms, the flow downstream is moving away from the shock at speed U_D. So the drift speed is U_U minus U_D.



With words, formulas and sketches, describe how you would calculate the lift and drag on a flat plate supersonic foil.

The foil has an expansion wave at the leading edge at the top and an oblique shock wave at the leading edge on the bottom.

With known upstream Mach Number M_U and attack angle Θ, the oblique shock plot gives the shock angle β. Substitution into the normal Mach Number equation gives N_U:

$$N_U = M_U \operatorname{Sin}\beta$$

Substitution into the pressure ratio equation gives P_B:

$$P_B/P_U = 1 + [2k/(k+1)]\ (N_U N_U - 1)$$

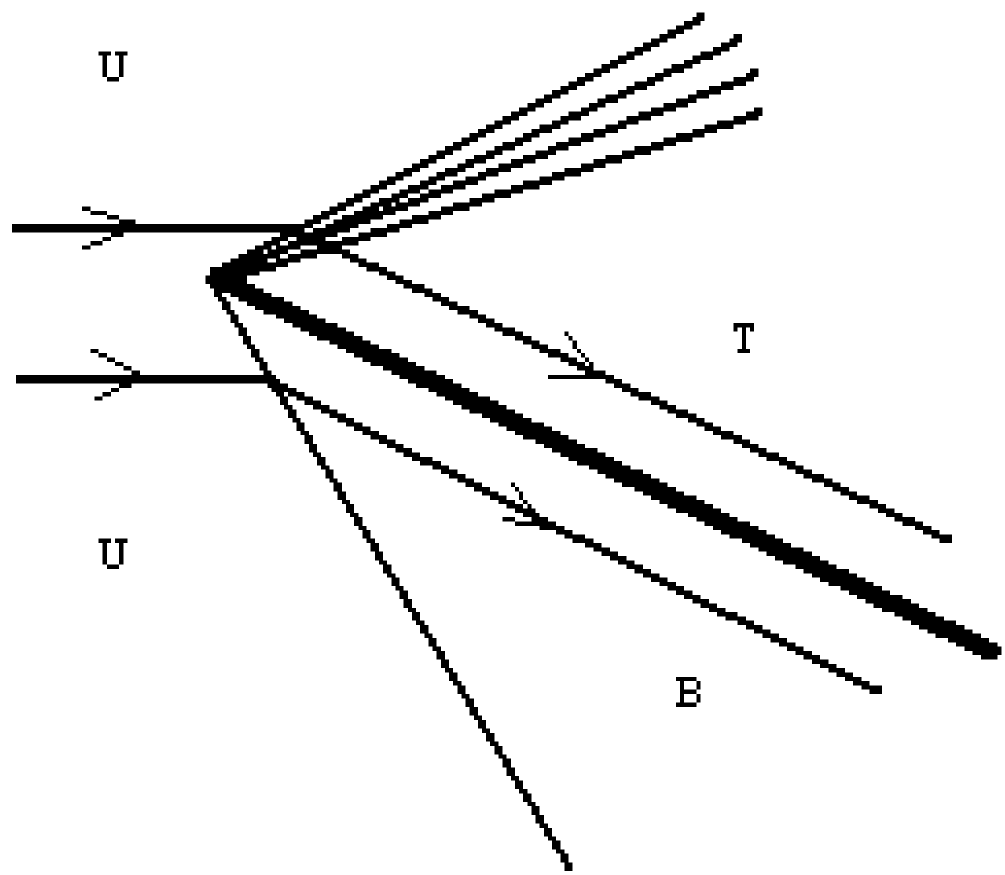
With known upstream Mach Number M_U and attack angle Θ, the expansion wave plot gives M_T. The isentropic pressure ratio equation then gives P_T.

$$T_T/T_U = [\ (1 + [(k-1)/2]\ M_U M_U) \ /\ (1 + [(k-1)/2]\ M_T M_T)\ ]$$

$$P_T/P_U = [T_T/T_U]^x \qquad x = k/(k-1)$$

The lift is:

$$P_B\ C\ \operatorname{Cos}\Theta\ - P_T\ C\ \operatorname{Cos}\Theta$$


U
T
U
B

With words, formulas and sketches, describe how you would calculate the properties of a high speed gas flow down a pipe.

Gas dynamics theory gives

$$\Delta M^2/M^2 = kM^2[1+[(k-1)/2]M^2] /[1-M^2] \ f\Delta x/D$$

$$\Delta P/P = -kM^2[1+(k-1) M^2]/[2(1-M^2)] \ f\Delta x/D$$

$$\Delta T/T = -k(k-1)M^4/[2(1-M^2)] \ f\Delta x/D$$

These can be put into the form:

$$\Delta M^2 = A \ \Delta x$$

$$\Delta P = B \ \Delta x$$

$$\Delta T = C \ \Delta x$$

For moving a step down a pipe, these equations give:

$$[M^2]_{NEW} = [M^2]_{OLD} + A_{OLD} \ \Delta x$$

$$P_{NEW} = P_{OLD} + B_{OLD} \ \Delta x$$

$$T_{NEW} = T_{OLD} + C_{OLD} \ \Delta x$$

A B C is each a function of M^2. Since M^2 changes during each step, A B C must be updated after each step down the pipe.

