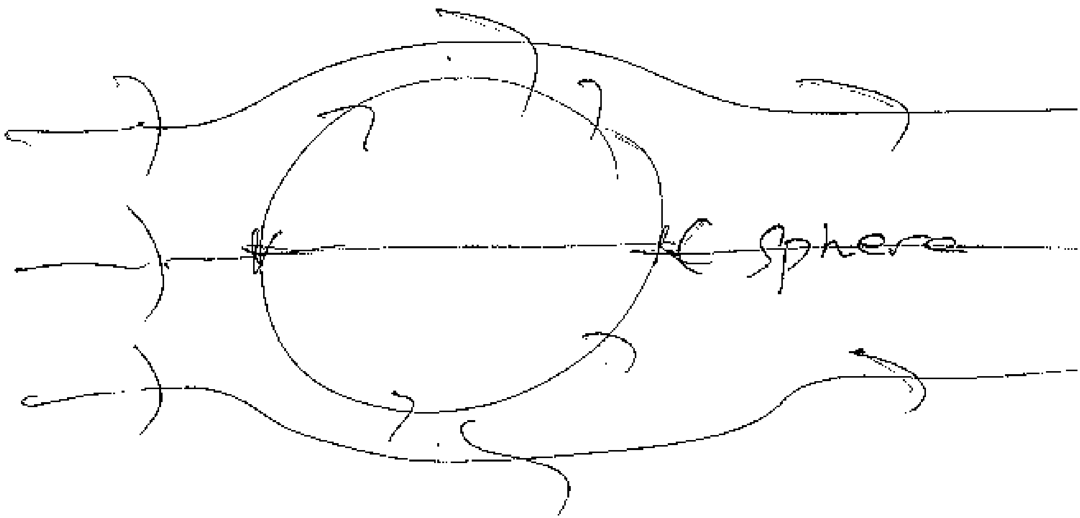
LOADS ON SPHERES

Superposition of a 3D stream with a 3D doublet gives

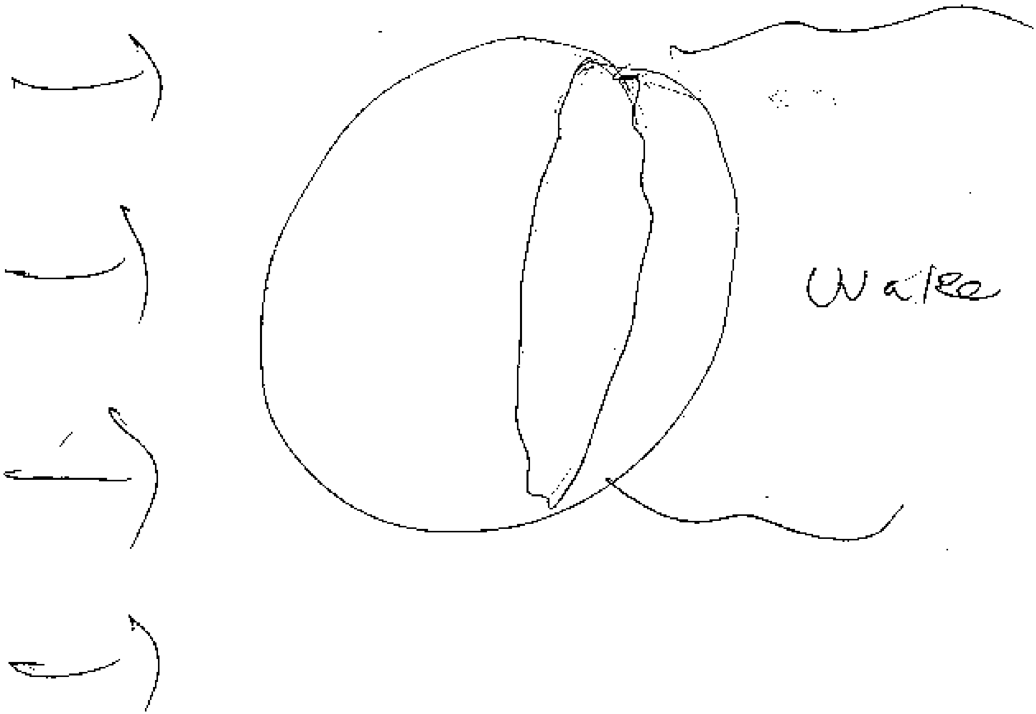
$$\Phi = -U r \cos\theta - \frac{U}{2} \frac{R^3}{r^2} \cos\theta$$

Stream Doublet

Superposition creates a spherical closed stream surface.



