IMPORTANT INFORMATION

PLEASE READ

There are no numbers on this exam. However you can use sample calculations with your own simple numbers to show you know how to insert numbers.

There is choice in all questions. A few bonus marks will be given if you answer correctly extra questions. Please identify them as bonus.

The exam is 180 minutes long. This means a 10 marks question should take no more than 18 minutes. Use the time wisely. Do not get stuck on a question. Give answers in step format. Be concise.

ENGINEERING 6961

FLUID MECHANICS II

FINAL EXAMINATION

FALL 2013

INSTRUCTIONS

PENS PENCILS ERASERS ALLOWED

NO NOTES TEXTS PAPERS ALLOWED
NO ELECTRONIC DEVICES ALLOWED

GIVE ANSWERS IN STEP FORMAT
LEAVE SPACE BETWEEN STEPS
GIVE CONCISE ANSWERS

HAND IN ALL SHEETS
DO NOT REMOVE STAPLES

ASK NO QUESTIONS

[THIS QUESTION IS WORTH 16 MARKS: EACH PART IS WORTH 4%]

FIVE double sided derivation sheets are attached. Identify any **FOUR** derivations and add statements to the sheets to explain in detail each step of the derivation.

[THIS QUESTION IS WORTH 15 MARKS: PART MARKS IN [] BRACKETS]

Identify and state the main observations from any **FOUR** of the **SIX** labs [8]. Identify and state the main observations from any **THREE** of the **FIVE** demos [3]. Identify and state the main observations from any **TWO** of the **FOUR** efluids videos [4].

[THIS QUESTION IS WORTH 20 MARKS: EACH PART IS WORTH 5%]

Write brief notes on any **FOUR** of the following **SEVEN** topics: (1) Turbulent Wake Flows (2) Fluid Structure Interactions (3) Conservation Laws (4) Porous Media Flows (5) Two Phase Flows (6) Wide Hydrodynamic Bearings (7) Viscometers. In each case, identify the important formulas on the formula sheets.

[THIS QUESTION IS WORTH 49 MARKS: EACH PART IS WORTH 7%]

With words, formulas and sketches, describe briefly the steps you would take to calculate any **SEVEN** of the following **ELEVEN** things: (1) the pressures in a pipe tank system following a sudden pump start up (2) the thrust of a rocket nozzle (3) the influence of friction on compressible pipe flow (4) the drift speed generated by an explosion (5) the stagnation pressure on a supersonic blunt object (6) the pressures on a supersonic foil (7) the flow rate through a choked valve (8) the pressures on a subsonic foil (9) the lift and drag on a hemispherical hut (10) the load supported by a hydrodynamic lubrication thrust bearing (11) the motion statistics of a floating body in random waves. In each case, identify the important formulas used at each step in the calculation. Also list the assumptions and the things that are known.

BONUS QUESTION [5]

Give a TRUE or FALSE answer to following statements.
Give a single sentence explanation for each answer.

(1) Shock waves cannot occur in water.

(2) Flow speed at a singularity is zero.

(3) Small eddies in a flow diffuse mass.

(4) Speed at a stagnation point is infinite.

(5) A single Mach wave cannot be heard.

(6) Lift can be generated by a doublet.

(7) A potential vortex is a rotational flow.

(8) Bernoulli is valid for viscous flows.

(9) An expansion shock wave is not possible.

(10) Flow in hydrodynamic bearings is turbulent.

TITLE

$$\partial U/\partial x + \partial V/\partial y + \partial W/\partial z = 0$$

$$\partial P/\partial x = \mu \partial^2 U/\partial z^2$$

$$\partial P/\partial y = \mu \partial^2 V/\partial z^2$$

$$0 = \mu \partial^2 W/\partial z^2$$

$$\int [\partial U/\partial x + \partial V/\partial y + \partial W/\partial z]\, dz = 0$$

$$I = \int Udz \qquad J = \int Vdz \qquad K = \int Wdz$$

$$\partial I/\partial x + \partial J/\partial y + \partial K/\partial z = 0$$

$$U = \partial P/\partial x \; (z^2 - zh)/2\mu + (U_T - U_B) z/h + U_B$$

$$V = \partial P/\partial y \; (z^2 - zh)/2\mu + (V_T - V_B) z/h + V_B$$

$$W = (W_T - W_B) z/h + W_B$$

$$I = \partial P/\partial x \; (-h^3/12\mu) + (U_T - U_B) h/2 + U_B h$$

$$J = \partial P/\partial y \; (-h^3/12\mu) + (V_T - V_B) h/2 + V_B h$$

$$K = (W_T - W_B) z^2/2h + W_B z$$

$$\partial/\partial x \; (h^3/12\mu \; \partial P/\partial x) \; + \; \partial/\partial y \; (h^3/12\mu \; \partial P/\partial y)$$

$$= \; \partial[h(U_T + U_B)/2]/\partial x \; + \; \partial[h(V_T + V_B)/2]/\partial y \; + \; (W_T - W_B)$$

