# UNSTEADY FLOW

# IN

# PIPE NETWORKS

HINCHEY

# PREAMBLE

Unsteady flow in pipe networks can be caused by a number of factors. A turbomachine with blades can send pressure waves down a pipe. If the period of these waves matches a natural period of the pipe wave speed resonance develops. A piston pump can send similar waves down a pipe. Waves on the surface of a water reservoir can also excite resonance of inlet pipes. Sudden valve or turbomachine changes can send waves up and down pipes. These can cause the pipes to explode or implode. In some cases interaction between pipes and devices is such that oscillations develop automatically. Examples include oscillations set up by leaky valves and those set up by slow turbomachine controllers.

# WAVE PROPAGATION IN PIPES

Consider flow in a rigid pipe with a valve at its downstream end and a reservoir at its upstream end. Assume that there are no friction losses. This implies that the pressure and flow speed are the same everywhere along the pipe.

Imagine now that the valve is suddenly closed. This causes a high pressure or surge wave to propagate up the pipe. As it does so, it brings the fluid to rest. The fluid immediately next to the valve is stopped first. The valve is like a wall. Fluid enters an infinitesimal layer next to this wall and pressurizes it and stops. This layer becomes like a wall for an infinitesimal layer just upstream. Fluid then enters that layer and pressurizes it and stops. As the surge wave propagates up the pipe, it causes an infinite number of these pressurizations. When it reaches the reservoir, all of the inflow has been stopped, and pressure is high everywhere along the pipe. The pipe resembles a compressed spring.

When the surge wave reaches the reservoir, it creates a pressure imbalance. The layer of fluid just inside the pipe has high pressure fluid downstream of it and reservoir pressure upstream. Fluid exits the layer on its upstream side and depressurizes it. The pressure drops back to the reservoir level. A backflow wave is created. The speed of the backflow is exactly the same as the speed of the original inflow. The pressure that was generated by taking the original inflow away is exactly what is available to generate the backflow. The backflow wave propagates down the pipe restoring pressure everywhere to its original level.

When the backflow wave reaches the valve, it creates a flow imbalance. This causes a low pressure or suction wave to propagate up the pipe. As it does so, it brings the fluid to rest. Again, the valve is like a wall. Because of backflow, fluid exits an infinitesimal layer next to this wall and depressurizes it and stops. The pressure drops below the reservoir level by exactly the amount it was above the reservoir level in the surge wave.

When the suction wave reaches the reservoir, all of the backflow has been stopped, and pressure is low everywhere along the pipe. The pipe resembles a stretched spring. At the reservoir, the suction wave creates a pressure imbalance. An inflow wave is created. The speed of the inflow is exactly the same as the speed of the backflow. The inflow wave travels down the pipe restoring pressure to its original level. Conditions in the pipe become what they were just before the valve was closed.

During one cycle of vibration, there are 4 transits of the pipe by pressure waves. This means that the natural period of the pipe is 4 times the length of the pipe divided by the wave speed. Without friction, the vibration cycle repeats over and over. With friction, it gradually dies away.

# BASIC WAVE EQUATIONS

Consider a wave travelling up a rigid pipe. In a reference frame moving with the wave, mass considerations give

$$\rho \; A \; (U+a) = (\rho+\Delta\rho) \; A \; (U+\Delta U+a)$$

where $\rho$ is density, A is pipe area, U is flow velocity and a is wave speed. When a >> U, this reduces to

$$\rho \; \Delta U = - \; a \; \Delta\rho$$

Momentum considerations give

$$\rho A(U+a) \; [(U+\Delta U+a) - (U+a)] = [P - [P+\Delta P]] \; A$$

where P is pressure. When a >> U, this reduces to

$$\rho \; a \; \Delta U = - \; \Delta P$$

Manipulations give

$$a = \sqrt{[\Delta P/\Delta\rho]}$$

For a gas such as air moving down a pipe, one can assume ideal gas behavior for which:

$$P/\rho = R \; T$$

R is the ideal gas constant and T is the absolute temperature of the gas. For a wave propagating through a gas, one can assume processes are isentropic: in other words, adiabatic and frictionless. The wave moves so fast through the gas that there is no time for heat transfer or friction. The isentropic equation of state is:

