

HYDRODYNAMIC LUBRICATION

ILLUSTRATION: JOURNAL BEARING CFD

A CFD template for a journal bearing is

$$P_P = \frac{(A P_E + B P_W + C P_N + D P_S + H)}{(A + B + C + D)}$$

$$A = [(h_E + h_P)/2]^3 / [\Delta x^2] \quad B = [(h_W + h_P)/2]^3 / [\Delta x^2]$$

$$C = [(h_N + h_P)/2]^3 / [\Delta y^2] \quad D = [(h_S + h_P)/2]^3 / [\Delta y^2]$$

$$H = - 6\mu S (h_E - h_W) / [2\Delta x]$$

Consider a case where the rolled out geometry is 0.12m long and 0.04m wide. For a 9 point CFD grid this gives Δx equal to 0.12/4 or 0.03m and Δy equal to 0.04/4 or 0.01m. The rolled out length is πR . Working back one finds that R is 0.038m. Let the speed S due to shaft rotation be 5m/s. This speed is $R 2\pi \text{ RPM}/60$. Working back one finds that RPM is 1256. Let the maximum gap be 1.0mm and the minimum gap be 0.2mm. Assume a linear gap variation from the maximum to the minimum. Let the oil viscosity μ be 0.1 Ns/m².

The CFD coefficients for point #1 are

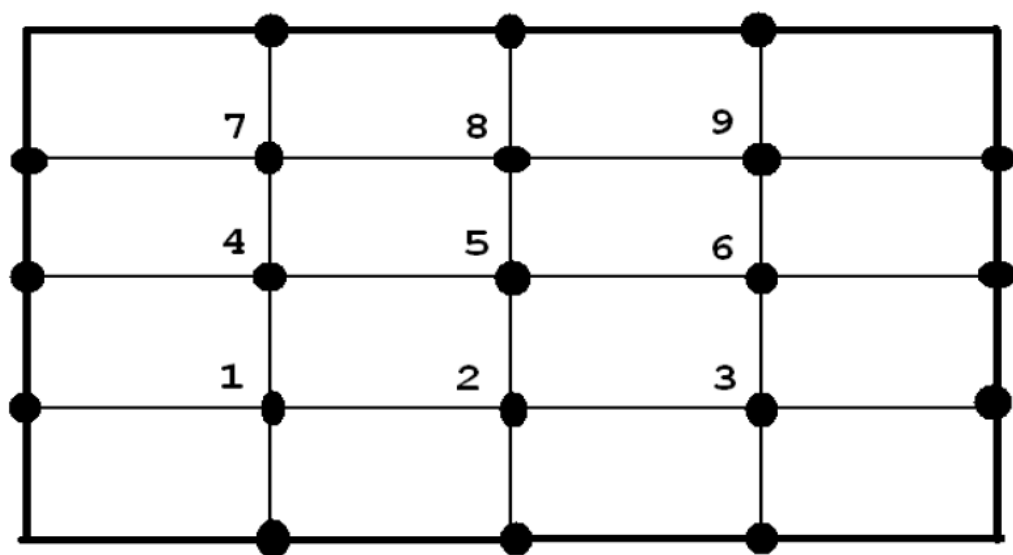
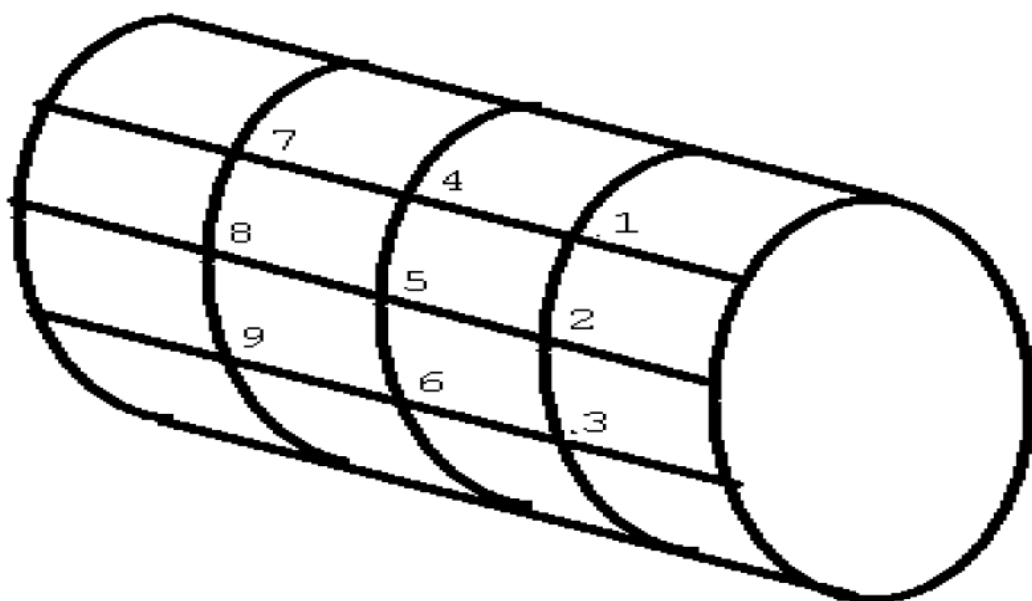
$$A = (0.7 \times 10^{-3})^3 / (0.03)^2 = 3.851 \times 10^{-7}$$

