MEMORIAL UNIVERSITY OF NEWFOUNDLAND

FACULTY OF ENGINEERING AND APPLIED SCIENCE

FLUIDS II 5913

FINAL EXAMINATION

WEDNESDAY 14 APRIL 2004
9:00 AM TO 11:30 AM

INSTRUCTOR
M. HINCHEY

Identify on the formula sheets the block of formulas for each of the following: (1) unsteady flow in long pipes (2) choked flow in long pipes (3) turbulent wake flows (4) boundary layer flows (5) water wave phenomena. Write brief notes on each topic.

A certain journal bearing when unfolded is 2 by 2. Its minimum gap is 2 and its maximum gap is 6. The shaft tangential speed due to rotation is 280. The viscosity of the bearing oil is 1/12. Develop a single point NSEW CFD template for the bearing and use it to estimate P at the center of the bearing. Based on the single point template where would the minimum gap be located on the bearing? Describe one way to measure the viscosity of the bearing oil.

In the foil lab you tested a foil generated from a circle with known radius and known offsets. Imagine that the wind speed and foil angle of attack are both set. What is the theoretical lift and drag that can be generated by the foil? How could lift and drag be determined from pressure measurements and the geometry of the foil? Sketch a velocity vector plot for flow around the foil when it has a large angle of attack. Draw streamlines on the plot.

In the shock tube lab a supersonic flow was generated for a very short period of time. With words and formulas describe how you would get the speed of this flow from a single pressure sensor measurement. How would you get the pressure at the stagnation point on a very small blunt object placed in the supersonic flow? Would the flow change if the end of the tube was blocked? Explain.

With words and formulas describe how you would get the lift and drag on a supersonic half diamond foil moving at a set MACH Number. How would you get the lift and drag if expansion waves had to be modelled as inverse oblique shock waves? Sketch the flow pattern around the foil when it is flying at a very large angle of attack.

With words and formulas describe how you would calculate the thrust of an ideal rocket nozzle. How would you estimate the upstream chamber MACH Number? How would you find the nozzle exit area? How would you get the thrust if the downstream cone was removed? How would a normal shock wave within the nozzle influence thrust?

JOURNAL BEARING QUESTION

Unfold converging half of bearing flat letting the minimum gap be at the bottom. Calculate pressure using the single point CFD template

$$[\ [(h_E+h_P)/2]^3\ (P_E-P_P)/\Delta x\ -\ [(h_P+h_W)/2]^3\ (P_P-P_W)/\Delta x\ ]\ /\ \Delta x$$
$$+$$
$$[\ [(h_N+h_P)/2]^3\ (P_N-P_P)/\Delta y\ -\ [(h_P+h_S)/2]^3\ (P_P-P_S)/\Delta y\ ]\ /\ \Delta y$$
$$=\ 3\mu S\ (h_E-h_W)/\Delta x$$

Knowns : Gaps h_N h_S h_E h_W h_P. Speed S. Viscosity μ and Pressures $P_N=P_S=P_E=P_W=0$. Unknown : P_P. Substitution into the template gives $P_P=1$. Fold pressure back onto bearing. This shows that net load is horizontal. But load must be vertical. So load must be rotated 90 degrees.

FOIL LAB QUESTION

The theoretical lift is $\rho S\Gamma$. The theoretical drag is 0. Lift and drag from pressure measurements are:

$$\text{Lift} = \Sigma\ P\Delta c\ \text{Sin}(\theta-\Theta) \qquad \text{Drag} = \Sigma\ P\Delta c\ \text{Cos}(\theta-\Theta)$$

where θ is the local inward pointing normal at points on the foil and Θ is the angle of attack of the foil.

SHOCK TUBE LAB QUESTION

Supersonic flow behind the shock which goes down the tube creates another shock in front of small stationary blunt object in flow. Let U indicate upstream of the main shock and D indicate downstream. Also let A indicate just upstream of the object shock and B indicate just downstream while C is the stagnation point on the object. Use main shock fixed UD frame to get the speed of the flow. Use object fixed ABC frame to get stagnation point pressure. Pressure sensor measurement gives M_U. Speed of sound equation gives C_U. Mach Number connection gives M_D. Temperature ratio equation gives T_D. Speed of sound equation gives C_D. Mach Number equation gives U_D. Speed of supersonic flow is $U_A=U_U-U_D$. Absolute Mach Number is $M_A=C_AU_A$ and $C_A=C_D$. Normal shock pressure ratio equation gives P_B. Mach Number connection gives M_B. The stagnation Point Mach Number is $M_C=0$. The pressure ratio equation gives pressure at the stagnation point.

SUPERSONIC FOIL QUESTION

Let 1 indicate upstream of the foil and 2 indicate below the foil. Let 3 indicate top of the foil at front and 4 indicate top of the foil to the rear. Lift and Drag are:

$$\text{Lift} = P_2C_2\text{Cos}\theta_2 - P_3C_3\text{Cos}\theta_3 - P_4C_4\text{Cos}\theta_4$$
$$\text{Drag} = P_2C_2\text{Sin}\theta_2 - P_3C_3\text{Sin}\theta_3 - P_4C_4\text{Sin}\theta_4$$

For region 2 oblique shock plot gives β. Oblique shock pressure ratio equation gives P_2. For region 3 expansion wave plot gives M_3. Isentropic pressure ratio equation gives P_3. For region 4 expansion wave plot gives M_4. Isentropic pressure ratio equation then gives P_4. Geometry gives angles θ_2 θ_3 θ_4 and chords C_2 C_3 C_4. At a large angle of attach a bow shock would form ahead of the foil: flow behind it would be subsonic. To model expansion wave as an inverse oblique shock we would use $\kappa=\beta+\theta$ instead of $\kappa=\beta-\theta$ in the $\tan\beta/\tan\kappa$ equation.

ROCKET NOZZLE QUESTION

Let U indicate the upstream combustion chamber and D indicate the nozzle exit. Let T indicate the nozzle throat. Knowns: P_U T_U P_D A_T M_U=0 M_T=1. Thrust for an ideal rocket nozzle is $\mathring{M}U_D$. Mass flow rate is

$$\mathring{M} = [P_T/RT_T]\ A_T\ \sqrt{kRT_T}$$

Isentropic equations connect point U to point T and gives P_T and T_T. Isentropic pressure ratio equation gives M_D. Isentropic temperature ratio equation gives T_D. Speed of sound equation gives C_D. Mach Number equation gives U_D. To get the upstream Mach Number one can use the fact that the mass flow is constant throughout the nozzle:

$$\mathring{M} = [P_U/RT_U]\ A_UM_U\ \sqrt{kRT_U}$$

Everything is known in this equation except M_U. To get the nozzle exit area we can use

$$\mathring{M} = [P_D/RT_D]\ A_DM_D\ \sqrt{kRT_D}$$

Everything is known in this equation except A_D. The thrust without a cone downstream of the throat is

$$\mathring{M}U_T + [P_T-P_D]A_T$$

A normal shock within the nozzle would cause the flow to go subsonic and this reduces U_D and thus thrust $\mathring{M}U_D$.

