BOUNDARY LAYER

FLOWS

HINCHEY

BOUNDARY LAYER PHENOMENA

When a body moves through a viscous fluid, the fluid at its surface moves with it. It does not slip over the surface. When a body moves at high speed, the transition between the surface and flow outside is known as a boundary layer. In a relative sense, it is a very thin layer. Within it, viscosity plays a dominant role because normal velocity gradients are very large. Gradients are responsible for skin friction drag on things like the wings and fuselage of aircraft. Boundary layer can be either laminar or turbulent: for a wing they are usually turbulent. Laminar boundary layers have a simple structure: turbulent boundary layers have a complex layered structure. Boundary layers grow in the downstream direction. For the wing of a 747, they are about 5cm at its trailing edge. Flow over the surface of the Earth also has a boundary layer structure. A boundary layer does not really have a sharp edge, and there are several ways to define its thickness. One common way puts its edge where the flow speed is 99% of the outer flow speed. Another thickness is known as the displacement thickness. This is the amount that the outer flow is displaced by the boundary layer. To the outer flow, the body appears to be thicker than it actually is. Another thickness is known as the momentum thickness. This is related to the wall drag on the body. Boundary layers can separate from surfaces and radically alter the surrounding flow pattern. This is what happens when a wing stalls. There is a large wake downstream of the separation point, and this greatly reduces lift and increases drag. For separation to occur, pressure must increase in the downstream direction. Such an adverse pressure gradient occurs downstream of the maximum suction point on the upper surface of a wing. The severity of this gradient increases with wing angle of attack. Separation occurs when the fluid in the boundary layer does not have the momentum to move into this gradient, and it is brought to rest. The lack of momentum is caused by wall drag. Turbulence delays separation: it makes it occur much farther downstream. Conventional wings operate with turbulent boundary layers. They would not be able to fly if they had laminar boundary layers. The fluid in a turbulent boundary layer is more energetic than that in a laminar boundary layer. This is because extra momentum from the high speed outer flow is diffused into it by eddy motion. Separation is delayed because, with extra momentum, fluid can move farther into the pressure gradient. The dimpled surface of a golf ball stimulates laminar to turbulent transition. This delays separation and gives a smaller wake and thus less drag.

SIDE WALL DRAG ON LONG SLENDER BODIES

Boundary layers on long bodies like ships and submarines are generally turbulent. Semi empirical formulas have been developed to calculate the side wall drag on such bodies. Development of these formulas makes use of velocity profile and wall shear data and the boundary layer thicknesses. The displacement thickness δ^* can be obtained by calculating the flow rate that would be next to a wall if there was no viscosity and subtracting from it the flow rate that is actually there:

$$\int \mathbf{U} dy \quad - \quad \int U dy$$

An equivalent flow rate is: $\delta^* \, \mathbf{U}$. Manipulation gives

$$\delta^* = \int (1 - U/\mathbf{U}) dy$$

The momentum thickness Θ can be obtained by calculating the flow momentum that would be next to a wall if there was no viscosity and subtracting from it the flow momentum that is actually there:

$$\rho\mathbf{U}\,\mathbf{U}h \quad - \quad \int \rho U \, U dy \quad = \quad \rho\mathbf{U} \int U dy \quad - \quad \int \rho U \, U dy$$

An equivalent momentum is: $\rho \, \mathbf{U}^2 \, \Theta$. Manipulation gives

$$\Theta = \int U/\mathbf{U} (1 - U/\mathbf{U}) dy$$

Momentum is missing next to the wall because of viscous stresses associated with the presence of the wall. The viscous stresses cause drag on the wall. A force balance on fluid immediately next to the wall gives

$$\mathbf{D}/b = \rho\, \mathbf{U}^2\Theta$$

where **D** is the drag on the wall and b is the width of the wall. For a submarine, b is the hull perimeter πD. The rate of change of drag in the streamwise direction is equal to the shear stress on the wall:

$$\tau = d[\mathbf{D}/b]/dx = \rho\mathbf{U}^2\, d\Theta/dx$$

For a smooth wall turbulent boundary layer, experiments suggest that velocity profiles have the power law form

$$U/\mathbf{U} = (y/\delta)^{1/n}$$

where δ is the 99% boundary layer thickness. Substitution into the thickness equations gives $\delta^*=I\delta$ and $\Theta=J\delta$ where I and J are constants which depend on the value of n. The value of n depends on the flow Reynolds Number. Experiments also suggest that the shear stress has the form

$$\tau = C\, \rho\mathbf{U}^2/(\mathbf{U}\delta/\nu)^{1/k}$$

where C and k are constants which depend on flow Reynolds Number. Substitution into the shear stress equation gives an equation for $d\delta/dx$. Integration gives

$$\delta = A\ x\ R_{EX}^{-1/a}$$

$$\mathbf{D} = M\ bx\ R_{EX}^{-1/m}\ \rho\mathbf{U}^2$$

where A a and M m are constants which depend on flow Reynolds Number. For a body with length L, the drag is:

$$\mathbf{D} = M\ bL\ R_{EL}^{-1/m}\ \rho\mathbf{U}^2$$

For a laminar boundary layer case, one gets

$$\mathbf{D} = N\ bL\ R_{EL}^{-1/2}\ \rho\mathbf{U}^2$$

Such a boundary layer can occur only at low flow Reynolds Number. For a rough wall boundary layer, one gets

$$\mathbf{D} = K\ bL\ \rho\mathbf{U}^2$$

where K is a function of flow Reynolds Number and the level of roughness. It can be obtained experimentally.