$$B = (0.9 \times 10^{-3})^3 / (0.03)^2 = 8.184 \times 10^{-7}$$

$$C = (0.8 \times 10^{-3})^3 / (0.01)^2 = 5.120 \times 10^{-6}$$

$$D = (0.8 \times 10^{-3})^3 / (0.01)^2 = 5.120 \times 10^{-6}$$

$$H = - [6 \times 0.1 \times 5 \times (0.6 - 1.0) \times 10^{-3}] / [2 \times 0.03] = 0.0201$$



The CFD coefficients for point #2 are

$$A = (0.5 \times 10^{-3})^3 / (0.03)^2 = 1.403 \times 10^{-7}$$

$$B = (0.7 \times 10^{-3})^3 / (0.03)^2 = 3.851 \times 10^{-7}$$

$$C = (0.6 \times 10^{-3})^3 / (0.01)^2 = 2.160 \times 10^{-6}$$

$$D = (0.6 \times 10^{-3})^3 / (0.01)^2 = 2.160 \times 10^{-6}$$

$$H = - [6 \times 0.1 \times 5 \times (0.4 - 0.8) \times 10^{-3}] / [2 \times 0.03] = 0.0201$$

The CFD coefficients for point #3 are

$$A = (0.3 \times 10^{-3})^3 / (0.03)^2 = 0.3031 \times 10^{-7}$$

$$B = (0.5 \times 10^{-3})^3 / (0.03)^2 = 1.403 \times 10^{-7}$$

$$C = (0.4 \times 10^{-3})^3 / (0.01)^2 = 0.640 \times 10^{-6}$$

$$D = (0.4 \times 10^{-3})^3 / (0.01)^2 = 0.640 \times 10^{-6}$$

$$H = - [6 \times 0.1 \times 5 \times (0.2 - 0.6) \times 10^{-3}] / [2 \times 0.03] = 0.0201$$

Points 4 and 7 have the same coefficients as point 1. Points 5 and 8 have the same coefficients as point 2. Points 6 and 9 have the same coefficients as point 3. The CFD template gives

$$\begin{aligned} P_1 &= \frac{0.3851 P_2 + 5.12 P_4 + 0.0201 \times 10^{+6}}{0.3851 + 0.8184 + 5.12 + 5.12} \\ &= 0.0337 P_2 + 0.4474 P_4 + 1757 \end{aligned}$$

$$\begin{aligned} P_2 &= \frac{0.1403 P_3 + 0.3851 P_1 + 2.16 P_5 + 0.0201 \times 10^{+6}}{0.1403 + 0.3851 + 2.16 + 2.16} \\ &= 0.029 P_3 + 0.0795 P_1 + 0.4458 P_5 + 4149 \end{aligned}$$

$$\begin{aligned}
 P_3 &= \frac{0.1403 P_2 + 0.64 P_6 + 0.0201 \times 10^6}{0.03031 + 0.1403 + 0.64 + 0.64} \\
 &= 0.0967 P_2 + 0.4412 P_6 + 13859
 \end{aligned}$$

$$\begin{aligned}
 P_4 &= \frac{0.3851 P_5 + 5.12 P_7 + 5.12 P_1 + 0.0201 \times 10^6}{0.3851 + 0.8184 + 5.12 + 5.12} \\
 &= 0.0337 P_5 + 0.4474 P_7 + 0.4474 P_1 + 1757
 \end{aligned}$$

$$\begin{aligned}
 P_5 &= \frac{0.1403 P_6 + 0.3851 P_4 + 2.16 P_8 + 2.16 P_2 + 0.0201 \times 10^6}{0.1403 + 0.3851 + 2.16 + 2.16} \\
 &= 0.029 P_6 + 0.0795 P_4 + 0.4458 P_8 + 0.4458 P_2 + 4149
 \end{aligned}$$

$$\begin{aligned}
 P_6 &= \frac{0.1403 P_5 + 0.64 P_9 + 0.64 P_3 + 0.0201 \times 10^6}{0.03031 + 0.1403 + 0.64 + 0.64} \\
 &= 0.0967 P_5 + 0.4412 P_9 + 0.4412 P_3 + 13859
 \end{aligned}$$

$$\begin{aligned}
 P_7 &= \frac{0.3851 P_8 + 5.12 P_4 + 0.0201 \times 10^6}{0.3851 + 0.8184 + 5.12 + 5.12} \\
 &= 0.0337 P_8 + 0.4474 P_4 + 1757
 \end{aligned}$$

$$\begin{aligned}
 P_8 &= \frac{0.1403 P_9 + 0.3851 P_7 + 2.16 P_5 + 0.0201 \times 10^6}{0.1403 + 0.3851 + 2.16 + 2.16} \\
 &= 0.029 P_9 + 0.0795 P_7 + 0.4458 P_5 + 4149
 \end{aligned}$$

$$\begin{aligned}
 P_9 &= \frac{0.1403 P_8 + 0.64 P_6 + 0.0201 \times 10^6}{0.03031 + 0.1403 + 0.64 + 0.64} \\
 &= 0.0967 P_8 + 0.4412 P_6 + 13859
 \end{aligned}$$