FLUIDS II QUIZ

Do you agree or disagree with each of the following statements? Briefly explain your answer.

A turbulent wake flow can be solved analytically. Disagree! A turbulent wake flow is too complicated to solve analytically: CFD is the only option. In irrotational flow each fluid particle spins on its own internal axis. Disagree! Viscosity makes particles spin. In irrotational flow viscosity is zero. Boundary layer theory can be used to estimate wake drag. Disagree! Boundary layer theory is used to estimate wall drag not wake drag. In hydrodynamic lubrication devices viscous forces are insignificant relative to inertia forces. Disagree! Because velocity gradients are very large across small gaps viscous forces are very strong. Processes within shock waves are isentropic. Disagree! Isentropic means adiabatic and frictionless. Mechanical energy is dissipated by friction inside shock waves. Also there is significant heat conduction within them.

Answer any 4 of the following 5 questions. In each case list the things that you would assume to be known.

With words and formulas describe how you would calculate the lift and drag on a V shaped flat plate supersonic foil. You would assume you know M_U P_U T_U upstream of the foil. You would also assume you know the geometry of the foil and its angle of attack. There would be two oblique shock waves on the bottom of the foil and two expansion waves on the top of the foil. To get across an oblique shock wave you use upstream M and plate angle to get β from oblique shock plot A. You would then calculate the normal mach numbers N either side of the shock. N upstream gives the pressure downstream of the shock while N downstream gives M downstream. To get across an expansion wave you use upstream M and plate angle to get downstream M from expansion wave plot B. The isentropic pressure ratio equation then gives P downstream. Knowing the pressure on each flat section of the plate we multiply by chord to get load on it. We then break each load down into vertical and horizontal components to get lift and drag. Pressures on the top of the foil give negative lift and negative drag. Pressures on the bottom of the foil give positive lift and positive drag.

With words and formulas describe how you would calculate the thrust of an ideal rocket nozzle. You would assume the following are known: P_U T_U P_D $M_U=0$ $M_T=1$ A_T. You would also assume processes isentropic throughout nozzle. For an ideal nozzle the thrust is $\dot{M}U_D$. The surrounding pressure is P_D: so ΔP is zero. The mass flow rate $\dot{M}$ is $\rho_T A_T U_T$. The density ρ_T is P_T/RT_T. The speed U_T is $\sqrt{kRT_T}$. The isentropic ratios with known P_U T_U M_U M_T give P_T and T_T. The isentropic pressure ratio equation with known P_U M_U P_D gives M_D. The temperature ratio equation with known T_U M_U M_D gives T_D. The speed C_D is $\sqrt{kRT_D}$. The speed U_D is $C_D M_D$. So thrust $\dot{M}U_D$ can now be calculated.

With words and formulas describe how you would calculate the mass flow rate through a valve in a high pressure gas line. You would assume that the ratio of high pressure P_H to low pressure P_L across the valve is large enough to make flow through it choked. You would assume that the valve has a minimum area or throat and the following are known: P_H T_H $M_H=0$ $M_T=1$ A_T. The mass flow rate is $\rho_T A_T U_T$. The density ρ_T is P_T/RT_T. The speed U_T is $\sqrt{kRT_T}$. The isentropic ratios with known P_H T_H M_H M_T give P_T and T_T. So the mass flow rate can now be calculated.

With words and formulas describe how you would get the pressure and temperature at the stagnation point on a blunt object moving faster than the speed of sound. You would assume that P_U T_U M_U $M_S=0$ are known. The normal shock pressure ratio equation would allow you to get P_B just behind the shock. The normal shock mach number equation would give M_B just behind the shock. The isentropic pressure ratio equation with M_B and M_S known would then give the stagnation pressure P_S while the temperature ratio equation with T_U M_U and M_S known would give the stagnation temperature T_S.

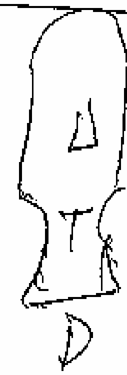
With words and formulas describe how you would calculate the speed of the shock wave generated by an explosion in air. You would assume the explosion takes place in initially calm air. In a shock reference frame the air would approach the shock at the speed of the shock. You would assume you know P_A and T_A for the air and P_G for the gas ball. The normal shock equation with P_G and P_A known would give M_A. The sound speed in air C_A would be $\sqrt{kRT_A}$. The shock speed U_A would be C_A times M_A.

D1

ROCKET NOZZLE PROBLEM

KNOWNS: P_U T_U P_D

$M_U = 0$ $M_T = 1$ A_T



IMPORTANT EQUATIONS

ISENTROPIC RATIOS

∘ J ∘ I

$$\frac{P_I}{P_J} = \left(\frac{1 + \frac{k-1}{2} M_J^2}{1 + \frac{k-1}{2} M_I^2} \right)^{\frac{k}{k-1}}$$

$$\frac{T_I}{T_J} = \frac{1 + \frac{k-1}{2} M_J^2}{1 + \frac{k-1}{2} M_I^2}$$

Ideal Rocket Thrust $\dot{M}\omega_D$. (D2)

Pressure ratio equation applied between ω and T gives P_T.

Temperature ratio equation applied between ω and T gives T_T.

Mass flow rate ω

$$\dot{M} = \rho_T A_T \omega_T = \frac{P_T}{R T_T} A_T \sqrt{k R T_T}$$

Pressure ratio equation applied between U and D gives M_D.

(D3)

Temperature ratio equation applied between U and D gives T_D.

Sound speed equation gives C_D.

Mach Number equation gives M_D.