TITLE: ____________________

$$\dot{M} = \rho_1 U_1 \Delta A = \rho_2 U_2 \Delta A$$

$$\dot{M} (U_2 - U_1) = (P_1 - P_2) \Delta A$$

$$\rho_2 U_2 \Delta A U_2 - \rho_1 U_1 \Delta A U_1 = (P_1 - P_2) \Delta A$$

$$P_1 + \rho_1 U_1U_1 = P_2 + \rho_2 U_2U_2$$

$$P_1 (1 + k M_1M_1) = P_2 (1 + k M_2M_2)$$

$$h_1 + [U_1U_1]/2 = h_2 + [U_2U_2]/2$$

$$T_1 (1 + (k-1)/2 M_1M_1) = T_2 (1 + (k-1)/2 M_2M_2)$$

$$\rho_1 U_1 \Delta A = \rho_2 U_2 \Delta A$$

$$\rho_2 / \rho_1 = U_1 / U_2$$

$$= [M_1 a_1] / [M_2 a_2] = [M_1 \sqrt{T_1}] / [M_2 \sqrt{T_2}]$$

$$= \sqrt{\{[1 + (k-1)/2 M_2M_2] / [1 + (k-1)/2 M_1M_1]\}} \quad M_1/M_2$$

$$P_1 / [\rho_1 R T_1] = P_2 / [\rho_2 R T_2]$$

$$\rho_2/\rho_1 T_2/T_1 = P_2/P_1$$

$$M_2M_2 = [(k-1) M_1M_1 + 2] / [2k M_1M_1 - (k-1)]$$

$$P_2/P_1 = 1 + 2k/(k+1) (M_1M_1 - 1)$$

$$T_2/T_1 = ([1+(k-1)/2 M_1M_1][2k M_1M_1-(k-1)])/[(k+1)^2/2 M_1M_1]$$

$$\rho_2/\rho_1 = [(k+1) M_1M_1] / [2 + (k-1) M_1M_1]$$

TITLE: ______________________________

$$\rho\ \partial U/\partial t + \rho U\ \partial U/\partial x + \partial P/\partial x + \rho C = 0$$

$$C = f/D\ U|U|/2 - g\ \mathrm{Sin}\alpha$$

$$\partial P/\partial t + U\ \partial P/\partial x + \rho a^2\ \partial U/\partial x = 0$$

$$\rho\ \partial U/\partial t + \partial P/\partial x = 0$$

$$\partial P/\partial t + \rho a^2\ \partial U/\partial x = 0$$

$$\partial^2 P/\partial t^2 = a^2\ \partial^2 P/\partial x^2$$

$$\partial^2 U/\partial t^2 = a^2\ \partial^2 U/\partial x^2$$

$$P - P_o = f(N) + F(M)$$

$$U - U_o = [f(N) - F(M)] / [\rho a]$$

$$N = x - a\,t \qquad M = x + a\,t$$

$$[P-P_o] - \rho a[U-U_o] = 2F(M)$$

$$\Delta P = + \rho a\ \Delta U$$

$$[P-P_o] + \rho a[U-U_o] = 2f(N)$$

$$\Delta P = - \rho a\ \Delta U$$

TITLE: ____

$$\nabla.\mathbf{v} = 0$$

$$\mathbf{v} = U\ \mathbf{i} + V\ \mathbf{j} + W\ \mathbf{k}$$

$$\nabla = \partial/\partial x\ \mathbf{i} + \partial/\partial y\ \mathbf{j} + \partial/\partial z\ \mathbf{k}$$

$$\partial U/\partial x + \partial V/\partial y + \partial W/\partial z = 0$$

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}.\nabla\mathbf{v} + \nabla P + \nabla\rho gz - \mu\nabla^2\mathbf{v} = 0$$

$$\rho\partial\mathbf{v}/\partial t + \rho\mathbf{v}.\nabla\mathbf{v} + \nabla P + \nabla\rho gz = 0$$

$$\rho\ (\ \partial U/\partial t + U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z\ ) = -\ \partial P/\partial x$$

$$\rho\ (\ \partial V/\partial t + U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z\ ) = -\ \partial P/\partial y$$

$$\rho\ (\ \partial W/\partial t + U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z\ ) = -\ \partial P/\partial z - \rho g$$

$$\boldsymbol{\omega} = 2\boldsymbol{\Omega} = \nabla \times \mathbf{v} = 0$$

$$\nabla \times \nabla\varphi = 0 \qquad \mathbf{v} = \nabla\varphi$$

$$\nabla . \mathbf{v} = 0 \qquad \nabla . \nabla\varphi = 0$$

$$\nabla^2\varphi = \partial^2\varphi/\partial X^2 + \partial^2\varphi/\partial Y^2 + \partial^2\varphi/\partial Z^2 = 0$$

$$\mathbf{v}.\nabla\mathbf{v} = \nabla [\mathbf{v}.\mathbf{v}]/2 - \mathbf{v}\times\boldsymbol{\omega}$$

$$\rho\partial\mathbf{v}/\partial t + \rho\nabla[\mathbf{v}.\mathbf{v}]/2 + \nabla P + \nabla\rho g z = 0$$

$$\partial\varphi/\partial t + (\nabla\varphi.\nabla\varphi)/2 + P/\rho + gz = C$$

TITLE: ______________________________

$$r\ \partial/\partial c\ (h^3/12\mu\ \partial P/\partial c) + \partial/\partial r\ (rh^3/12\mu\ \partial P/\partial r)$$

$$= \partial[h(V_T+V_B)/2]/\partial\Theta + \partial[rh(U_T+U_B)/2]/\partial r + r(W_T-W_B)$$

$$r\ \partial/\partial c\ (h^3\ \partial P/\partial c) + \partial/\partial r\ (rh^3\ \partial P/\partial r) = 6\mu\ S\ \partial h/\partial\Theta$$