With words, formulas and sketches, describe how you would calculate the lift and the drag on an igloo.

For a real igloo, flow separates at some point on its surface and there is an approximately constant pressure wake downstream. Potential flow theory gives the pressure upstream of the wake and the pressure in the wake. The potential is

$$\varphi = - S\ r\ \mathrm{Cos}\sigma \ - \ S/2\ R^3/r^2\ \mathrm{Cos}\sigma$$

On the surface of the igloo where r=R and c=Rσ, this becomes

$$\varphi = - S\ R\ \mathrm{Cos}[c/R] \ - \ S/2\ R\ \mathrm{Cos}[c/R]$$

Differentiation gives the speed

$$\partial\varphi/\partial c = 3/2\ S\ \mathrm{Sin}\sigma$$

Application of Bernoulli gives pressure

$$P = \rho/2\ [\ S^2 - (\partial\varphi/\partial c)^2\ ]$$

$$P = \rho/2\ [\ S^2 - 9/4\ S^2\ \mathrm{Sin}^2\sigma\ ]$$

In the wake, the pressure is

$$P = \rho/2\ [\ S^2 - 9/4\ S^2\ \mathrm{Sin}^2\sigma_S\ ]$$

An incremental area is

$$dA = R \sin\sigma \, d\Theta \, R \, d\sigma$$

An incremental force is

$$dF = P \, dA = P \, R^2 \sin\sigma \, d\sigma \, d\Theta$$

Incremental lift and drag are

$$dL = - dF \sin\sigma \cos\Theta$$

$$dD = dF \cos\sigma$$

Integration gives the total lift and drag

$$\int\int [- P \, R^2 \sin\sigma \sin\sigma \cos\Theta] \, d\sigma \, d\Theta$$

$$\int\int [+ P \, R^2 \sin\sigma \cos\sigma] \, d\sigma \, d\Theta$$

Numerical integration gives

$$\Sigma\Sigma [- P \, R^2 \sin\sigma \sin\sigma \cos\Theta] \, \Delta\sigma \, \Delta\Theta$$

$$\Sigma\Sigma [+ P \, R^2 \sin\sigma \cos\sigma] \, \Delta\sigma \, \Delta\Theta$$

TITLE: DERIVATION OF THE POTENTIAL FLOW EQUATIONS

The derivation starts with the conservation laws in terms of velocities and pressure. Conservation of Mass is

$$\nabla . \mathbf{v} = 0$$

where

$$\mathbf{v} = U\ \mathbf{i} + V\ \mathbf{j} + W\ \mathbf{k} \qquad \nabla = \partial/\partial x\ \mathbf{i} + \partial/\partial y\ \mathbf{j} + \partial/\partial z\ \mathbf{k}$$

Manipulation gives

$$\partial U/\partial x + \partial V/\partial y + \partial W/\partial z = 0$$

Conservation of Momentum is

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}.\nabla\mathbf{v} + \nabla P + \nabla\rho g z - \mu\nabla^2\mathbf{v} = 0$$

For a zero viscosity or inviscid fluid, it reduces to

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}.\nabla\mathbf{v} + \nabla P + \nabla\rho g z = 0$$

$$\rho\ (\partial U/\partial t + U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z) = - \partial P/\partial x$$

$$\rho\ (\partial V/\partial t + U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z) = - \partial P/\partial y$$

$$\rho\ (\partial W/\partial t + U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z) = - \partial P/\partial z - \rho g$$

In a flow of an inviscid fluid, fluid particles do not spin. This implies that the vorticity is zero. This implies that

$$\boldsymbol{\omega} = 2\boldsymbol{\Omega} = \nabla \mathrm{x} \mathbf{v} = 0$$

$$\mathbf{v}.\nabla\mathbf{v} = \nabla[\mathbf{v}.\mathbf{v}]/2 - \mathbf{v} \mathrm{x}\, \boldsymbol{\omega} = \nabla[\mathbf{v}.\mathbf{v}]/2$$

For any scalar φ, $\nabla \mathrm{x} \nabla\varphi=0$. Comparison with $\nabla \mathrm{x}\mathbf{v}=0$ suggests that $\mathbf{v}=\nabla\varphi$. Substitution into Conservation of Mass gives

$$\nabla.\mathbf{v} = \nabla.\nabla\varphi = 0$$

$$\nabla^2\varphi = \partial^2\varphi/\partial X^2 + \partial^2\varphi/\partial Y^2 + \partial^2\varphi/\partial Z^2 = 0$$

So Conservation of Mass becomes the Laplace Equation.