For nozzle without cone thrust is

$$\dot{M} V_T + (P_T - P_D) A_T$$

Following equation gives M_U

$$\dot{M} = \frac{P_U}{R T_U} A_U M_U \sqrt{k R T_U}$$

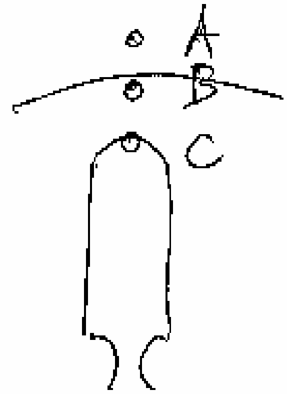
Following equation gives A_D

$$\dot{M} = \frac{P_D}{R T_D} A_D M_D \sqrt{k R T_D}$$

(E1)

ROCKET STAGNATION POINT PRESSURE

KNOWNS



P_A T_A M_A $M_C = 0$

Stagnation Point => C

For rocket moving at low subsonic speed S stagnation point pressure is $P_C = \frac{\rho S^2}{2}$, For rocket moving at supersonic speed normal shock equations are first used to get P_B and M_B just downstream of bow shock. (12)

$$\frac{P_B}{P_A} = 1 + \frac{2k}{k+1}\left(M_A^2 - 1\right)$$

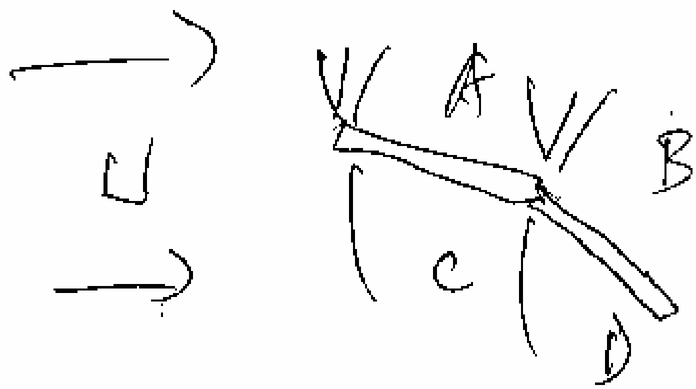
$$M_B^2 = \frac{(k-1) M_A^2 + 2}{\left[2k M_A^2 - (k-1)\right]}$$

Then isentropic equation is used to get P_c!

$$\frac{P_c}{P_B} = \left(\frac{1 + \frac{k-1}{2} M_B^2}{1 + \frac{k-1}{2} M_c^2}\right)^{\frac{k}{k-1}}$$

P1

SUPERSONIC V SHAPED FOIL



KNOWNS: M_U P_U T_U

ALL GEOMETRY

Oblique shock equations are used to get P_C & P_D

$$\frac{P_C}{P_U} = 1 + \frac{2k}{k+1}\left(N_U^2 - 1\right)$$

$$\frac{P_D}{P_C} = 1 + \frac{2k}{k+1}\left(N_C^2 - 1\right)$$

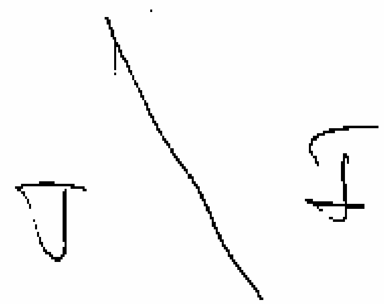
(f2)

Isentropic equations are used to get P_A & P_B

$$\frac{P_A}{P_U} = \left(\frac{1 + \frac{k-1}{2} M_U^2}{1 + \frac{k-1}{2} M_A^2} \right)^{\frac{k}{k-1}}$$

$$\frac{P_B}{P_A} = \left(\frac{1 + \frac{k-1}{2} M_A^2}{1 + \frac{k-1}{2} M_B^2} \right)^{\frac{k}{k-1}}$$