$$r\ \partial I/\partial c + \partial J/\partial r = K$$

$$I = h^3\ \partial P/\partial c$$

$$J = rh^3\ \partial P/\partial r$$

$$K = 6\mu\ r\omega\ \partial h/\partial\Theta$$

$$r\ \Delta I/\Delta c + \Delta J/\Delta r = K$$

$$\Delta I/\Delta c = [I_A - I_B]/\Delta c$$

$$I_A = [(h_E+h_P)/2]^3 \; [P_E-P_P]/\Delta c$$

$$I_B = [(h_W+h_P)/2]^3 \; [P_P-P_W]/\Delta c$$

$$\Delta J/\Delta r = [J_C - J_D]/\Delta r$$

$$J_C = [h_P]^3 \; [(r_N+r_P)/2] \; [P_N-P_P]/\Delta r$$

$$J_D = [h_P]^3 \; [(r_S+r_P)/2] \; [P_P-P_S]/\Delta r$$

$$P_P = \frac{A\ P_E + B\ P_W + C\ P_N + D\ P_S + H)}{(A + B + C + D)}$$

$$A = r_P \; [(h_E+h_P)/2]^3 \; / \; [\Delta c]^2$$

$$B = r_P \; [(h_W+h_P)/2]^3 \; / \; [\Delta c]^2$$

$$C = [h_P]^3 \; [(r_N+r_P)/2] \; / \; [\Delta r]^2$$

$$D = [h_P]^3 \; [(r_S+r_P)/2] \; / \; [\Delta r]^2$$

$$H = -\ 6\mu \; r_P \; \omega \; (h_E-h_W)/[2\Delta\Theta]$$

$$\Delta F = P \; \Delta c \; \Delta r$$

WATERHAMMER QUESTIONS

A pipe connecting two large tanks has a constant flow positive displacement pump at its upstream end and a valve at its downstream end. Initially, the valve is fully open, the pump is stopped and the conditions in the pipe are P_o=20BAR U_o=0m/s. The ρa of the pipe is 10 BAR/[m/s]. Then, the pump suddenly starts and generates a velocity of 1 m/s. At the instant the pump starts, the valve suddenly closes. Using algebraic water hammer analysis, determine the pressure and velocity at the ends of the pipe for 3 steps in time. [30] Using graphical waterhammer analysis, determine the pressure and velocity at the ends of the pipe for 3 steps in time. [30] Explain what happens in the pipe.

The starting conditions are:

$$P_m = 20 \quad U_m = 0 \qquad P_n = 20 \quad U_n = 0$$

The stepping equations are:

$$\leftarrow F : \Delta P = + \rho a \; \Delta U$$

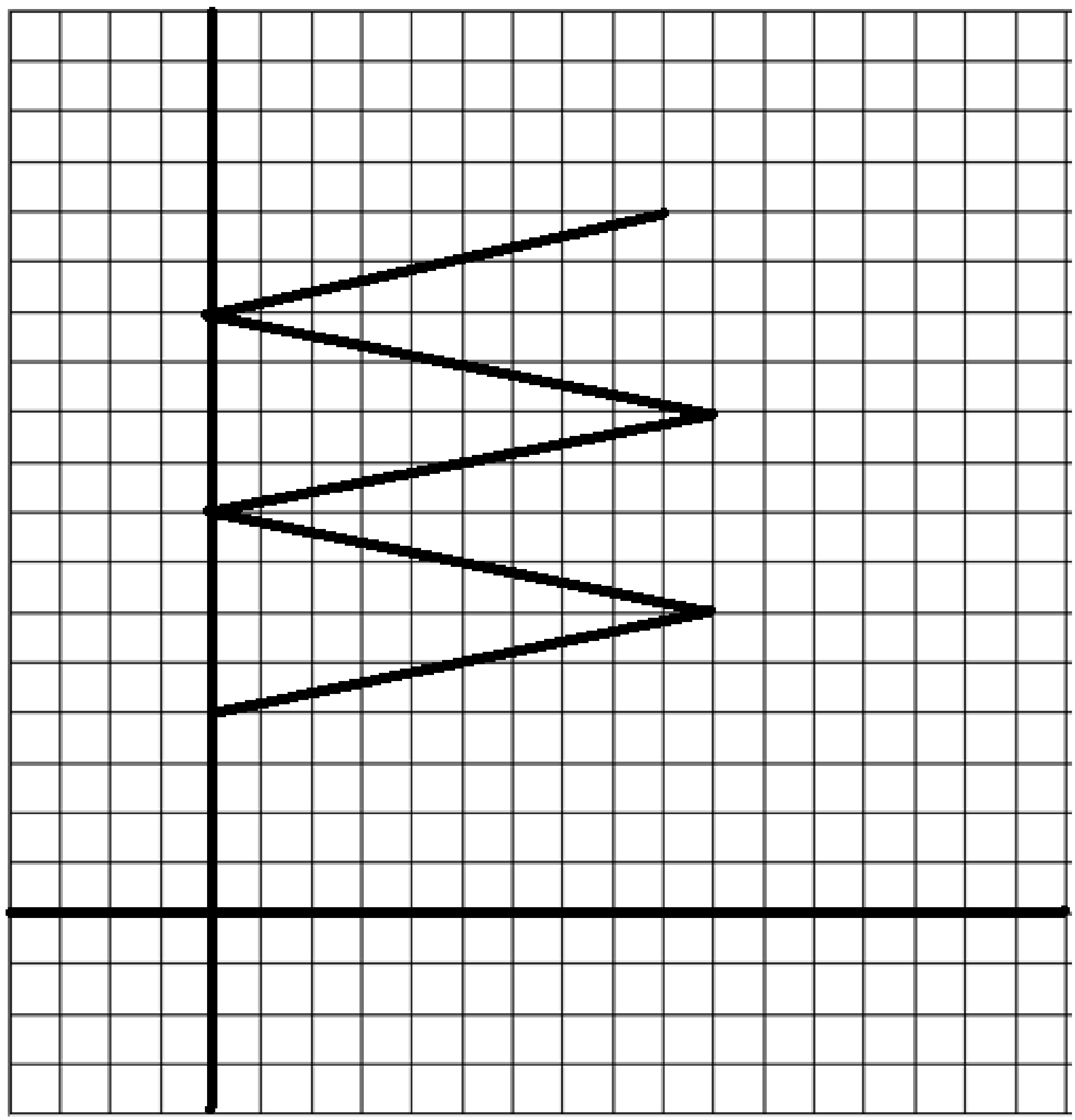
$$P_M - P_n = + [\rho a] \; [U_M - U_n]$$

$$U_M = 1 \qquad P_M = P_n + 10 \; [U_M - U_n]$$

$$\rightarrow f : \Delta P = - \rho a \; \Delta U$$

$$P_N - P_m = - [\rho a] \; [U_N - U_m]$$

$$U_N = 0 \qquad P_N = P_m - 10 \; [U_N - U_m]$$



A pipe has a tank at its upstream end and a valve at its downstream end. Initially, the valve is closed, and conditions in the pipe are $P_o=30$ $U_o=0$. The ρa of the pipe is 10. Then, the valve is suddenly opened. Its pressure flow characteristic is $P_N=20U_N$. Using algebraic water hammer analysis, determine the pressure and velocity at the ends of the pipe for 2 steps in time. [30] Using graphical waterhammer analysis, determine the pressure and velocity at the ends of the pipe for 2 steps in time. [30] Explain what happens in the pipe.