Real Flow



On sphere

$r = R \qquad R\theta = c$

Speed on sphere

$$\frac{\partial \phi}{\delta c} = \frac{3}{2} s \sin\theta$$

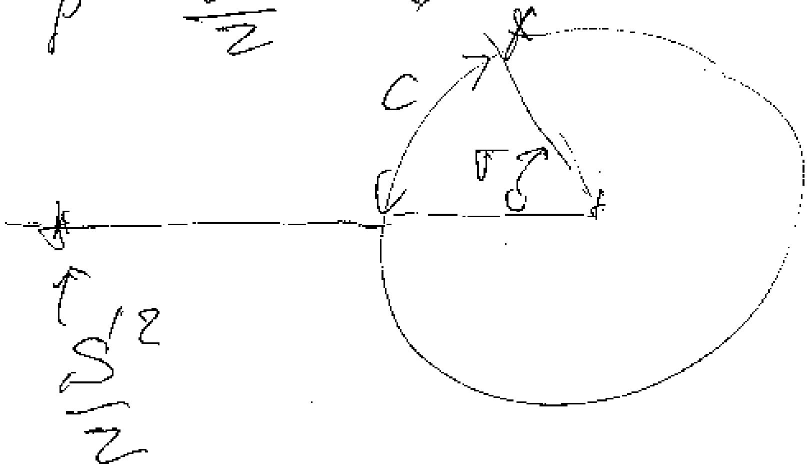
$$\phi = - S' R \cos \frac{c}{R} - \frac{S'}{2} \frac{R^3}{R^2} \cos \frac{c}{R}$$

$$= - \frac{3 S'}{2} R \cos \frac{c}{R}$$

$$\frac{\partial \phi}{\partial c} = \frac{3 S'}{2} \sin \tau$$

Bernoulli gives pressure

$$\frac{p}{\rho} + \frac{\left(\frac{\partial \phi}{\partial c}\right)^2}{2}$$



$$\frac{\delta^2}{2} = \frac{p}{\rho} + \frac{\left(\frac{\partial\phi}{\partial c}\right)^2}{2}$$

Manipulation gives

$$p = \frac{\rho}{2}\left[\delta^2 - \left(\frac{3}{2}\delta\sin\theta\right)^2\right]$$

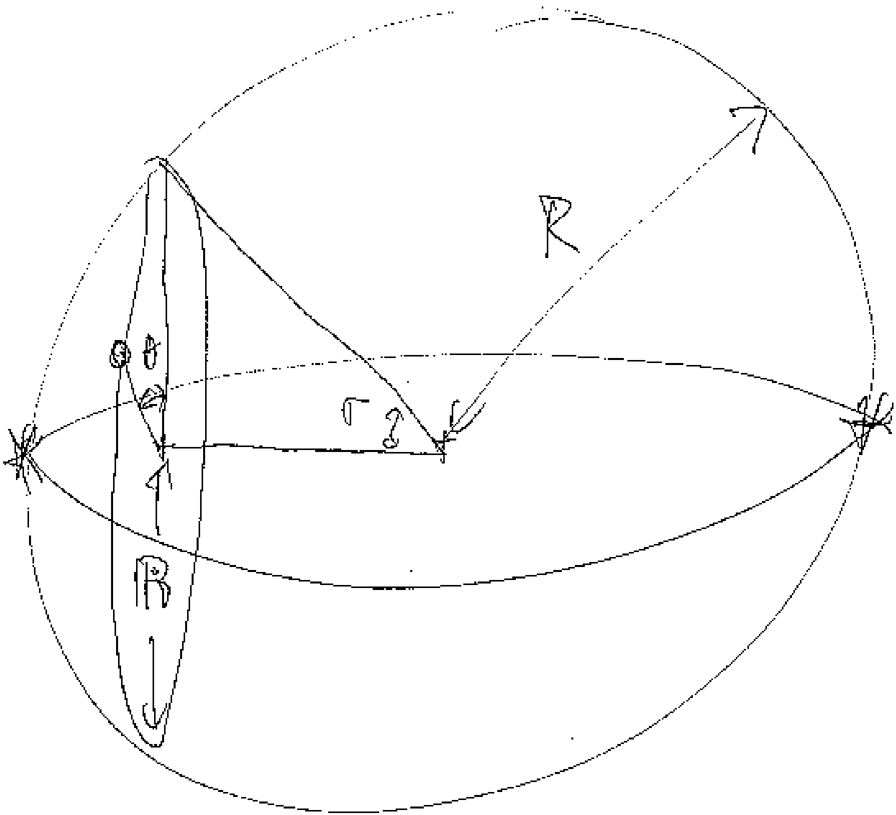
$$= \frac{\rho\delta^2}{2}\left[1 - \frac{9}{4}\sin^2\theta\right]$$