For a typical shock



P3

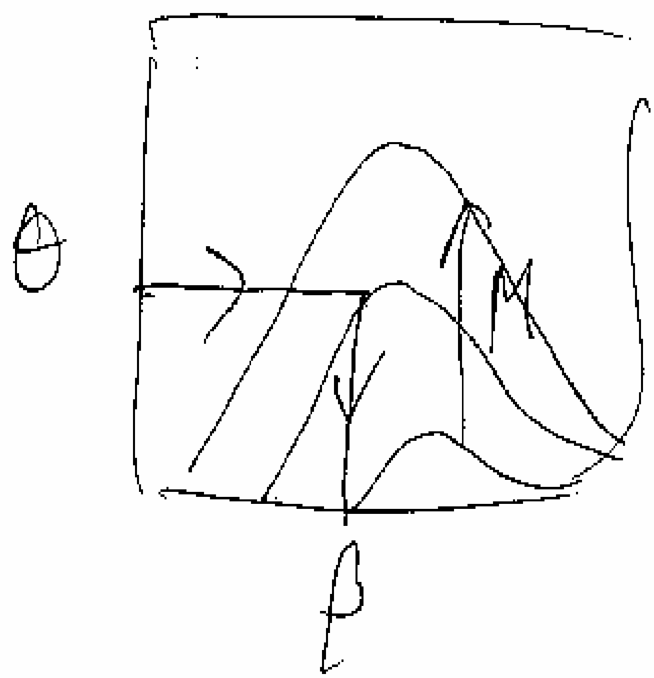
$$N_J = M_J \sin\beta$$

$$N_I^2 = \frac{(k-1)N_J^2 + 2}{\left[2k N_J^2 - (k-1)\right]}$$

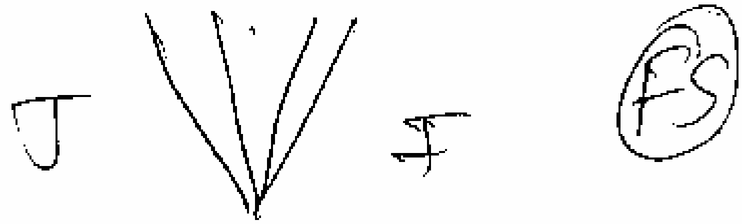
$$N_I = M_I \sin(\beta - \theta)$$

P4

Get β from oblique shock plot

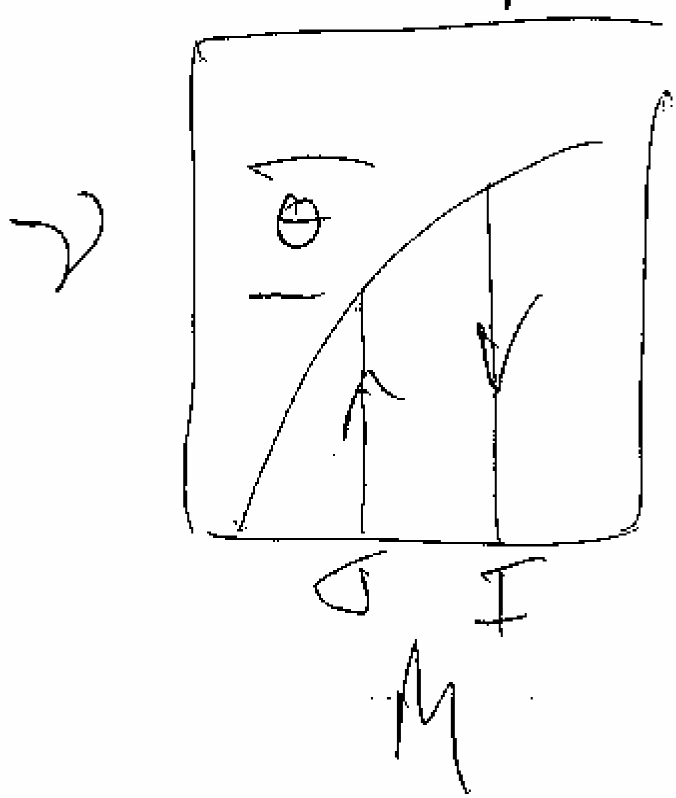


For a typical expansion

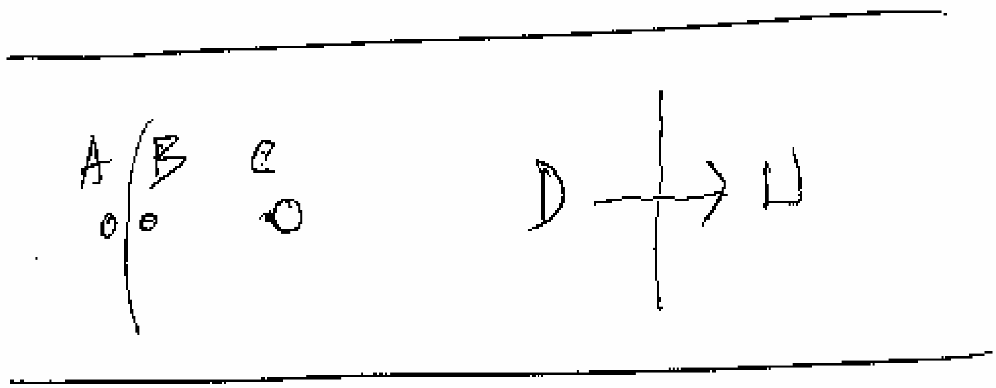


$$\nu_I = \nu_J + \theta$$

Get M from expansion plot



BLUNT OBJECT IN SHOCK TUBE



KNOWNS: P_U T_U $\frac{P_D}{P_U}$ $M_C = 0$

STAGNATION POINT PRESSURE ?

M_U C_U U_U P_D M_D T_D

C_D U_D U_A $C_A = C_D$ M_A

$P_A = P_D$ P_B M_B P_C

JOUKOWSKY FOIL

(9)

Theoretical lift and drag are

$L = \rho \Gamma S \quad D = 0$

Lift and Drag from summed incremental lift and drag are

$L = \Sigma \delta L \quad D = \Sigma \delta D$

(32)

ΔL_s and ΔD_s can be obtained from experiments or foil theory

$$\Delta L = P \Delta c \sin(\theta - \theta)$$

$$\Delta D = P \Delta c \cos(\theta - \theta)$$

Get P theoretically from Bernoulli equation

$$P = \frac{\rho}{2}\left[S^2 - \left(\frac{\partial \phi}{\partial c}\right)^2\right] \approx \frac{\rho}{2}\left[S^2 - \left(\frac{\Delta \phi}{\Delta c}\right)^2\right]$$

Get Δc from mapping from circle to foil plane

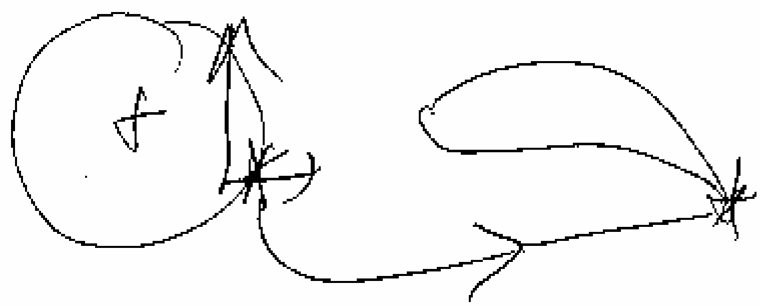
$$\Delta c = \sqrt{(\Delta\alpha)^2 + (\Delta\beta)^2}$$

Get $\Delta\phi$ from circle plane

$$\phi = 2SX + \frac{1}{2\pi}\sigma$$

Corresponding points in circle and foil planes have same ϕ.

Proper circulation Γ is that which make trailing edge stagnation point



Manipulation gives

$$\Gamma = 4\pi S R \sin K$$

$$K = \theta + \varepsilon$$

$$\varepsilon = \tan^{-1}\left(\frac{m}{n+a}\right)$$

TORPEDO PROBLEM (A1)

The turbulent smooth wall drag is $D = N\, b\, L\, Re_L^{-1/k} \rho U^2$.

Use Re_L to get N and k.

The turbulent rough wall drag is $D = K\, b\, L\, \rho U^2$.

Use roughness plot to get K.

The laminar side wall drag is $D = M\, b\, L\, Re_L^{-1/2} \rho U^2$.

Use data to get M.

(A2)

Wake drag is $W = C_D B \rho \bar{U}^2/2$,

Use data to get C_D,

Power is $P = (D + W)\bar{U}$,

For curved geometry external pressure gradient acts on fluid in the boundary layer

$$\frac{d}{dx}(\rho \bar{U}^2 \theta) = \tau + \frac{dp}{dx}\delta^*$$

$$= \tau - \rho \bar{U} \frac{d\bar{U}}{dx} \delta^*$$

(A3)

Data relates τ and θ and δ^{*} so δ. Get an equation of form

$$\frac{d\delta}{dc} = f$$

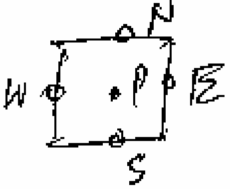
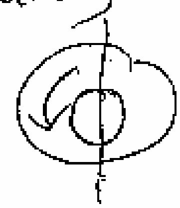
Numerical integration gives

$$\delta_{NEW} = \delta_{OLD} + \Delta c \, f_{OLD}$$

Another integration gives

$$D = \int_0^L \tau_b \, dc = \Sigma \tau_b \Delta c$$

JOURNAL BEARING PROBLEM

When unrolled bearing looks like  - .