The starting conditions are:

$$P_m = 30 \quad U_m = 0 \qquad P_n = 30 \quad U_n = 0$$

The stepping equations are:

$$\leftarrow F : \Delta P = + \rho a \ \Delta U$$

$$P_M - P_n = + [\rho a] \ [U_M - U_n]$$

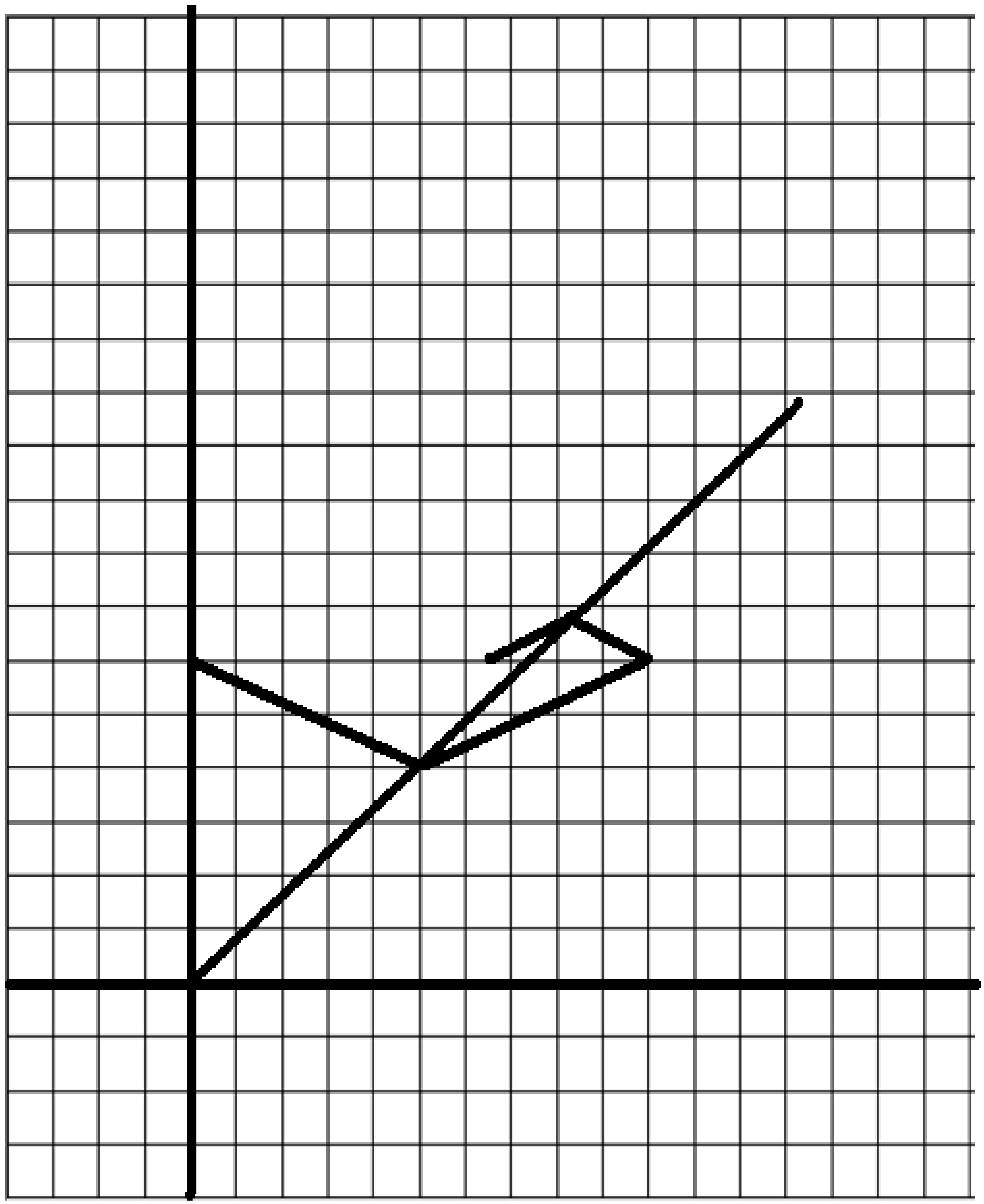
$$P_M = 30 \qquad U_M = U_n + [P_M - P_n]/10$$

$$\rightarrow f : \Delta P = - \rho a \ \Delta U$$

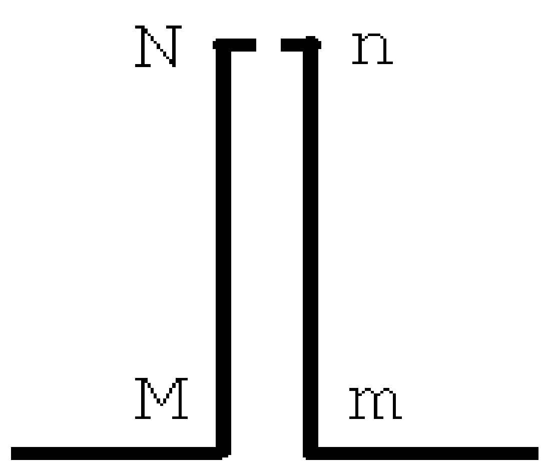
$$P_N - P_m = - [\rho a] \ [U_N - U_m] \qquad P_N = 20 \ U_N$$

$$20 \ U_N - P_m = - 10 \ [U_N - U_m]$$

$$U_N = P_m/30 + U_m/3$$



A small pipe is attached to a large pipe. The large pipe acts like a tank at the upstream end of the small pipe. There is a leak at the downstream end of the small pipe. The leak has the pressure flow characteristic is $P_N=20U_N$. Initially pressure is 20 BAR everywhere. Then suddenly a surge wave in the large pipe passes by the entrance of the small pipe. The pressure of the surge wave is 30 BAR. The ρa of the pipes is 10 BAR/[m/s]. Using algebraic water hammer analysis, determine the pressure and velocity at the ends of the small pipe for 2 steps in time. [30] Using graphical waterhammer analysis, determine the pressure and velocity at the ends of the small pipe for 2 steps in time. [30] Explain what happens in the pipe.