The equations for pressure can be solved using Gauss Seidel iteration. The iteration starts with all pressures set to zero. The first sweep through the equations gives:

$$\begin{aligned} P_1 &= 0.0337 P_2 + 0.4474 P_4 + 1757 \\ &= 0 + 0 + 1757 = 1757 \end{aligned}$$

$$\begin{aligned} P_2 &= 0.029 P_3 + 0.0795 P_1 + 0.4458 P_5 + 4149 \\ &= 0 + 0.0795 * 1757 + 0 + 4149 = 4289 \end{aligned}$$

$$\begin{aligned} P_3 &= 0.0967 P_2 + 0.4412 P_6 + 13859 \\ &= 0.0967 * 4289 + 0 + 13859 = 14274 \end{aligned}$$

$$\begin{aligned} P_4 &= 0.0337 P_5 + 0.4474 P_7 + 0.4474 P_1 + 1757 \\ &= 0 + 0 + 0.4474 * 1757 + 1757 = 2543 \end{aligned}$$

$$\begin{aligned} P_5 &= 0.029 P_6 + 0.0795 P_4 + 0.4458 P_8 + 0.4458 P_2 + 4149 \\ &= 0 + 0.078 * 2543 + 0 + 0.4458 * 4289 + 4149 = 6263 \end{aligned}$$

$$\begin{aligned} P_6 &= 0.0967 P_5 + 0.4412 P_9 + 0.4412 P_3 + 13859 \\ &= 0.0967 * 6263 + 0 + 0.4412 * 14274 + 13859 = 20762 \end{aligned}$$

$$\begin{aligned} P_7 &= 0.0337 P_8 + 0.4474 P_4 + 1757 \\ &= 0 + 0.4474 * 2543 + 1757 = 2895 \end{aligned}$$

$$\begin{aligned} P_8 &= 0.029 P_9 + 0.0795 P_7 + 0.4458 P_5 + 4149 \\ &= 0 + 0.0795 * 2895 + 0.4458 * 6263 + 4149 = 7171 \end{aligned}$$

$$\begin{aligned} P_9 &= 0.0967 P_8 + 0.4412 P_6 + 13859 \\ &= 0.0967 * 7171 + 0.4412 * 20762 + 13859 = 23712 \end{aligned}$$

After 100 sweeps through the equations one gets

$$\begin{array}{lll}
P_1 = 5483 & P_2 = 13866 & P_3 = 36191 \\
P_4 = 7284 & P_5 = 18468 & P_6 = 47519 \\
P_7 = 5483 & P_8 = 13866 & P_9 = 36191
\end{array}$$

The cell area is $\Delta x \Delta y = 0.03 \times 0.01 = 0.0003$.

The incremental loads $\Delta F = P \Delta x \Delta y$ are

$$\begin{array}{lll}
\Delta F_1 = 1.64 & \Delta F_2 = 4.14 & \Delta F_3 = 10.80 \\
\Delta F_4 = 2.17 & \Delta F_5 = 5.51 & \Delta F_6 = 14.20 \\
\Delta F_7 = 1.64 & \Delta F_8 = 4.14 & \Delta F_9 = 10.80
\end{array}$$

The horizontal components $\Delta F_H = \Delta F \sin \theta$ are

$$\begin{array}{lll}
\Delta F_{1H} = +1.16 & \Delta F_{2H} = +4.14 & \Delta F_{3H} = +7.64 \\
\Delta F_{4H} = +1.54 & \Delta F_{5H} = +5.51 & \Delta F_{6H} = +10.04 \\
\Delta F_{7H} = +1.16 & \Delta F_{8H} = +4.14 & \Delta F_{9H} = +7.64
\end{array}$$

The vertical components $\Delta F_V = \Delta F \cos \theta$ are

$$\begin{array}{lll}
\Delta F_{1V} = -1.16 & \Delta F_{2V} = 0.0 & \Delta F_{3V} = +7.64 \\
\Delta F_{4V} = -1.54 & \Delta F_{5V} = 0.0 & \Delta F_{6V} = +10.04 \\
\Delta F_{7V} = -1.16 & \Delta F_{8V} = 0.0 & \Delta F_{9V} = +7.64
\end{array}$$

The total loads are

$$F_H = \sum \Delta F_H = +42.96 \quad F_V = \sum \Delta F_V = +21.46$$

$$F = \sqrt{[F_H]^2 + [F_V]^2} = 48.02$$

The load angle $\theta = \tan^{-1} [F_H / F_V] = 63.4^\circ$