Reynolds equation is

$$\frac{\partial}{\partial x}\left(h^3 \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3 \frac{\partial p}{\partial y}\right) = 6\mu S \frac{\partial h}{\partial x}$$

Application of CFD gives

$$\left[\left(\frac{h_E + h_P}{2}\right)^3 \left(\frac{h_E - h_P}{\Delta x}\right) - \left(\frac{h_P + h_W}{2}\right)^3 \left(\frac{P_P - P_W}{\Delta x}\right)\right] \Big/ \Delta x$$

$$\left[\left(\frac{h_N + h_P}{2}\right)^3 \left(\frac{P_N - P_P}{\Delta y}\right) - \left(\frac{h_P + h_S}{2}\right)^3 \left(\frac{P_P - P_S}{\Delta y}\right)\right] \Big/ \Delta y$$

$$= 6\mu S \left(\frac{h_E - h_W}{2\Delta x}\right)$$

(B2)

Manipulation gives

$$P_p = \frac{AP_E + BP_W + CP_N + DP_S + kt}{A+B+C+D}$$

Substitution into this gives P_p.

When folded back onto shaft load looks like . However load must be vertical so must rotate 90° .

If ends blocked $\frac{\partial P}{\partial y} = 0$ ∴ C and D both zero ∴

$$P_p = \frac{AP_E + BP_W + kt}{A+B}$$

This gives higher P_p.

THRUST BEARING PROBLEM (Q1)

Reynolds equation is

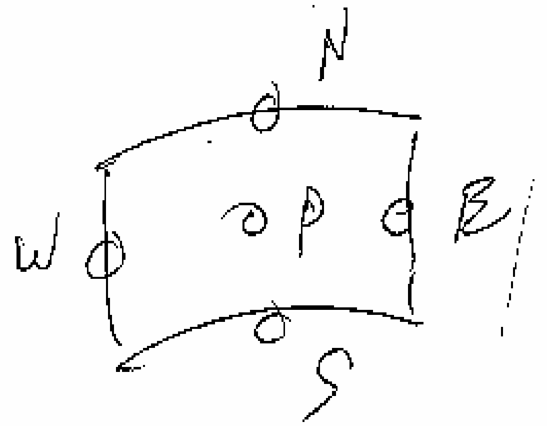
$$\frac{1}{r}\frac{\partial}{\partial r}\left(rh^3\frac{\partial p}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{h^3}{r}\frac{\partial p}{\partial \theta}\right)=6\mu\frac{s}{r}\frac{\partial h}{\partial \theta}$$

For CFD this can be rewritten in following forms:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rh^3\frac{\partial p}{\partial r}\right)+\frac{\partial}{\partial c}\left(h^3\frac{\partial p}{\partial c}\right)=6\mu s\frac{\partial h}{\partial c} \quad (I)$$

$$r\frac{\partial}{\partial r}\left(rh^3\frac{\partial p}{\partial r}\right)+\frac{\partial}{\partial \theta}\left(h^3\frac{\partial p}{\partial \theta}\right)=6\mu s r\frac{\partial h}{\partial \theta} \quad (II)$$

Application of CFD to (II) gives



C2

$$\left[\left(\frac{h_N+h_P}{2}\right)^3\left(\frac{r_N+r_P}{2}\right) r_P \left(\frac{P_N-P_P}{\Delta r}\right) - \left(\frac{h_P+h_S}{2}\right)^3\left(\frac{r_P+r_S}{2}\right) r_P \left(\frac{P_P-P_S}{\Delta r}\right)\right] \Big/ \Delta r$$

$$+ \left[\left(\frac{h_E+h_P}{2}\right)^3\left(\frac{P_E-P_P}{\Delta\theta}\right) - \left(\frac{h_P+h_W}{2}\right)^3\left(\frac{P_P-P_W}{\Delta\theta}\right)\right] \Big/ \Delta\theta$$

$$= 6 \mu S r_P \left(\frac{h_E-h_W}{2\Delta\theta}\right)$$

(C3)

Manipulation gives

$$P_p = \frac{A P_E + B P_w + C P_N + D P_S + H}{A + B + C + D}$$

Substitution into this gives P_p, if ends of bearing blocked C and D both zero & $\frac{\partial P}{\partial r} = 0$,

$$P_p = \frac{A P_E + B P_w + H}{A + B}$$

This gives higher P_P.

FLUIDS II 3913

QUIZ #2 2007

A) Consider a circle with radius R=1m and offsets n=0 and m=0. 1) Map 4 points on the circle to a Joukowsky foil plane. The foil is moving through STP air at speed S equal to 500m/s and angle of attach θ equal to 10 degrees. 2) Estimate pressure at 4 points on the foil. 3) Use this to estimate the lift on the foil. 4) This lift is not realistic. Why? Describe how you would get a more realistic estimate of lift.

B) Imagine that you know the pressure ratio across a shock wave generated by an explosion in still air. 1) With words and formulas describe how you would get the speed of the supersonic flow behind the shock wave. 2) With words and formulas describe how you would get the pressure at the stagnation point on a small blunt object placed in the flow. 3) With words and formulas describe choked flow. 4) Describe two situations where choked flow is important.