The starting conditions are:

$$P_m = 20 \quad U_m = 1 \qquad P_n = 20 \quad U_n = 1$$

The stepping equations are:

$$\leftarrow F : \Delta P = + \rho a \; \Delta U$$

$$P_M - P_n = + [\rho a] \; [U_M - U_n]$$

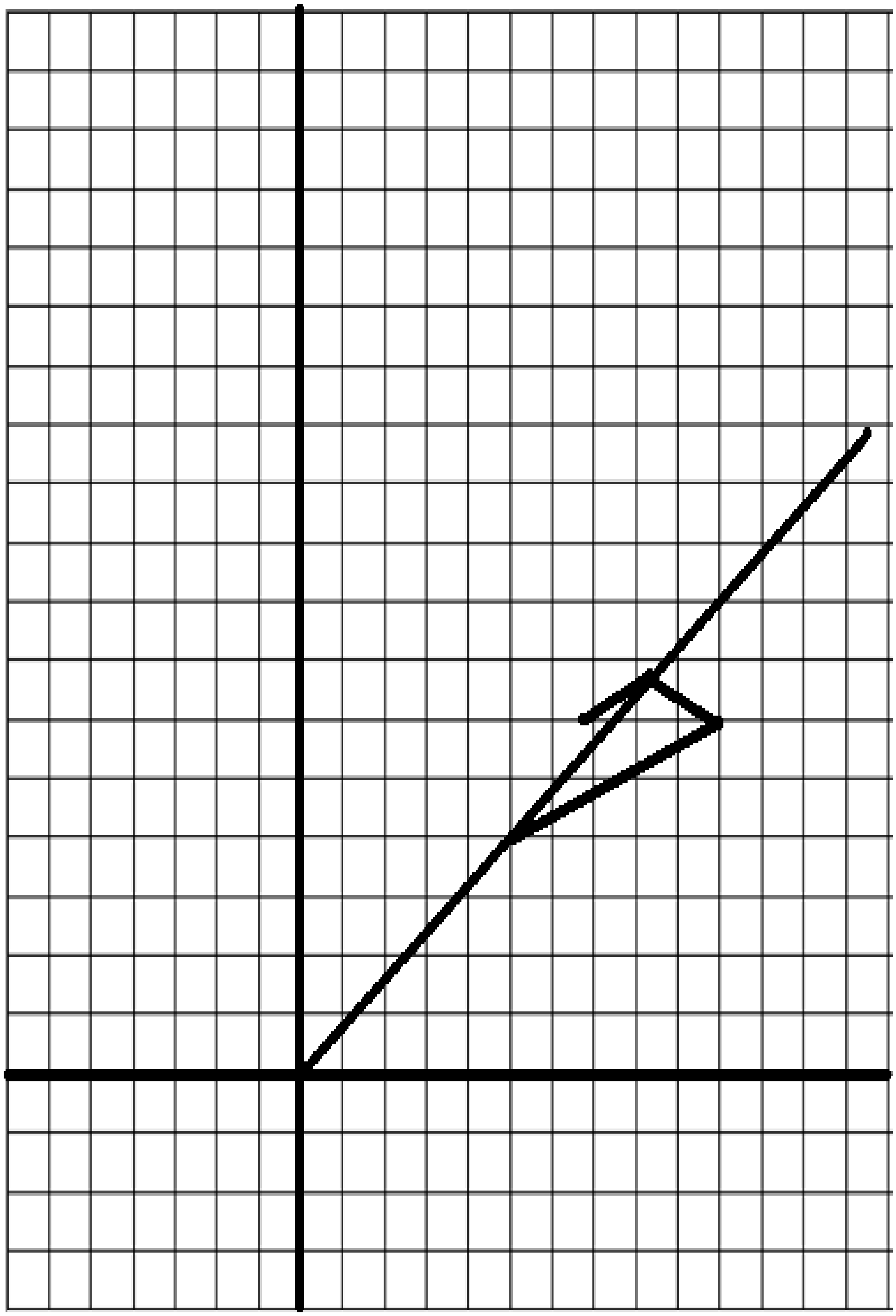
$$P_M = 30 \qquad U_M = U_n + [P_M - P_n]/10$$