Substitution into Conservation of Momentum gives

$$\rho\partial\mathbf{v}/\partial t + \rho\nabla[\mathbf{v}.\mathbf{v}]/2 + \nabla P + \nabla\rho g z = 0$$

$$\rho \nabla \partial\varphi/\partial t + \rho \nabla (\nabla\varphi.\nabla\varphi)/2 + \nabla P + \nabla\rho g z = 0$$

Integration gives

$$\partial\varphi/\partial t + (\nabla\varphi.\nabla\varphi)/2 + P/\rho + gz = C$$

This equation is known as the Unsteady Bernoulli equation.

BONUS QUESTION [10]

Identify the equations listed below. Outline briefly how they can be solved using Computational Fluid Dynamics.

CONSERVATION OF MOMENTUM

$$\rho \left(\partial U/\partial t + U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z \right) + A = - \partial P/\partial x$$
$$+ \left[\partial/\partial x \left(\mu\ \partial U/\partial x\right) + \partial/\partial y \left(\mu\ \partial U/\partial y\right) + \partial/\partial z \left(\mu\ \partial U/\partial z\right) \right]$$

$$\rho \left(\partial V/\partial t + U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z \right) + B = - \partial P/\partial y$$
$$+ \left[\partial/\partial x \left(\mu\ \partial V/\partial x\right) + \partial/\partial y \left(\mu\ \partial V/\partial y\right) + \partial/\partial z \left(\mu\ \partial V/\partial z\right) \right]$$

$$\rho \left(\partial W/\partial t + U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z \right) + C = - \partial P/\partial z - \rho g$$
$$+ \left[\partial/\partial x \left(\mu\ \partial W/\partial x\right) + \partial/\partial y \left(\mu\ \partial W/\partial y\right) + \partial/\partial z \left(\mu\ \partial W/\partial z\right) \right]$$

CONSERVATION OF MASS

$$\partial P/\partial t + \rho\ c^2 \left(\partial U/\partial x + \partial V/\partial y + \partial W/\partial z \right) = 0$$

WATER SURFACE TRACKER

$$\partial F/\partial t + U\partial F/\partial x + V\partial F/\partial y + W\partial F/\partial z = 0$$

TURBULENCE KINETIC ENERGY

$$\partial k/\partial t + U\partial k/\partial x + V\partial k/\partial y + W\partial k/\partial z = T_P - T_D$$
$$+ \left[\partial/\partial x \left(\mu/a\ \partial k/\partial x\right) + \partial/\partial y \left(\mu/a\ \partial k/\partial y\right) + \partial/\partial z \left(\mu/a\ \partial k/\partial z\right) \right]$$

TURBULENCE DISSIPATION RATE

$$\partial\varepsilon/\partial t + U\partial\varepsilon/\partial x + V\partial\varepsilon/\partial y + W\partial\varepsilon/\partial z = D_P - D_D$$
$$+ \; [\; \partial/\partial x\; (\mu/b\; \partial\varepsilon/\partial x) + \partial/\partial y\; (\mu/b\; \partial\varepsilon/\partial y) + \partial/\partial z\; (\mu/b\; \partial\varepsilon/\partial z)\;]$$

PRODUCTION AND DISSIPATION

$$T_P = G\; \mu_t \;/\; \rho \qquad D_P = T_P\; C_1\; \varepsilon \;/\; k$$

$$T_D = C_D\; \varepsilon \qquad D_D = C_2\; \varepsilon^2 \;/\; k$$

VISCOSITIES

$$\mu_t = C_3\; k^2 \;/\; \varepsilon \qquad \mu = \mu_t + \mu_l$$

CFD TEMPLATE

$$\partial M/\partial t = N \qquad M_{NEW} = M_{OLD} + \Delta t\; N_{OLD}$$

The region of interest is divided by a CFD grid. CFD cells surround each point where grid lines cross. Each PDE is put into the form: $\partial M/\partial t = N$. The template $M_{NEW} = M_{OLD} + \Delta t\; N_{OLD}$ is applied to each PDE at each point in the grid to get flows step by step in time. Finite differences are used to approximate the various derivatives in N. Central differences are used to approximate the diffusion terms. Upwind differences are used to approximate the convective terms. The eddy viscosity concept is used to model turbulence. The volume of fluid concept is used to track the water surface. The function F is 1 inside water and 0 outside it: cells with F between 1 and 0 contain the water surface. The Semi Implicit Method for Pressure Linked Equations or SIMPLE procedure is used to update pressure and correct velocities so that they satisfy mass and momentum.