$$P = K \rho^k$$

where K is another constant and k is the ratio of specific heats. Differentiation of this equation gives

$$\Delta P/\Delta\rho = k P/\rho$$

The ideal gas law into this gives

$$\Delta P/\Delta\rho = k R T$$

So wave speed for a gas becomes

$$a = \sqrt{[k R T]}$$

For a liquid, fluid mechanics shows that

$$\Delta P = - K \Delta V/V$$

where K is the bulk modulus of the liquid. It is a measure of its compressibility. For a bit of fluid mass

$$\Delta M = \Delta [\rho V] = V \Delta\rho + \rho \Delta V = 0$$

This implies that

$$\Delta P = K \Delta\rho/\rho \qquad \Delta P/\Delta\rho = K/\rho$$

So wave speed for a liquid becomes

$$a = \sqrt{[K/\rho]}$$

The bulk modulus of a gas follows from

$$a = \sqrt{[k R T]} = \sqrt{[K/\rho]}$$

$$K/\rho = k R T \qquad K = k \rho R T \qquad K = k P$$

The wave speed for a mixture is

$$a = \sqrt{[K_M/\rho_M]}$$

$$\rho_M = \sum[\rho_C V_C]/V_M \qquad K_M = V_M/\sum[V_C/K_C]$$

The wave speed for a flexible pipe is

$$a = \sqrt{[\mathbf{K}/\rho]} \qquad \mathbf{K} = K / [1 + [DK]/[Ee]]$$

where E is the Elastic Modulus of the pipe wall material, e is the wall thickness and D is the pipe diameter.

## WATERHAMMER ANALYSIS

Waterhammer analysis allows one to connect unknown pressure and flow velocity at one end of a pipe to known pressure and velocity at the other end of the pipe one transit time back in time. The derivation of the waterhammer equations starts with the conservation of momentum and mass equations for unsteady flow in a pipe. These are:

$$\rho\ \partial U/\partial t + \rho U\ \partial U/\partial x + \partial P/\partial x - \rho g\ \mathrm{Sin}\alpha + f/D\ \rho U|U|/2 = 0$$

$$\partial P/\partial t + U\ \partial P/\partial x + \rho a^2\ \partial U/\partial x = 0$$

where P is pressure and U is velocity. For the case where gravity and friction are insignificant and the mean flow speed is approximately zero, these reduce to:

$$\rho\ \partial U/\partial t + \partial P/\partial x = 0$$

$$\partial P/\partial t + \rho a^2\ \partial U/\partial x = 0$$

Manipulation gives the wave equations:

$$\partial^2 P/\partial t^2 = a^2\ \partial^2 P/\partial x^2$$

$$\partial^2 U/\partial t^2 = a^2\ \partial^2 U/\partial x^2$$

The general solution consists of two waves: one wave which travels up the pipe known as the F wave and the other which travels down the pipe known as the f wave.

In terms of these waves, pressure and velocity are:

$$P - P_o = f(N) + F(M)$$

$$U - U_o = [f(N) - F(M)] / [\rho a]$$

where N and M are wave fixed frames given by:

$$N = x - a\ t \qquad M = x + a\ t$$

For a given point N on the f wave, the N equation shows that x must increase as time increases, which means the wave must be moving down the pipe. For a given point M on the F wave, the M equation shows that x must decrease as time increases, which means the wave must be moving up the pipe. Substitution of the general solution into mass or momentum or the wave equations shows that they are valid solutions.

Multiplying U by ρa and subtracting it from P gives:

$$[P-P_o] - \rho a[U-U_o] = 2F(M)$$

Let the F wave travel from the downstream end of the pipe to the upstream end. For a point on the wave, the value of F would be the same. This implies

$$\Delta P = + \rho a \, \Delta U$$

Multiplying U by $\rho a$ and adding it to P gives:

$$[P-P_o] + \rho a[U-U_o] = 2f(N)$$

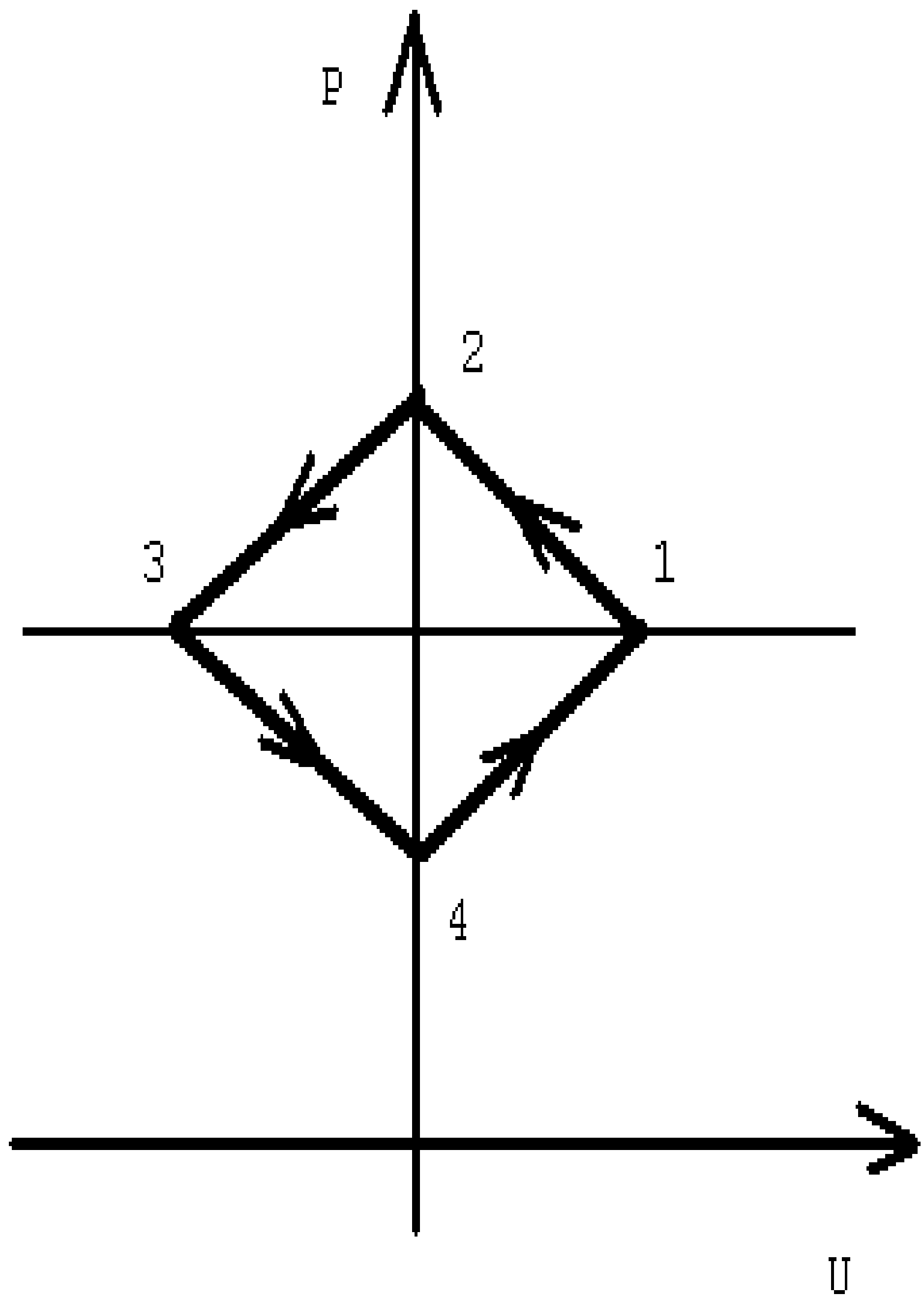
Let the f wave travel from the upstream end of the pipe to the downstream end. For a point on the wave, the value of f would be the same. This implies

$$\Delta P = - \rho a \, \Delta U$$

The $\Delta P$ vs $\Delta U$ equations allow us to connect unknown conditions at one end of a pipe at some point in time to known conditions at the other end back in time. They are known as the waterhammer equations.