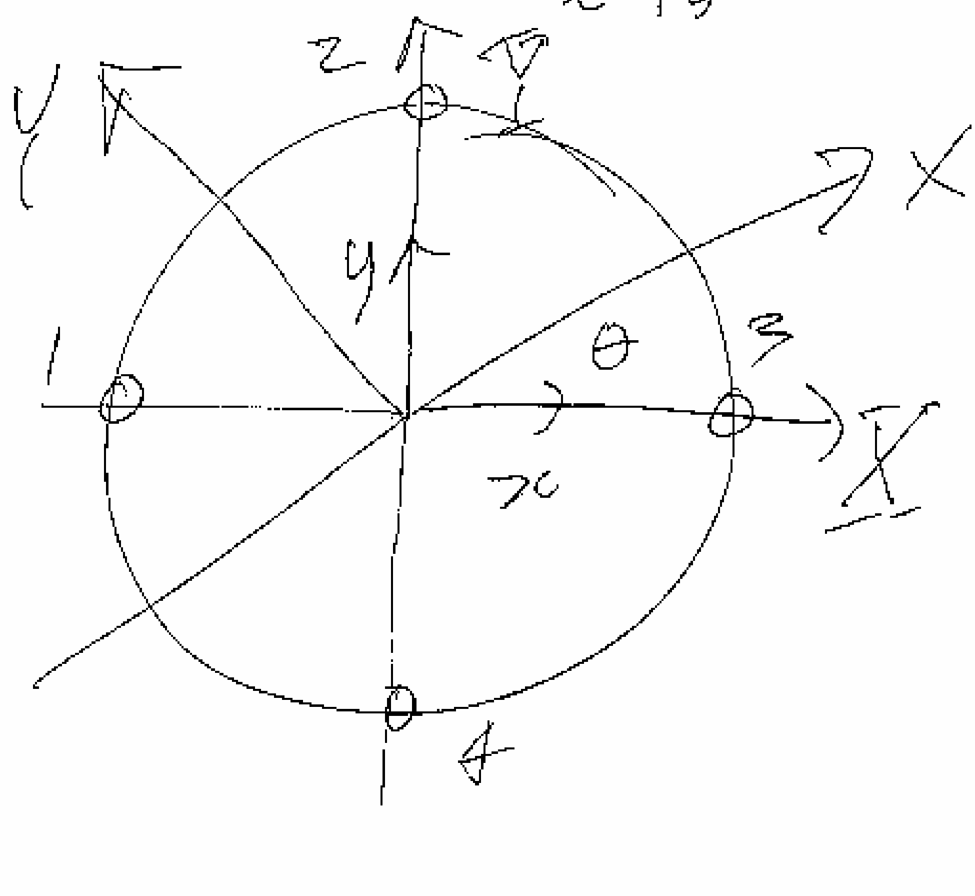
C) Answer TRUE or FALSE to each of the following and briefly explain each answer: 1) Conservation Laws are ODEs. (2) Shock waves cannot occur in water. (3) Lubrication flow is high Reynolds Number flow. (4) Eddies in a flow make it appear more dense locally. (5) A potential vortex is a rotational flow. (6) Expansion shock waves are impossible. (7) The wave equation governs creeping flows. (8) Boundary layers occur in ideal flows. (9) A single Mach wave cannot be heard. (10) Capillary flow is flow of an inviscid fluid.

(A1) Geometry of Foil

Mapping Function

$$\alpha = x + \frac{x a^2}{x^2 + y^2}$$

$$\beta = y - \frac{y a^2}{x^2 + y^2}$$



$a = R = 1$

$x^2 + y^2 = R^2 = 1$

① $\alpha = x + x = -2.0$

$\beta = y - y = 0.0$

② $\alpha = x + x = 0.0$

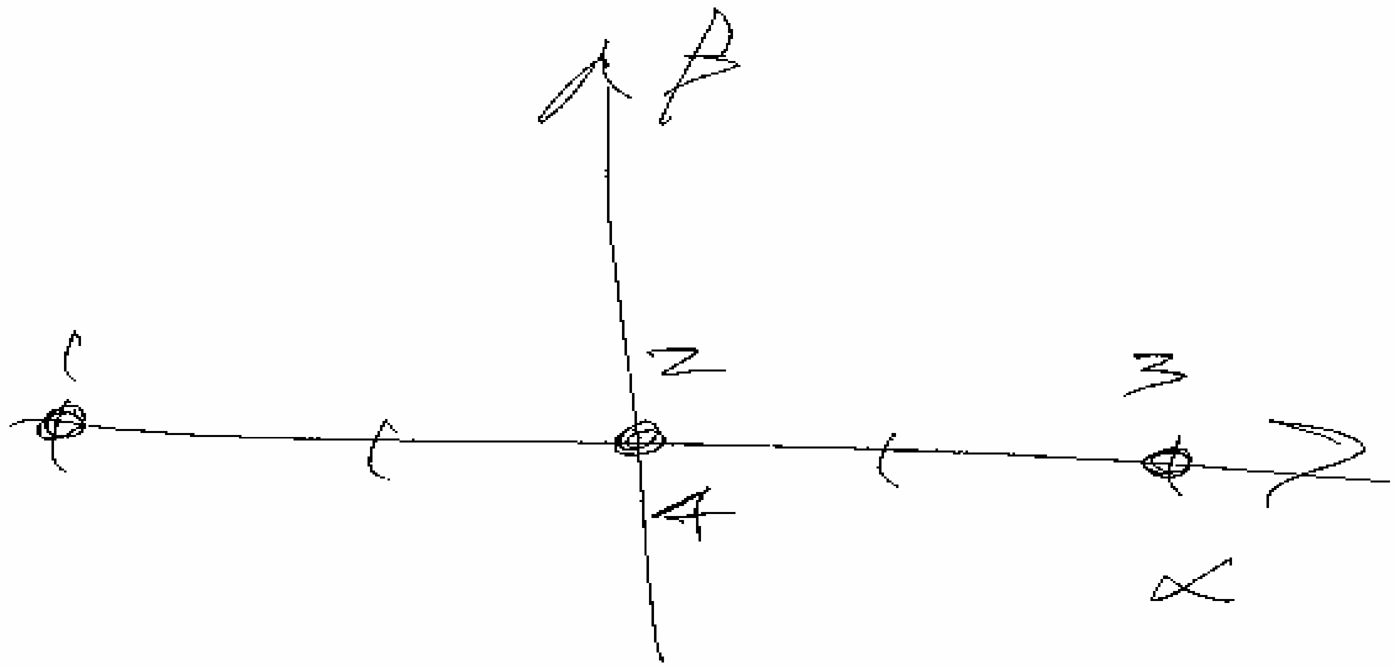
$\beta = y - y = 0.0$

③ $\alpha = x + x = +2.0$

$\beta = y - y = 0.0$

④ $\alpha = x + x = 0.0$

$\beta = y - y = 0.0$



TOP	1	2	3
BOTTOM	3	4	1

(A2) Pressure on foil

$$P = \frac{\rho}{2}\left[S^2 - \left(\frac{\delta\phi}{\delta c}\right)^2\right]$$

$$= \frac{\rho}{2}\left[S^2 - \left(\frac{\Delta\phi}{\Delta c}\right)^2\right]$$

$$\Delta\phi = 2S\,\Delta x + \frac{\Gamma}{2\pi}\Delta\Gamma$$

$$X = \bar{X}\cos\theta + \bar{Y}\sin\theta$$

$$\Delta c = \sqrt{(\Delta\alpha)^2 + (\Delta\beta)^2}$$

$$\Delta\Gamma = \frac{\pi}{2}$$

$X_1 = -0.98 \quad X_2 = +0.17$

$X_3 = +0.98 \quad X_4 = -0.17$

① → ② $\Delta X = X_2 - X_1 = +1.15$

② → ③ $\Delta X = X_3 - X_2 = +0.81$

③ → ④ $\Delta X = X_4 - X_3 = -1.15$

④ → ① $\Delta X = X_1 - X_4 = -0.81$

$\Gamma = 2\pi S R \sin \alpha$

$\alpha = \theta + \varepsilon = \theta$

$= 10$

$\Gamma = 4 * \pi * 500 * 1 * \sin 10$

$\fallingdotseq 1068$

① → ② $\Delta\phi = 2 \times 500 \times 1.15 + \frac{1068}{2\times\pi}\frac{\pi}{2}$