$$\rightarrow f : \Delta P = - \rho a \; \Delta U$$

$$P_N - P_m = - [\rho a] \; [U_N - U_m] \qquad P_N = 20 \; U_N$$

$$20 \; U_N - P_m = - 10 \; [U_N - U_m]$$

$$U_N = P_m/30 + U_m/3$$



$$\Delta M^2/M^2 = + kM^2[1+[(k-1)/2]M^2] /[1-M^2] f\Delta x/D$$

$$\Delta P/P = - kM^2[1+(k-1) M^2]/[2(1-M^2)] f\Delta x/D$$

$$\Delta T/T = - k(k-1)M^4/[2(1-M^2)] f\Delta x/D$$

$$\Delta\rho/\rho = - kM^2/[2(1-M^2)] f\Delta x/D$$

$$\Delta G = H \Delta x \qquad G_{NEW} = G_{OLD} + H_{OLD} [x_{NEW} - x_{OLD}]$$

$$fL^*/D = (1-M^2)/(kM^2) + [(k+1)/(2k)] \ln[(k+1)M^2/(2+(k-1)M^2)]$$

$$fL^*/D = (1-kM^2)/(kM^2) + \ln[kM^2]$$

$$T_D/T_U = [(1 + [(k-1)/2] M_U M_U) / (1 + [(k-1)/2] M_D M_D)]$$

$$P_D/P_U = [T_D/T_U]^x \qquad x = k/(k-1)$$

$$\dot{M} = \rho AU \qquad M = U/a \qquad a = \sqrt{kRT}$$

$$\rho = P/[RT] \qquad \dot{M} U + \Delta P A$$

$$d\rho/\rho + dA/A + dU/U = 0 \qquad UdU + a^2 d\rho/\rho = 0$$

$$dU = UdA / [A(M^2-1)]$$

$$P_D/P_U = [\ 1 + k\ M_U M_U\ ] / [\ 1 + k\ M_D M_D\ ]$$

$$T_D/T_U = [\ (1 + [(k-1)/2]\ M_U M_U) / (1 + [(k-1)/2]\ M_D M_D)\ ]$$

$$M_D M_D = [(k-1)\ M_U M_U + 2] / [2k\ M_U M_U - (k-1)]$$

$$P_D/P_U = 1 + [2k/(k+1)]\ (M_U M_U - 1)$$

$$P_D/P_U = 1 + [2k/(k+1)]\ (N_U N_U - 1)$$

$$N_D N_D = [(k-1)\ N_U N_U + 2] / [2k\ N_U N_U - (k-1)]$$

$$N_U = M_U \operatorname{Sin}\beta \qquad N_D = M_D \operatorname{Sin}\kappa \qquad \kappa=\beta-\Theta$$

$$\tan(\beta)/\tan(\kappa) = [(k+1)\ N_U\ N_U\ ] / [\ (k-1)\ N_U\ N_U + 2\ ]$$

$$\upsilon = \sqrt{[K]}\ \tan^{-1}\sqrt{[(M^2-1)/K]} - \tan^{-1}\sqrt{[M^2-1]}$$

$$K = (k+1)/(k-1) \qquad \upsilon_D = \upsilon_U + \Theta$$

$$P\,A\ \operatorname{Sin}(\boldsymbol{\theta}-\Theta) \qquad P\,A\ \operatorname{Cos}(\boldsymbol{\theta}-\Theta)$$

$$a = \sqrt{[\mathbf{K}/\rho]}$$

$$\mathbf{K} = K / [1 + [DK]/[Ee]]$$

$$a = \sqrt{[k R T]} = \sqrt{[K/\rho]}$$

$$K/\rho = k R T \qquad K = k \rho R T$$

$$K = k P$$

$$a_M = \sqrt{[K_M/\rho_M]}$$

$$\rho_M = \Sigma[\rho_S V_S]/V_M \qquad K_M = V_M/\Sigma[V_S/K_S]$$

$$[P - P_o] = [f(N) + F(M)]$$

$$[U - U_o] = [f(N) - F(M)] / [\rho a]$$

$$N = x - a t \qquad M = x + a t$$

$$\leftarrow F : \Delta P = + \rho a \Delta U$$

$$\rightarrow f : \Delta P = - \rho a \Delta U$$

$$P_M - P_n = + [\rho a] [U_M - U_n]$$

$$P_N - P_m = - [\rho a] [U_N - U_m]$$

$$\partial/\partial x \ (h^3 \ \partial P/\partial x) \ + \ \partial/\partial y \ (h^3 \ \partial P/\partial y) \ = \ 6 \mu S \ \partial h/\partial x$$

$$A = [(h_E+h_P)/2]^3 \ / \ [\Delta x]^2 \qquad B = [(h_W+h_P)/2]^3 \ / \ [\Delta x]^2$$

$$C = [h_P]^3 \ / \ [\Delta y]^2 \qquad D = [h_P]^3 \ / \ [\Delta y]^2$$

$$H = - \ 6\mu \ S \ [h_E-h_W]/[2\Delta x]$$

$$r \ \partial/\partial c \ (h^3 \ \partial P/\partial c) \ + \partial/\partial r \ (rh^3 \ \partial P/\partial r) \ = \ 6\mu \ S \ \partial h/\partial \Theta$$

$$A = r_P \ [(h_E+h_P)/2]^3 \ / \ [\Delta c]^2 \qquad B = r_P \ [(h_W+h_P)/2]^3 \ / \ [\Delta c]^2$$

$$C = [h_P]^3 \ [(r_N+r_P)/2] \ / \ [\Delta r]^2 \qquad D = [h_P]^3 \ [(r_S+r_P)/2] \ / \ [\Delta r]^2$$

$$H = - \ 6\mu \ r_P\omega \ [h_E-h_W]/[2\Delta\Theta]$$

$$P_P = \frac{(A \ P_E + B \ P_W + C \ P_N + D \ P_S + H)}{(A + B + C + D)}$$

$$\Delta F = P \ \Delta c \ \Delta r \qquad \Delta F = P \ \Delta x \ \Delta y$$

$$\Delta F_H = \Delta F \ \text{Sin}\theta \qquad \Delta F_V = \Delta F \ \text{Cos}\theta$$

