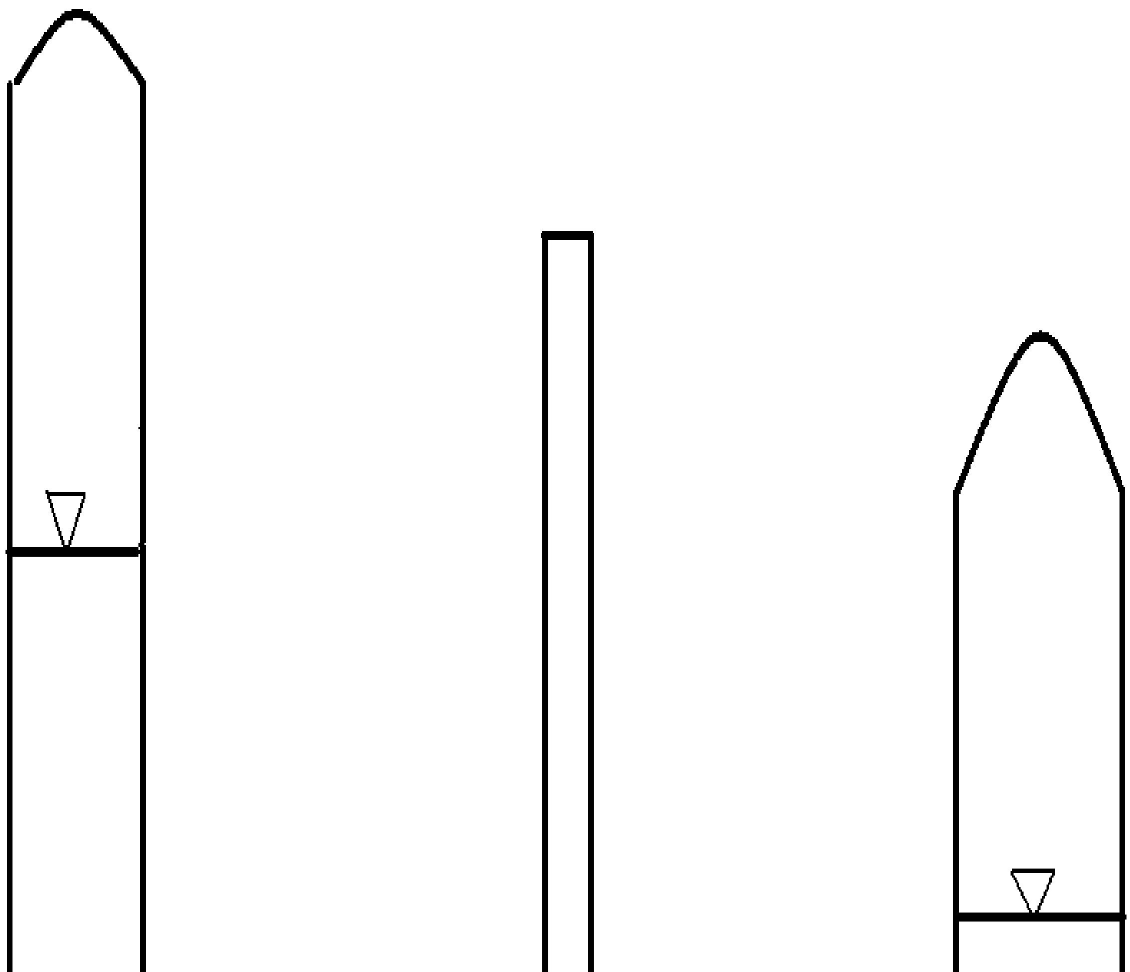
FLUID MECHANICS II

HOMEWORK #1

A small centrifugal pump rotates at 1800rpm. Its rotor has an inlet diameter of 5cm and an outlet diameter of 10cm. The depth of the rotor is 2cm at the inlet and at the outlet. The inlet blade angle is 120° and the outlet blade angle is 150°. Derive an equation for the pressure flow characteristic of the pump. What is the maximum pressure of the pump? [Hint: Set Q to zero] What is the maximum flow of the pump? [Hint: Set P to zero] Derive an equation for the characteristic of a pump twice the size of the given pump. Derive an equation for the characteristic of a pump half the size of the given pump. Use MATLAB to plot the characteristics showing the size effect. Derive an equation for the characteristic of a pump with rotational speed twice that of the given pump. Derive an equation for the characteristic of a pump with rotational speed half that of the given pump. Use MATLAB to plot the characteristics showing the speed effect. Derive an equation for the pressure coefficient versus flow coefficient of the pumps. Use MATLAB to plot the coefficient characteristic of each pump. Derive an equation for the characteristic of a pump with blade angles both 90°. What is the maximum pressure of the pump? What is the maximum flow of the pump?