$= 1150 + 267 = +1417$

② → ③ $\Delta\phi = 2 \times 500 \times 0.81 + \frac{1068}{2\times\pi}\frac{\pi}{2}$

$= 810 + 267 = 1077$

③ → ④ $\Delta\phi = 2 \times 500 \times (-1.15) + \frac{1068}{2\pi}\frac{\pi}{2}$

$= -1150 + 267 = -883$

④ → ① $\Delta\phi = 2 \times 500 \times (-0.81) + \frac{1068}{2\times\pi}\frac{\pi}{2}$

$= -810 + 267 = -543$

$\frac{1.2}{2}\left[500^2 - \left(\frac{1417}{2}\right)^2\right]$

①→② $= \frac{1.2}{2}(250000 - 501972)$

$= -151183$

p $\frac{1.2}{2}\left[500^2 - \left(\frac{1077}{2}\right)^2\right]$

②→③ $= \frac{1.2}{2}(250000 - 289982)$

$= -23987$

p $\frac{1.2}{2}\left[500^2 - \left(\frac{883}{2}\right)^2\right]$

③→④ $= \frac{1.2}{2}(250000 - 194922)$

$= +33307$

p $\frac{1.2}{2}\left[500^2 - \left(\frac{543}{2}\right)^2\right]$

④→① $= \frac{1.2}{2}(250000 - 73712)$

$= +105773$

(A3) Lift on Roll

Lift $P \Delta c \sin(\theta - \phi)$

① → ② = −15483 * 2 * Sin (270 − 10)

= + 29777 2

Lift $P \Delta c \sin(\theta - \theta)$

② → ③ = − 23989 * 2 * Sin (270 − 10)

= + 47249

Lift $P \Delta c \sin(\theta - \theta)$

③ → ④ = + 33307 * 2 * Sin (90 − 10)

= + 65602

Lift $P \Delta c \sin(\theta - \theta)$

④ → ① = + 105723 * 2 * Sin (90 − 10)

= + 208234

Lift = Σ Lift = 619857

Lift ρ V S

= 1.2 * 1068 * 500

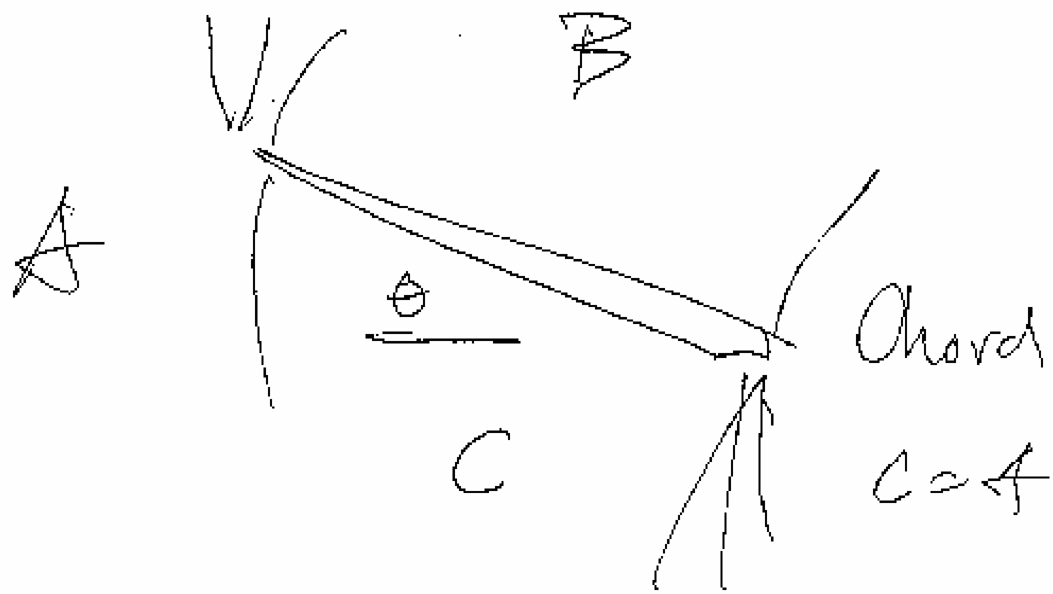
= 640800

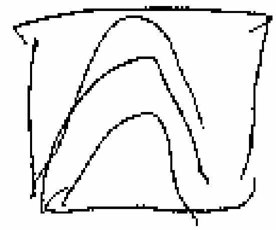
(A7) Realistic Lift

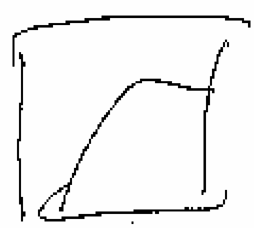
Speed of Boil is Supersonic

Subsonic Lift Invalid

$$M = \frac{500}{343} = 1.46$$



Oblique Shock Theory gives P_C. 