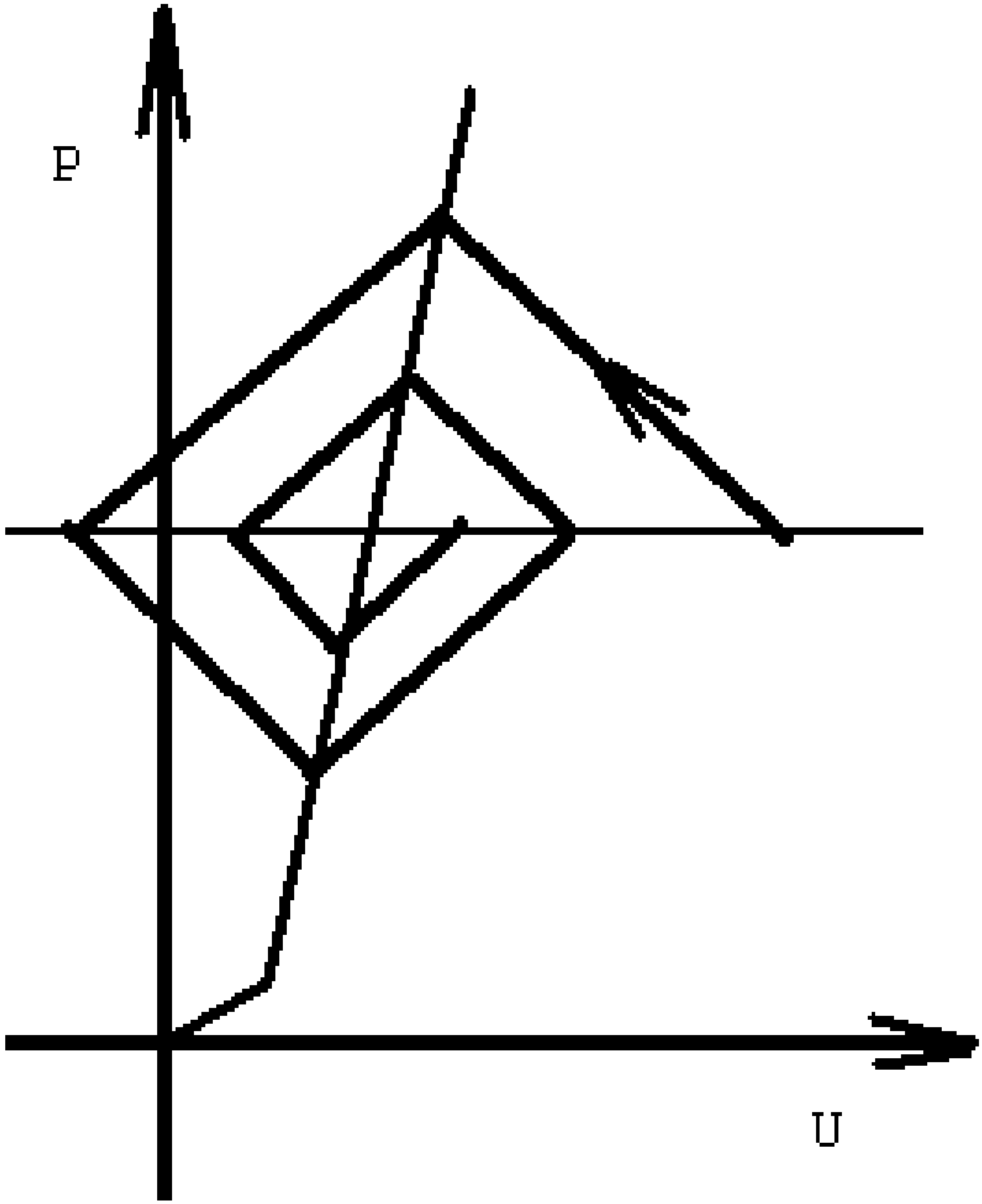
# SUDDEN VALVE CLOSURE

Imagine a pipe with a reservoir at its upstream end and a valve at its downstream end. The valve is initially open. Then it is suddenly shut. From that point onward, the velocity at the valve is zero. We ignore losses. Because of this, the pressure at the reservoir is fixed at its initial level. We start at point 1 which is at the reservoir and move along an f wave to point 2 which is at the valve. A surge wave is created at the valve. We then move from the valve along an F wave to point 3 which is at the reservoir. A backflow wave is created at the reservoir. We then move from the reservoir along an f wave to point 4 which is at the valve. A suction wave is created at the valve. We then move from the valve along an F wave to point 1 which is at the reservoir. An inflow wave is created at the reservoir. From this point onward the cycle repeats. Friction gradually dissipates the waves and the velocity homes in on zero.