A piston pump is used to pump water down the simple pipe network shown on the back of this page. The pump has the following flow characteristic: $U = A + B \cos[2\pi t/T]$. The A term gives the mean flow and the B term accounts for pulsations due to the pistons. Assume that A=1, B=0.1 and T=0.4. The pressure downstream of the pump is initially 50 BAR and there is no flow. Use algebraic water hammer analysis to determine pressure and velocity at the ends of each pipe in the network for 8 short pipe transit times when the pump is suddenly started. The two short pipes are 100m long and the long deadend pipe is 200m long. The wave speed for each pipe in the network is 1000m/s. The diameter of each pipe is 0.25m. Write a MATLAB code to check the predictions and go more steps in time. Use the code to get the PU plot at each end of each pipe. Use the code to do cases T=0.2 and T=0.8.



QUESTION ONE

The pressure of a pump is

$$P = \rho \ [\ (V_T \, V_B)_{OUT} - (V_T \, V_B)_{IN} \ ]$$

The tangential and blade velocities are:

$$V_T = V_B + V_N \, Cot[\beta] \qquad V_B = R\omega$$

$$V_N = Q/A \qquad A = 2\pi R \, h$$

At the outlet the radius R is equal to 5cm while at the inlet it is equal to 2.5cm. At the outlet the blade angle β is equal to 150^o while at the inlet β is equal to 120^o. The rotor depth h at the outlet and the inlet is 2cm. Substitution into the pressure equation gives an equation of the form:

$$P = A + B \ Q$$

The coefficients for a pump are:

$$C_P = P \ / \ [\rho N^2 D^2] \qquad C_Q = Q \ / \ [N D^3]$$

Manipulation gives:

$$P = [\rho N^2 D^2] \ C_P \qquad Q = [N D^3] \ C_Q$$

Substitution into the pressure equation gives

$$C_P = X + Y \ C_Q$$

QUESTION TWO

The waterhammer equations at the pump are:

$$U_M = A + B \cos[2\pi t/T]$$

$$P_M = P_n + [\rho a][U_M - U_n]$$

The waterhammer equations at the tank are:

$$P_H = 50 \qquad U_H = U_g - [P_H - P_g]/[\rho a]$$

The waterhammer equations at the deadend are:

$$U_Z = 0 \qquad P_Z = P_y - [\rho a]\ [U_Z - U_y]$$

The waterhammer equations at the midpoint are:

$$P_Y = (P_z+P_x)/2 - [\rho a][U_z-U_x]/2$$

$$U_Y = (U_z+U_x)/2 - [P_z-P_x]/[2\rho a]$$

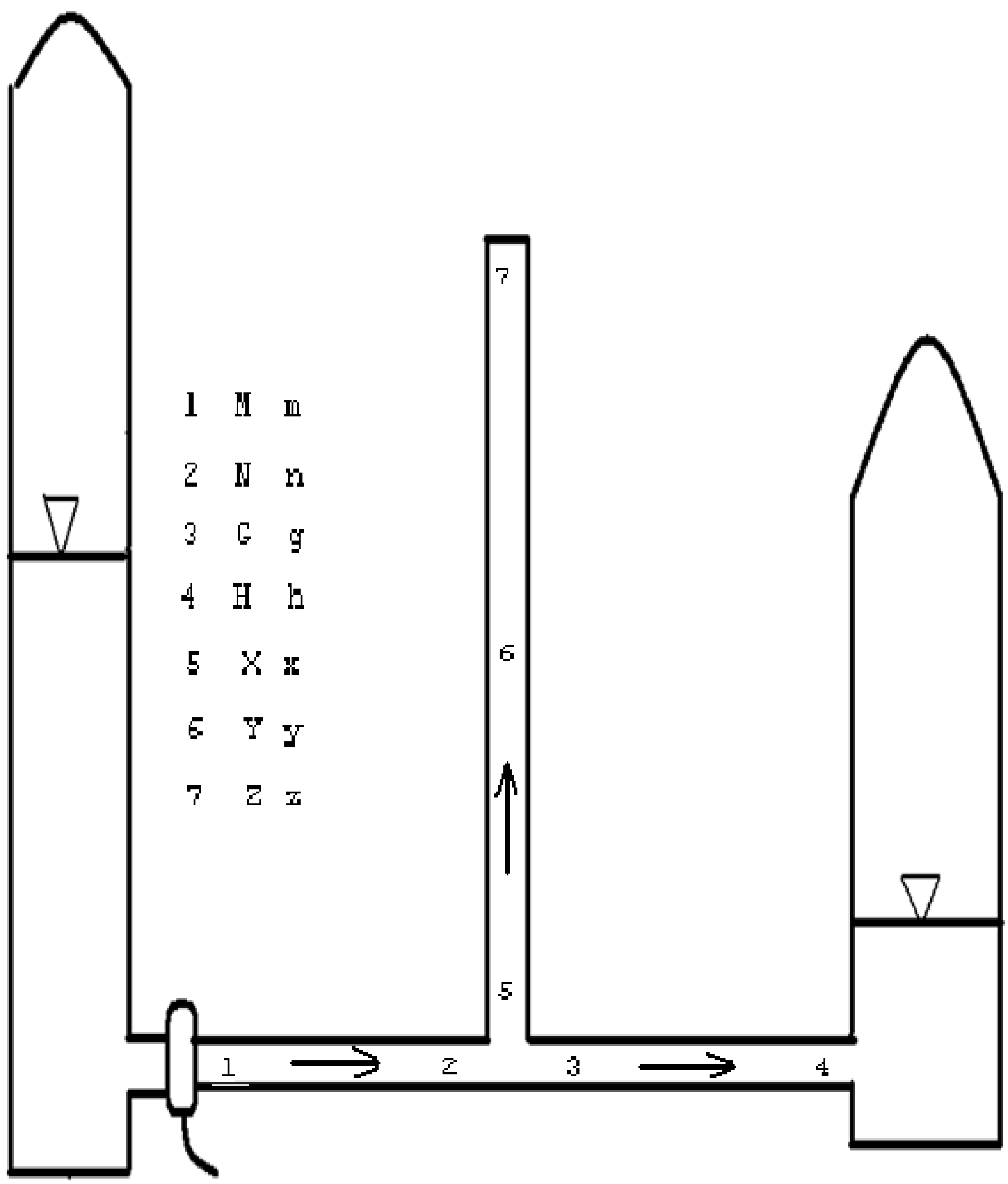
The waterhammer equations at the junction are:

$$P_J = [P_m+P_h+P_y]/3 + [\rho a]\ [U_m-U_h-U_y]/3$$

$$U_N = U_m - (P_J - P_m)/[\rho a]$$

$$U_G = U_h + (P_J - P_h)/[\rho a]$$

$$U_X = U_y + (P_J - P_y)/[\rho a]$$



1 M m
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3 G g
4 H h
5 X x
6 Y y
7 Z z
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FLUID MECHANICS II

MECHANICAL CLASS

HOMEWORK #2

A divers air bottle when fully charged has pressure P=20MPa and temperature T=20°C. It weighs 18kg. Imagine that a nozzle with throat and downstream cone is installed at the bottle exit. Let the throat diameter be 12.7mm. Estimate the thrust generated when the nozzle is suddenly opened. Repeat thrust calculations for case where the cone downstream of the throat has been removed. How many Gs of force would be generated if a single bottle with downstream cone was used to propel a 100kg person vertically? How many Gs would be generated if it was used to propel the person horizontally?

Estimate the lift and drag on a V shaped flat plate foil travelling at M=4 through an atmosphere with a pressure of 1BAR and a temperature of 20°C. The angle of attack of the foil from tip to tip is θ=20 degrees. The angle at the V is £=160 degrees. Each leg of the V has chord C=1m.