Expansion Fan Theory gives P_B. 

$$Lift = (P_C - P_B) \cos\theta$$

Shock Plot with known

M_A and θ gives β. This

gives $N_A = M_A \sin\beta$ and

$$P_C = P_A\left[1 + \frac{2k}{k+1}\left(N_A^2 - 1\right)\right]$$

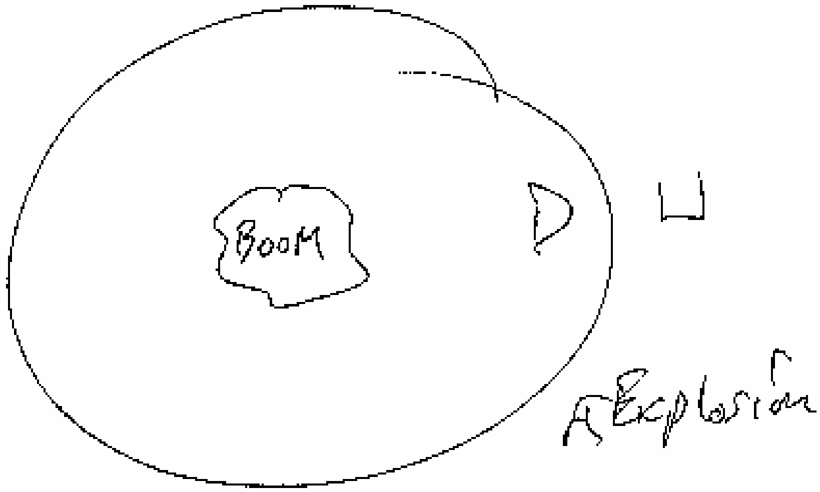
Expansion Plot with known

M_A and θ gives $\nu_B = \nu_A + \theta$.

This gives M_B and

$$P_B = P_A\left[\left(\frac{1 + \frac{k-1}{2} M_A^2}{1 + \frac{k-1}{2} M_B^2}\right)^{\frac{k}{k-1}}\right]$$

B1 Supersonic Flow



Pressure Ratio equation for Normal Shock Wave gives M_U in shock frame,

$$\frac{P_D}{P_U} = 1 + \frac{2k}{k+1}\left(M_U^2 - 1\right)$$

$\frac{P_D}{P_U}$ known

Speed of Sound equation with known T_U gives C_U.

$$C_U = \sqrt{kRT_U}$$

Mach Number equation gives U_U.

$$M_U = \frac{U_U}{C_U} \Rightarrow U_U = M_U C_U$$

Mach Number connection gives M_D in shock frame

$$M_D^2 = \frac{(k-1)M_U^2 + 2}{2k M_U^2 - (k-1)}$$

Temperature Ratio equation
gives T_D

$$T_D = T_U \frac{\left(1+\frac{(k-1)}{2} M_U^2\right)}{\left(1+\frac{(k-1)}{2} M_D^2\right)}$$

Speed of Sound equation
gives C_D

$$C_D = \sqrt{kRT_D}$$

Mach Number equation
gives U_D in shock frame.

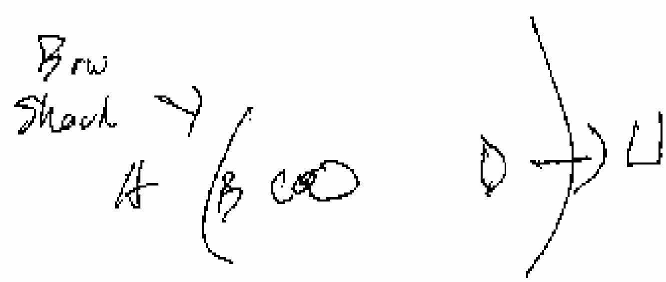
$$M_D = \frac{U_D}{C_D} \Rightarrow U_D = M_D C_D$$

Shock moves outward at U_U:
flow moves back from shock
at speed U_D ; So speed of
supersonic flow behind shock is

$$U_U - U_D$$

(B2) Stagnation Point Pressure

Blunt object in supersonic
flow behind explosion shock
generates bow shock



$$W_A = W_J - W_D$$

$$T_A = T_D \qquad C_A = C_D$$

$$M_C = 0$$

Mach Number of Supersonic flow ahead of object is:

$$M_A = \frac{W_A}{C_A}$$

Pressure Ratio equation for normal shock wave gives P_B:

$$\frac{P_B}{P_A} = 1 + \frac{2k}{k+1}\left(M_A^2 - 1\right)$$

Mach Number Connection for normal shock wave gives M_B

$$M_B^2 = \frac{(k-1) M_A^2 + 2}{2k M_A^2 - (k-1)}$$

Isentropic Pressure Ratio equation gives P_C :

$$\frac{P_C}{P_B} = \left(\frac{1 + \left(\frac{k-1}{2}\right) M_B^2}{1 + \left(\frac{k-1}{2}\right) M_C^2} \right)^{\frac{k}{k-1}}$$

(P3) Choked Flow

Conservation laws give

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dU}{U} = 0$$

$$U\,dU + c^2 \frac{d\rho}{\rho} = 0$$

Manipulation gives

$$dU = \frac{U\,dA}{A(M^2-1)}$$

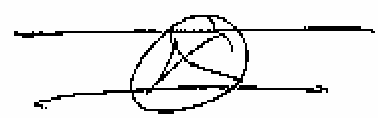
This equation shows that for a nozzle

$M < 1$ $\quad u \uparrow$ as $A \downarrow$

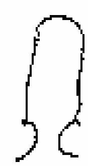
$M > 1$ $\quad u \uparrow$ as $A \uparrow$

$M = 1$ $\quad$ Throat

(B4) Choked Flow Applications

Valve Flow 

$$\dot{m} = \rho_T A_T u_T$$

Rocket Nozzle 

$$\dot{m} u_D \qquad \dot{m} = \rho_T A_T u_T$$

Isentropic Ratio gives

P_T and T_T at throat,

$$\rho_T = \frac{P_T}{RT_T}$$

Temperature gives C_T,

$$C_T = \sqrt{kRT_T} \quad U_T = C_T$$

Equations gives M_D

then T_D then C_D then U_D.

(C) TRUE OR FALSE

(1) FALSE: Conservation Laws are PDEs not ODEs.

(2) FALSE: Shock wave can occur in any fluid with a speed of sound.

(3) FALSE: Lubrication flow is low Re flow because gap is small; $Re = \frac{Uh}{\nu}$.

(4) FALSE: Eddies in a flow make it appear more viscous locally, like molecules eddies diffuse momentum.

(5) FALSE; A potential vortex like any potential flow is irrotational; fluid particles in vortex do not spin.

(6) TRUE; Second law shows that for expansion shock $\Delta S < 0$ which is impossible.

(7) FALSE; wave equation governs wave propagation in enclosures, viscosity not important for wave propagation but it is dominant for creeping flows.

(8) FALSE; Boundary layers are a viscous phenomenon, Ideal flows have zero viscosity.

(9) TRUE; A single MACH wave is generated by an infinitesimal disturbance, Sound generation is infinitesimal so it cannot be heard.

(10) FALSE; Capillary flow is flow of a viscous fluid through a small tube, An inviscid fluid has zero viscosity.