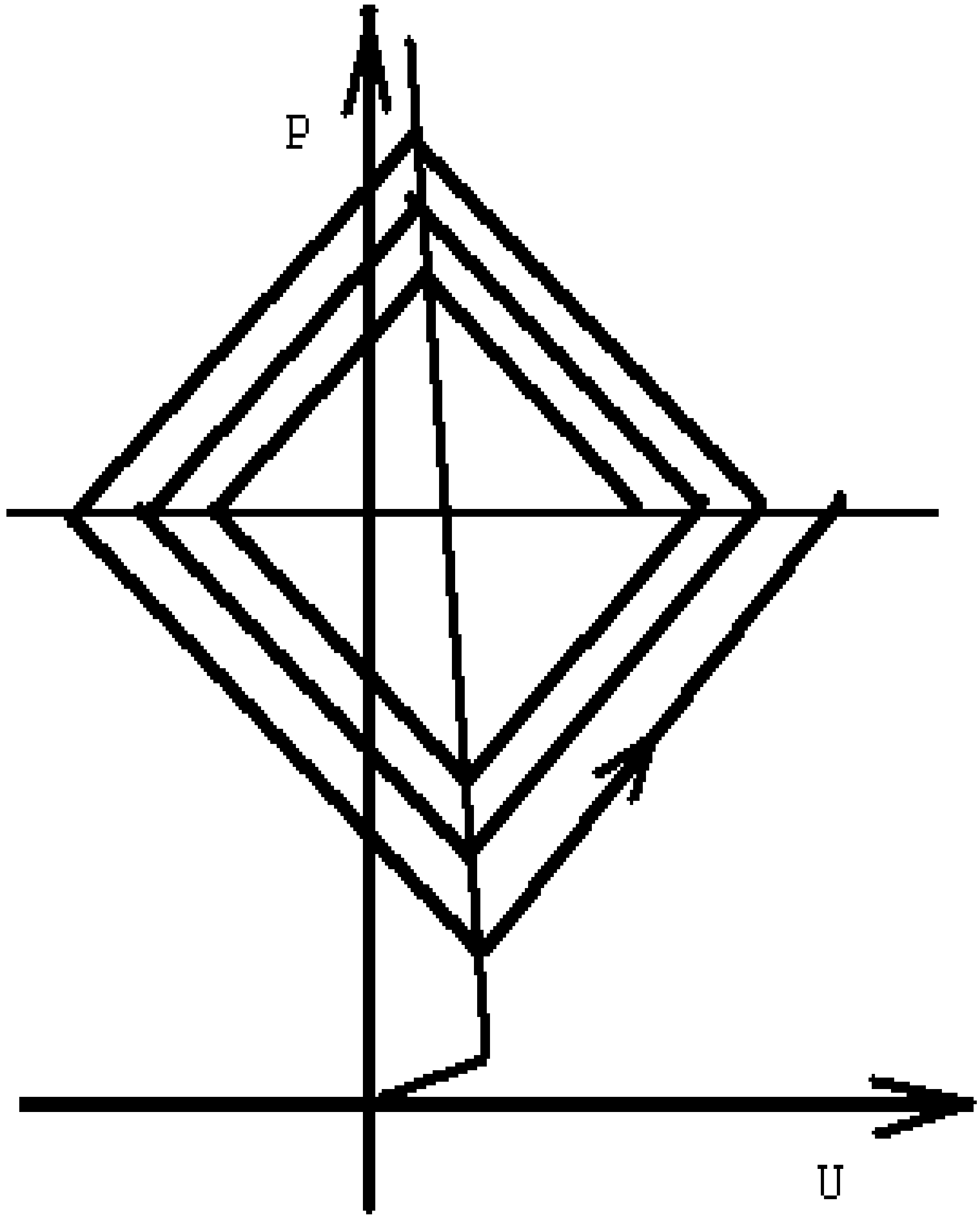

P
2
3
1
4
U

# LEAKY VALVES

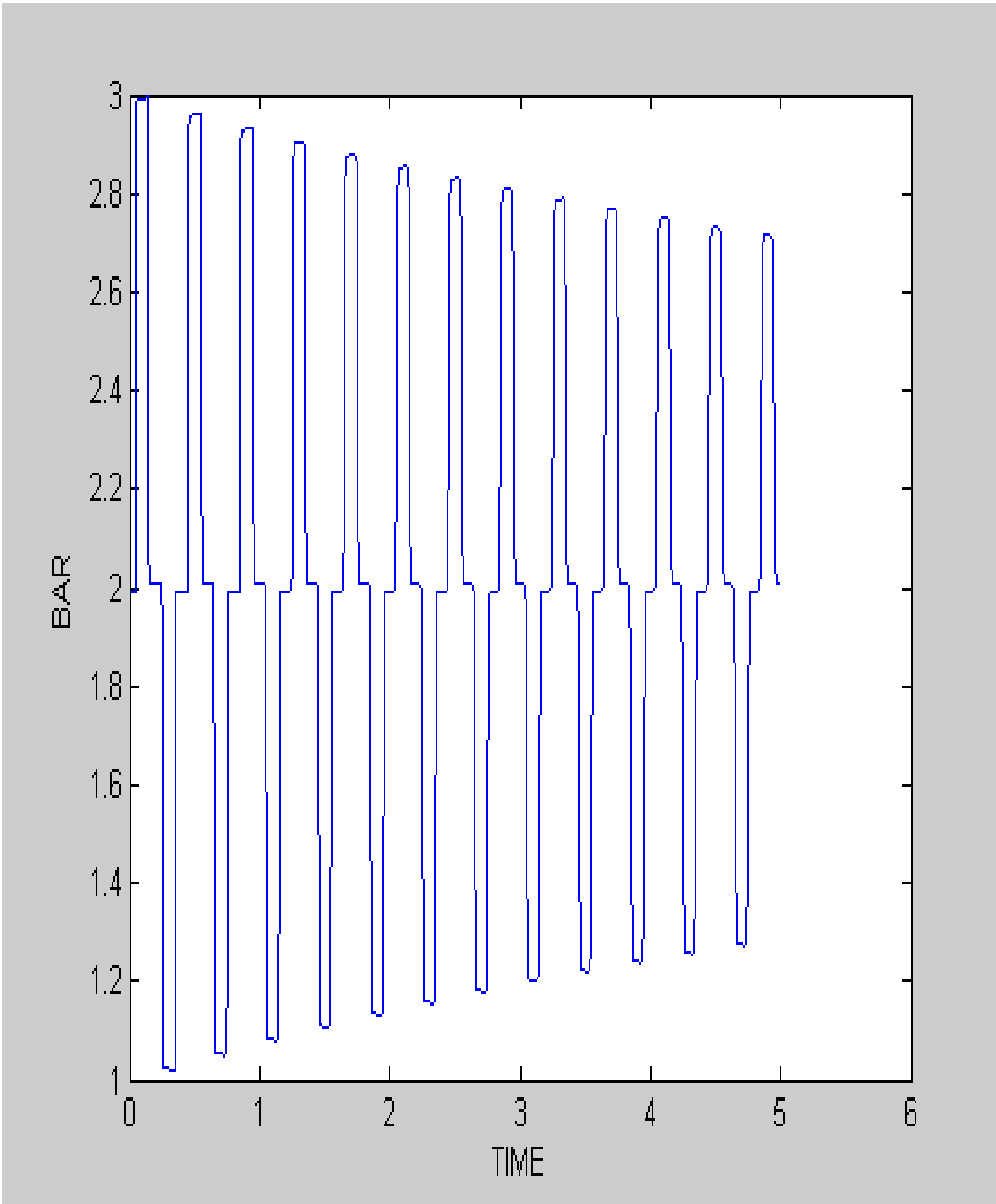
A stable leaky valve is basically one that has a P versus U characteristic which resembles that of a wide open valve. This has a parabolic shape with positive slope throughout. An unstable leaky valve has a characteristic that has a positive slope at low pressure but negative slope at high pressure. Basically, the valve tries to shut itself at high pressure. The flow rate just upstream of a valve is pipe flow speed times pipe area. The flow rate within the valve is valve flow speed times valve area. In a stable leaky valve, the areas are both constant. The valve flow speed increases with pipe pressure so the pipe flow speed also increases. In an unstable leaky valve, the flow speed within the valve also increases with pipe pressure but the valve area drops because of suction within the valve. The suction is generated by high speed flow through the small passageway within the valve. It pulls on flexible elements within the valve and attempts to shut it. Graphical waterhammer plots for stable and unstable leaky valves are given below. As can be seen, they both resemble the sudden valve closure plot, but the stable one is decaying while the unstable one is growing. In the unstable case, greater suction is needed each time a backflow wave comes up to the valve because the flow requirements of the valve keep getting bigger. In the stable case, less suction is needed because the flow requirements keep getting smaller.