An explosion in the atmosphere generates a shock wave with a pressure ratio of 6. The pressure of the air in the atmosphere is 1BAR and its temperature is 20°. What would be the drift speed behind the shock wave? What would be the pressure and temperature at the stagnation point on a small blunt object placed in the supersonic flow behind the shock wave?

FLUID MECHANICS II

PROCESS CLASS

HOMEWORK #2

A divers air bottle when fully charged has pressure P=20MPa and temperature T=20°C. It weighs 18kg. Imagine that a nozzle with throat and downstream cone is installed at the bottle exit. Let the throat diameter be 12.7mm. Estimate the thrust generated when the nozzle is suddenly opened. Repeat thrust calculations for case where the cone downstream of the throat has been removed. How many Gs of force would be generated if a single bottle with downstream cone was used to propel a 100kg person vertically? How many Gs would be generated if it was used to propel the person horizontally?

A smooth plastic tube has an ID of 1cm. The friction factor of the tube is f=0.005. At the tube entrance, air with temperature T=20°C and pressure P=10BAR is flowing with MACH number M=0.5. How far down the tube would the air have to flow to reach sonic conditions? Repeat the calculations for the case where M=2. Do the calculations for both adiabatic and isothermal cases. Write a MATLAB code to determine the variation of P and T and ρ of the flow along the tube. Calculate the mass flow rate down the tube for each M.

An explosion in the atmosphere generates a shock wave with a pressure ratio of 6. The pressure of the air in the atmosphere is 1BAR and its temperature is 20°C. What would be the drift speed behind the shock wave? What would be the pressure and temperature at the stagnation point on a small blunt object placed in the supersonic flow behind the shock wave?

QUESTION #1

For the ideal nozzle with cone the thrust is

$$\dot{M}\,U_D$$

For the nozzle without the cone the thrust is

$$\dot{M}\,U_T + (P_T - P_D)\ A_T$$

The mass flow rate is

$$\dot{M} = \rho\ A\ U$$

At the throat

$$\rho_T = P_T\ /[R\ T_T] \qquad U_T = a_T \qquad a_T = \sqrt{[k\ R\ T_T]}$$

With this the mass flow rate is

$$\dot{M} = P_T\ /[R\ T_T]\ A_T\ \sqrt{[k\ R\ T_T]}$$

The isentropic ratios give P_T and T_T

$$T_T/T_U = (1 + [(k-1)/2] M_U M_U) / (1 + [(k-1)/2] M_T M_T)$$

$$P_T/P_U = [T_T/T_U]^x \qquad x = k/(k-1)$$

The isentropic pressure ratio gives M_D

$$P_D/P_U = [T_D/T_U]^x \qquad x = k/(k-1)$$

$$T_D/T_U = [(1 + [(k-1)/2] M_U M_U) / (1 + [(k-1)/2] M_D M_D)]$$

The isentropic temperature ratio gives T_D

$$T_D/T_U = [(1 + [(k-1)/2] M_U M_U) / (1 + [(k-1)/2] M_D M_D)]$$

The speed of sound equation gives a_D

$$a_D = \sqrt{[k\ R\ T_D]}$$

The Mach number equation gives U_D

$$U_D = M_D\ a_D$$

QUESTION TWO

For flow of a gas down a pipe

$$\Delta M^2/M^2 = kM^2[1+[(k-1)/2]M^2] /[1-M^2] f\Delta x/D$$

$$\Delta P/P = -kM^2[1+(k-1) M^2]/[2(1-M^2)] f\Delta x/D$$

$$\Delta T/T = -k(k-1)M^4/[2(1-M^2)] f\Delta x/D$$

$$\Delta\rho/\rho = -kM^2/[2(1-M^2)] f\Delta x/D$$

Each equation is of the form

$$\Delta A = B \Delta x$$

Numerical integration gives

$$A_{NEW} = A_{OLD} + B_{OLD} \Delta x$$

This can be used to get changes along the pipe.

For adiabatic flow

$$fL^*/D = (1-M^2)/(kM^2) + [(k+1)/(2k)] \ln[(k+1)M^2/(2+(k-1)M^2)]$$

For isothermal flow

$$fL^*/D = (1-kM^2)/(kM^2) + \ln[kM^2]$$

These equations give the distance L^* to choking.