$$F_H = \sum \Delta F_H \qquad F_V = \sum \Delta F_V$$

$$F = \sqrt{\ } \ [F_H * F_H + F_V * F_V]$$

$$\theta = \tan^{-1} \ [F_H \ / \ F_V]$$

$$d/dx\ (h^3\ dP/dx) = 6\mu\ S\ dh/dx = H\ dh/dx$$

$$h^3\ dP/dx = H\ h + A \qquad dP/dx = H/h^2 + A/h^3$$

$$dP/dx = H/(sx+b)^2 + A/(sx+b)^3 \qquad s=[a-b]/d$$

$$P = -H/[s(sx+b)] - A/[2s(sx+b)^2] + B$$

$$A = \mathbf{[P_I-P_O}][2sa^2b^2]/[b^2-a^2] - 2Hba/[b+a]$$

$$B = \mathbf{[P_I}b^2\mathbf{-P_O}a^2]/[b^2-a^2] + H/[s(b+a)]$$

$$\Delta[h^3\ dP/dx] = H\ \Delta h$$

$$a^3\ [P_O-\mathbf{P}]/v - b^3\ [\mathbf{P}-P_I]/w = H\ [a-b]$$

$$\mathbf{P} = [\ a^3/v\ P_O + b^3/w\ P_I + H\ [b-a]\ ]\ /\ [\ a^3/v + b^3/w\ ]$$

$$d/dy\ (h^3\ dP/dy) = 6\mu\ S\ dh/dx = H\ dh/dx$$

$$d/dy\ (dP/dy) = H/h^3\ dh/dx = G$$

$$P = G/2\ y^2 + Ay + B$$

$$\varphi = S\ [\ X + XR^2/(X^2+Y^2)\ ]\ +\ \Gamma/[2\pi]\ \sigma$$

$$\varphi = 2\ S\ X\ +\ \Gamma/[2\pi]\ \sigma$$

$$\rho/2\ [\ S^2 - (\partial\varphi/\partial c)^2\ ] \qquad \rho/2\ [\ S^2 - (\Delta\varphi/\Delta c)^2\ ]$$

$$\alpha = x + xa^2/(x^2+y^2) \qquad \beta = y - ya^2/(x^2+y^2)$$

$$m^2 + (a+n)^2 = R^2$$

$$\Delta\varphi = 2\ S\ \Delta X\ +\ \Gamma/[2\pi]\ \Delta\sigma$$

$$\Delta c = \sqrt{[\Delta\alpha^2+\Delta\beta^2]}$$

$$X = \mathbf{X}\ \mathrm{Cos}\Theta + \mathbf{Y}\ \mathrm{Sin}\Theta \qquad Y = \mathbf{Y}\ \mathrm{Cos}\Theta - \mathbf{X}\ \mathrm{Sin}\Theta$$

$$\mathbf{X} = x + n \qquad \mathbf{Y} = y - m$$

$$\rho S\Gamma \qquad \Gamma = 4\pi SR\ \mathrm{Sin}\kappa$$

$$\kappa = \Theta + \varepsilon \qquad \varepsilon = \tan^{-1}\ [m/(n+a)]$$

$$P\Delta c\ \mathrm{Sin}(\boldsymbol{\theta}-\Theta) \qquad P\Delta c\ \mathrm{Cos}(\boldsymbol{\theta}-\Theta)$$

$$\boldsymbol{\theta} = \tan^{-1}[-\Delta\alpha/+\Delta\beta]$$

$$L=\Sigma\Delta L \qquad D=\Sigma\Delta D$$

$$\varphi = S\ X + S\ XR^2/[X^2+Y^2]$$

$$\varphi = 2\ S\ X = -2\ S\ R\ Cos[c/R]$$

$$\partial\varphi/\partial c = 2\ S\ Sin\sigma$$

$$P = \rho/2\ [\ S^2 - (\partial\varphi/\partial c)^2\ ] \qquad P = \rho/2\ S^2\ [\ 1 - 4\ Sin^2\sigma\ ]$$

$$- \int\ P\ L\ R\ Sin\sigma\ d\sigma \qquad + \int\ P\ L\ R\ Cos\sigma\ d\sigma$$

$$\varphi = -\ S\ r\ Cos[\sigma\ ] -\ S/2\ R^3/r^2\ Cos[\sigma]$$

$$\varphi = -\ S\ R\ Cos[c/R]\ -\ S/2\ R\ Cos[c/R]$$

$$\partial\varphi/\partial c = 3/2\ S\ Sin\sigma$$

$$P = \rho/2\ [\ S^2 - (\partial\varphi/\partial c)^2\ ] \qquad P = \rho/2\ S^2\ [\ 1 - 9/4\ Sin^2\sigma\ ]$$

$$dA = R\ Sin\sigma\ d\Theta\ \ R\ d\sigma$$

$$dF = P\ dA = P\ R^2\ Sin\sigma\ d\sigma\ d\Theta$$

$$\int\int - P\ R^2\ Sin\sigma\ Sin\sigma\ Cos\Theta\ d\sigma\ d\Theta$$

$$\int\int + P\ R^2\ Sin\sigma\ Cos\sigma\ d\sigma\ d\Theta$$

$$U = U_\circ \, M/M_\circ \, \zeta \, a$$

$$U_\circ = D/\mathbf{T} \qquad M_\circ = \rho D^2$$

$$U = \beta/\mathbf{T} \, \sqrt{[M\delta/\rho]}$$

$$U = \beta U_\circ \, \sqrt{[\delta M/M_\circ]}$$

$$U = D/[ST]$$

$$T = \mathbf{T}$$

$$U^2 = [\ EI/[\rho A] \ \pi^2/L^2 + T/[\rho A] - P/\rho\ ]$$

$$U = [4 + 14 \, M_\circ/M] \, U_\circ$$

$$U_\circ = \sqrt{[EI]/[M_\circ L^2]} \qquad M_\circ = \rho A$$

$$\mathbf{T}_n = [2L/n] \, \sqrt{[m/T]}$$

$$\mathbf{T}_n = [L/n]^2 \, [2/\pi] \, \sqrt{[m/EI]}$$

$$\mathbf{T}_n = 2\pi L^2/K_n \, \sqrt{[m/EI]}$$

$$\frac{D}{Dt} \int_{V(t)} \rho \, dV = 0$$

$$\partial U/\partial x + \partial V/\partial y + \partial W/\partial z = 0$$

$$\frac{D}{Dt} \int_{V(t)} \rho \mathbf{v} \, dV = \int_{S(t)} \boldsymbol{\sigma} \, dS + \int_{V(t)} \rho \mathbf{b} \, dV$$