Outside Wake

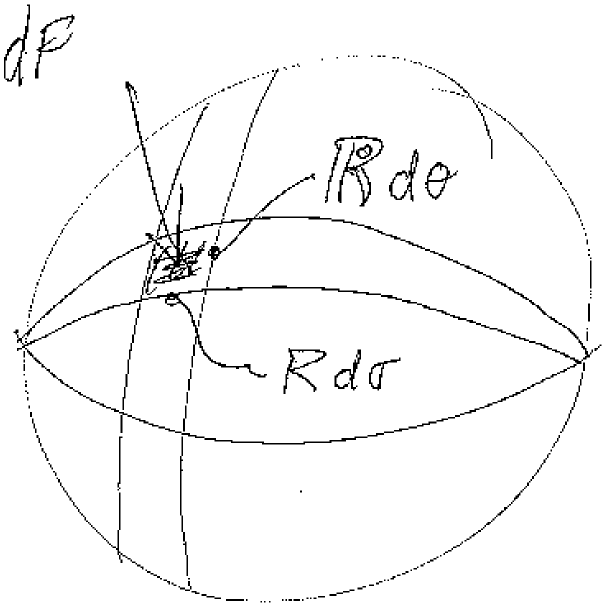
$$P = \frac{\rho S^2}{2}\left[1 - \frac{9}{4}\sin^2\theta\right]$$

Inside Wake

$$P = \frac{\rho S^2}{2}\left[1 - \frac{9}{4}\sin^2\theta_s\right]$$



R
θ
r
R



dF
$R\,d\theta$
$R\,d\sigma$

Elemental Area. dA

$$dA = R\, d\sigma \; \rho \, d\theta$$

$$\rho = R \sin\sigma$$

$$dA = R^2 \sin\sigma \, d\sigma \, d\theta$$

Elemental Force dF

$$dF = P\,dA$$

$$dF = P\,R^2 \sin\phi \, d\phi \, d\theta$$

Elemental Lift dL

$$dL = -dF \sin\sigma \cos\theta$$

$$= -P R^2 \sin^2\sigma \cos\theta \, d\sigma \, d\theta$$

Elemental Drag dD

$$dD = +dF \cos\sigma$$

$$= +P R^2 \sin\sigma \cos\sigma \, d\sigma \, d\theta$$

Lift on Sphere

$$L = \int_0^{2\pi} \int_0^{\pi} \left[-P R^2 \sin^2\sigma \cos\theta \right] d\sigma \, d\theta$$

Drag on Sphere

$$D = \int_0^{2\pi} \int_0^{\pi} \left[+P R^2 \sin\sigma \cos\sigma \right] d\sigma \, d\theta$$

APPROXIMATIONS

Lift on Sphere

$$\sum\sum\left(-PR^2\sin^2\sigma\cos\theta\right)\Delta\sigma\Delta\theta$$

Drag on Sphere

$$\sum\sum\left(+PR^2\sin\sigma\cos\sigma\right)\Delta\sigma\Delta\theta$$