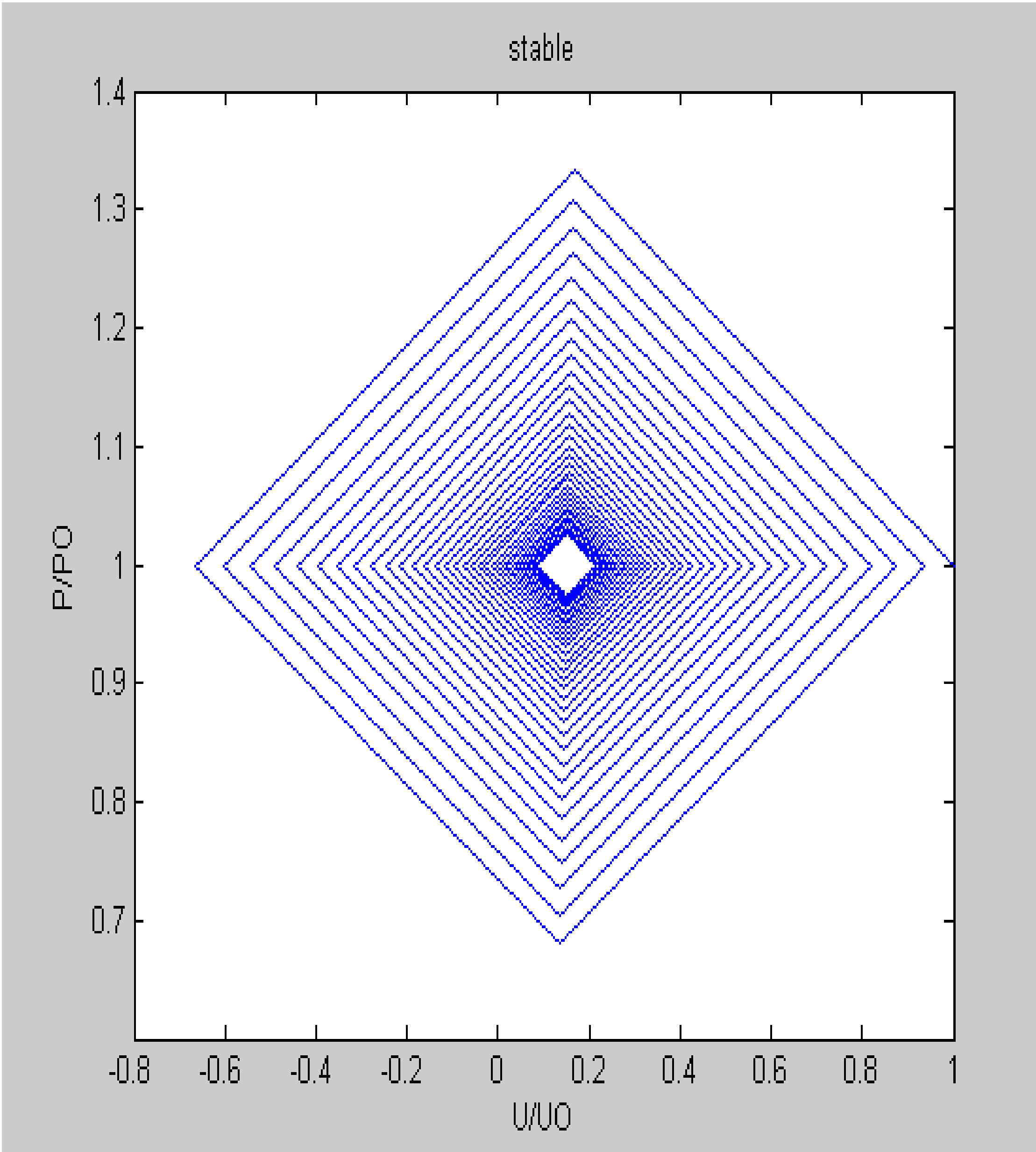

STABLE LEAKY VALVE



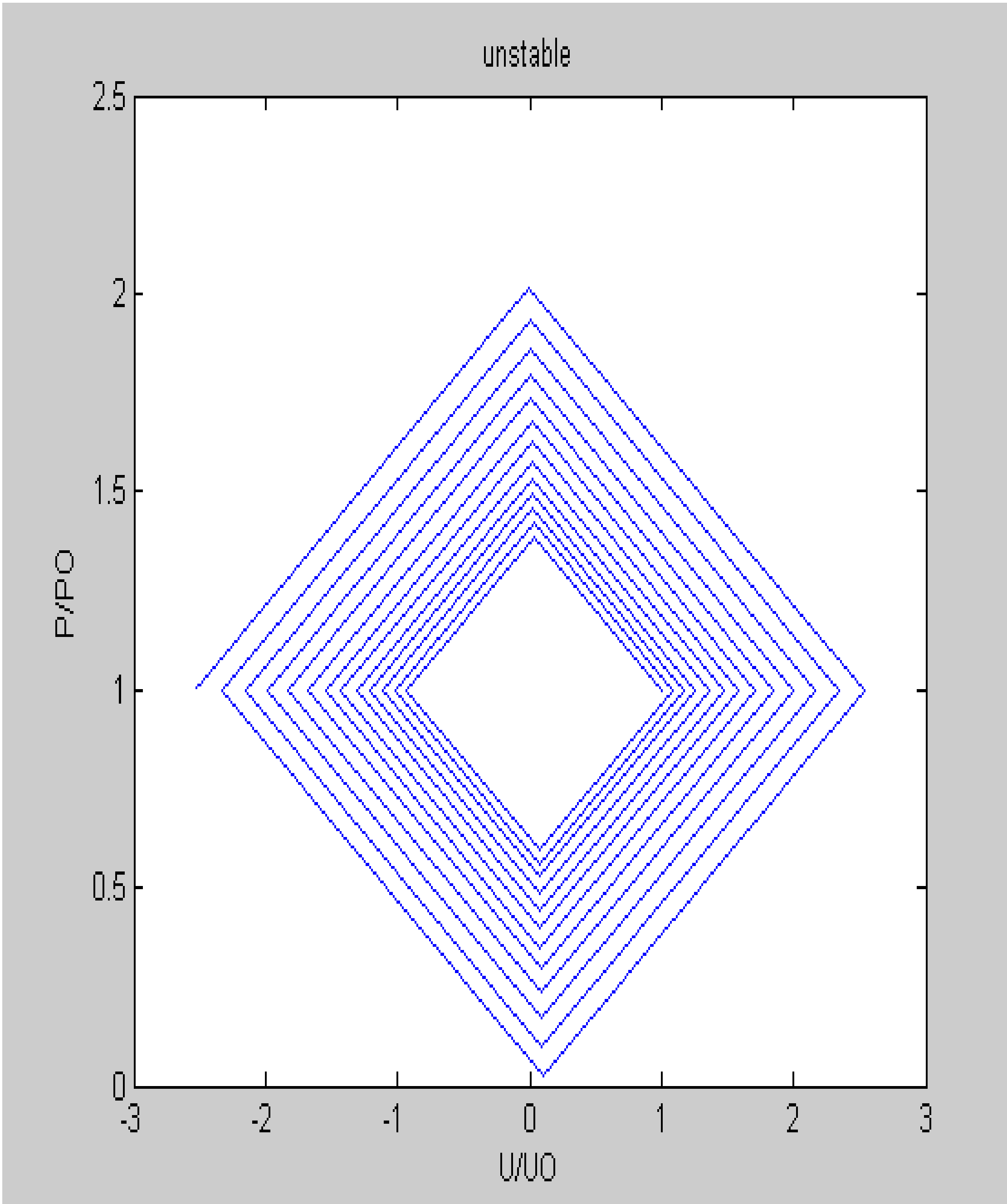

UNSTABLE LEAKY VALVE




SUDDEN VALVE CLOSURE




STABLE LEAKY VALVE




UNSTABLE LEAKY VALVE