Wake drag at the rear of a body would have the form

$$\mathbf{W} = C\ B\ \rho\mathbf{U}^2/2$$

where B is body profile area and C is a constant which depends on flow Reynolds Number and body shape.

The power required to overcome these drags is

$$\mathbf{P} = [\ \mathbf{D} + \mathbf{W}\ ]\ \mathbf{U}$$

For a curved wall, the shear stress is

$$\tau = d[\rho\mathbf{U}^2\Theta]/dc - dP/dc\ \delta^*$$

$$= d[\rho\mathbf{U}^2\Theta]/dc + \rho\mathbf{U}d\mathbf{U}/dc\ \delta^*$$

Experiments connect Θ and δ^* and τ to δ. Potential flow theory gives **U**. One gets an equation of the form

$$d\delta/dc = H$$

This can be integrated numerically as follows

$$\delta_{NEW} = \delta_{OLD} + \Delta c\ H_{OLD}$$

This allows us to calculate the drag

$$\mathbf{D} = \int \tau b d\mathbf{c} = \Sigma\, \tau b \Delta\mathbf{c}$$

This is only good upstream of separation.

As noted above, for long slender bodies with turbulent boundary layers, the important equations are:

$$U/\mathbf{U} = (y/\delta)^{1/n} \qquad \delta^* = I\delta \qquad \Theta = J\delta$$

$$\tau = C\ \rho\mathbf{U}^2/(\mathbf{U}\delta/\nu)^{1/k} \qquad \delta = A\ L\ R_{EL}^{-1/a}$$

$$\mathbf{D} = M\ bL\ R_{EL}^{-1/m}\ \rho\mathbf{U}^2$$

For R_{EL} less than 10^7, the constants are

n=7 I=1/8 J=7/12 C=0.0253 k=4

A=0.382 a=5 M=0.037 m=5

For R_{EL} greater than 10^7 but less than 10^8, they are:

n=8 I=1/9 J=8/90 C=0.0142 k=5

A=0.253 a=6 M=0.0225 m=6

For R_{EL} greater than 10^8, they are:

n=9 I=1/10 J=9/110 C=0.0100 k=6

A=0.186 a=7 M=0.0152 m=7

DRAG ON A SUBMARINE

Consider a submarine with an outer diameter D equal to 10m and a length L equal to 100m. It is travelling through water at 10 knots. Calculate the drag on the submarine and the power required to push it through the water.

The drag is made up of two components: wall drag and wake drag. Wall drag is due to viscous shear stresses in the wall boundary layer. Wake drag is due to boundary layer separation at the rear. The drags are given by:

$$\text{Wall Drag :} \quad M \, \pi D \, L \, R_{EL}^{\,-1/m} \, \rho U^2$$

$$\text{Wake Drag :} \quad C_D \, \pi D^2/4 \, \rho U^2/2$$

The Reynolds Number R_{EL} is UL/υ. The constants M and m depend on R_{EL}. One knot is approximately 0.5m/s. So speed U is approximately 5m/s. The viscosity of water is approximately 10^{-6}. So R_{EL} is approximately 5 times 10^{+8}. For R_{EL} greater than 10^{+8} M is 0.0152 and m is 7. The suction created by the propellor would tend to make the wake drag coefficient C_D low: let it be 0.05. With this the drags are:

$$\text{Wall Drag: } 0.0152 * \pi*10 * 100 * 0.061 * 1000 * 25 = 72864 \text{ N}$$

$$\text{Wake Drag: } 0.05 * \pi*10*10/4 * 1000 * 25 / 2 = 49088 \text{ N}$$

The power needed to overcome the drag is drag times speed:

$$(72864 + 49088) * 5 = 0.61 \text{ MW}$$

BOUNDARY LAYER EQUATIONS

For boundary layer flow along a flat plate, the conservation laws for an incompressible fluid reduce to:

$$\text{MASS} \qquad \partial U/\partial x + \partial V/\partial y = 0$$

$$\text{MOMENTUM} \qquad \rho \left(U\, \partial U/\partial x + V\, \partial U/\partial y \right) = \mu\, \partial^2 U/\partial y^2$$

$$\text{ENERGY} \qquad \rho\, C_P \left(U\, \partial T/\partial x + V\, \partial T/\partial y \right) = \mu\Phi + k\, \partial^2 T/\partial y^2$$

The mass and momentum equations are known as the Prandtl boundary layer equations. The energy equation is uncoupled from these equations. So, one can solve them separately. For laminar boundary layers, Blasius developed an analytic solution. Most boundary layers are turbulent. Also, most boundary layers move over curved surfaces. So, laminar solutions for flat plates are of limited value. Boundary layers could make CFD expensive because many computational cells would have to be put in the boundary layer to handle the near wall behavior. Codes get around this by using special wall functions based on boundary layer theory to approximate the near wall behavior. Development of these wall functions is beyond the scope of this note.