$$\rho \, \partial U/\partial t + \rho \, (U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z) = - \partial P/\partial x$$

$$+ \mu \, (\partial^2 U/\partial x^2 + \partial^2 U/\partial y^2 + \partial^2 U/\partial z^2)$$

$$\rho \, \partial V/\partial t + \rho \, (U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z) = - \partial P/\partial y$$

$$+ \mu \, (\partial^2 V/\partial x^2 + \partial^2 V/\partial y^2 + \partial^2 V/\partial z^2)$$

$$\rho \, \partial W/\partial t + \rho \, (U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z) = - \partial P/\partial z - \rho g$$

$$+ \mu \, (\partial^2 W/\partial x^2 + \partial^2 W/\partial y^2 + \partial^2 W/\partial z^2)$$

$$\frac{D}{Dt} \int_{V(t)} \rho e \, dV = - \int_{S(t)} \mathbf{q.n} \, dS + \int_{S(t)} \mathbf{v}.\boldsymbol{\sigma} \, dS$$

$$\rho C \, \partial T/\partial t + \rho C \, (U\partial T/\partial x + V\partial T/\partial y + W\partial T/\partial z) = \mu \, \Phi$$

$$+ \partial/\partial x(k\partial T/\partial x) + \partial/\partial y(k\partial T/\partial y) + \partial/\partial z(k\partial T/\partial z)$$

$$\rho \left(\partial U/\partial t + U\partial U/\partial x + V\partial U/\partial y + W\partial U/\partial z \right) + A = - \partial P/\partial x$$

$$+ \left[\partial/\partial x \left(\mu\, \partial U/\partial x\right) + \partial/\partial y \left(\mu\, \partial U/\partial y\right) + \partial/\partial z \left(\mu\, \partial U/\partial z\right) \right]$$

$$\rho \left(\partial V/\partial t + U\partial V/\partial x + V\partial V/\partial y + W\partial V/\partial z \right) + B = - \partial P/\partial y$$

$$+ \left[\partial/\partial x \left(\mu\, \partial V/\partial x\right) + \partial/\partial y \left(\mu\, \partial V/\partial y\right) + \partial/\partial z \left(\mu\, \partial V/\partial z\right) \right]$$

$$\rho \left(\partial W/\partial t + U\partial W/\partial x + V\partial W/\partial y + W\partial W/\partial z \right) + C = - \partial P/\partial z - \rho g$$

$$+ \left[\partial/\partial x \left(\mu\, \partial W/\partial x\right) + \partial/\partial y \left(\mu\, \partial W/\partial y\right) + \partial/\partial z \left(\mu\, \partial W/\partial z\right) \right]$$

$$\partial P/\partial t + \rho\, c^2 \left(\partial U/\partial x + \partial V/\partial y + \partial W/\partial z \right) = 0$$

$$\partial F/\partial t + U\partial F/\partial x + V\partial F/\partial y + W\partial F/\partial z = 0$$

$$\partial k/\partial t + U\partial k/\partial x + V\partial k/\partial y + W\partial k/\partial z = T_P - T_D$$

$$+ \left[\partial/\partial x \left(\mu/a\, \partial k/\partial x\right) + \partial/\partial y \left(\mu/a\, \partial k/\partial y\right) + \partial/\partial z \left(\mu/a\, \partial k/\partial z\right) \right]$$

$$\partial \varepsilon/\partial t + U\partial \varepsilon/\partial x + V\partial \varepsilon/\partial y + W\partial \varepsilon/\partial z = D_P - D_D$$

$$+ \left[\partial/\partial x \left(\mu/b\, \partial \varepsilon/\partial x\right) + \partial/\partial y \left(\mu/b\, \partial \varepsilon/\partial y\right) + \partial/\partial z \left(\mu/b\, \partial \varepsilon/\partial z\right) \right]$$

$$\partial M/\partial t = N \qquad M_{NEW} = M_{OLD} + \Delta t\ N_{OLD}$$

$$\mathbf{v} = -K \nabla P \qquad \nabla . \mathbf{v} = 0$$

$$\nabla . [K \nabla P] = 0$$

$$\partial/\partial x\ [K\ \partial P/\partial x] + \partial/\partial y\ [K\ \partial P/\partial y] + \partial/\partial z\ [K\ \partial P/\partial z] = 0$$

$$P_P = \frac{(A\ P_E + B\ P_W + C\ P_N + D\ P_S + G\ P_J + H\ P_I)}{(A + B + C + D + G + H)}$$

$$A = [(K_E+K_P)/2]/[\Delta x]^2 \qquad B = [(K_W+K_P)/2]/[\Delta x]^2$$

$$C = [(K_N+K_P)/2]/[\Delta y]^2 \qquad D = [(K_S+K_P)/2]/[\Delta y]^2$$

$$G = [(K_J+K_P)/2]/[\Delta z]^2 \qquad H = [(K_I+K_P)/2]/[\Delta z]^2$$

$$S_W = A/\omega^5\ e^{-B/\omega 4} \qquad A=346H^2/T^4 \qquad B=691/T^4$$

$$S_R = RAO^2\ S_W \qquad M_n = 1/2 \int S_R(\omega)\ \omega^n\ d\omega$$

$$H_R = 4\ \sqrt{M_0} \qquad T_R = 2\pi\ M_0/M_1$$

$$P(R_o > R_\bullet) = e^{-X} \qquad X = R_\bullet R_\bullet / [2M_0]$$

$$\mu = [\ T\ h\ ]\ /\ [\ 2\pi\ R^3\ L\ \omega\ ]$$

$$\mu = [\ 2\ T\ h\ ]\ /\ [\pi\ R^4\ \omega]$$

$$\mu = [\rho g H][\pi R^4]\ /\ [8QL]$$