The mass flow rate down the pipe is

$$\dot{M} = \rho\ U\ A$$

Gas dynamics gives

$$\rho = P\ /[R\ T] \qquad U = M\ a \qquad a = \sqrt{[k\ R\ T]}$$

With this the mass flow rate is

$$\dot{M} = P\ /[R\ T]\ \ M\ \sqrt{[k\ R\ T]}\ \ A$$

QUESTION TWO

There are two oblique shock waves on the bottom of the foil and two expansion waves on the top. The oblique shock plot gives the shock angle for each shock wave. The oblique shock angle is based on thermodynamics and geometry

$$\tan(\beta)/\tan(\kappa) = [(k+1)\ N_U\ N_U\ ] / [\ (k-1)\ N_U\ N_U + 2\ ]$$

$$N_D = M_D \operatorname{Sin}\kappa \qquad \kappa=\beta-\Theta$$

Substitution into the geometry equation gives N_U

$$N_U = M_U \operatorname{Sin}\beta$$

Substitution into the pressure equation gives P_D

$$P_D/P_U = 1 + [2k/(k+1)]\ (N_U N_U - 1)$$

Substitution into the Mach number connection gives N_D

$$N_D N_D = [(k-1)\ N_U N_U + 2]\ /\ [2k\ N_U N_U - (k-1)]$$

Substitution into the geometry equation gives M_D

$$N_D = M_D \, Sin\kappa$$

The expansion wave plot gives the flow angle and the Mach number downstream of each expansion wave. The flow angle is based on thermodynamics and geometry

$$\upsilon = \sqrt{[K]}\ tan^{-1}\sqrt{[(M^2-1)/K]} - tan^{-1}\sqrt{[M^2-1]}$$

$$K = (k+1)/(k-1) \qquad \upsilon_D = \upsilon_U + \Theta$$

The isentropic equation gives pressure downstream

$$T_D/T_U = [\ (1 + [(k-1)/2]\ M_U M_U)\ /\ (1 + [(k-1)/2]\ M_D M_D)\ ]$$

$$P_D/P_U = [T_D/T_U]^x \qquad x = k/(k-1)$$

The lift and drag on each leg are

$$P\ A\ Sin(\boldsymbol{\theta}-\Theta) \qquad P\ A\ Cos(\boldsymbol{\theta}-\Theta)$$

QUESTION THREE

For the explosion shock wave the pressure ratio equation gives the Mach number upstream M_U

$$P_D/P_U = 1 + [2k/(k+1)] (M_U M_U - 1)$$

The speed of sound equation gives a_U

$$a_U = \sqrt{[k R T_U]}$$

The Mach number equation gives U_U

$$U_U = M_U a_U$$

The Mach number connection gives M_D

$$M_D M_D = [(k-1) M_U M_U + 2] / [2k M_U M_U - (k-1)]$$

The temperature ratio equation gives T_D

$$T_D/T_U = [(1 + [(k-1)/2] M_U M_U) / (1 + [(k-1)/2] M_D M_D)]$$

The speed of sound equation gives a_D

$$a_D = \sqrt{[k\ R\ T_D]}$$

The Mach number equation gives U_D

$$U_D = M_D\ a_D$$

The drift speed is

$$U_A = U_U - U_D$$

For the bow shock wave

$$M_A = U_A / a_A$$

$$P_A = P_D \qquad T_A = T_D \qquad a_A = a_D$$

Substitution into the shock equation gives P_B

$$P_B/P_A = 1 + [2k/(k+1)]\ (M_A M_A - 1)$$

The Mach number connection gives M_B

$$M_B M_B = [(k-1)\ M_A M_A + 2]\ /\ [2k\ M_A M_A - (k-1)]$$

Substitution into the isentropic equation gives P_C

$$P_C/P_B = [T_C/T_B]^x \qquad x = k/(k-1)$$

$$T_C/T_B = [\ (1 + [(k-1)/2]\ M_B M_B)\ /\ (1 + [(k-1)/2]\ M_C M_C)\ ]$$

Substitution into the temperature ratio equation gives T_C

$$T_C/T_A = [\ (1 + [(k-1)/2]\ M_A M_A)\ /\ (1 + [(k-1)/2]\ M_C M_C)\ ]$$

FLUID MECHANICS II

HOMEWORK #3

A soccer ball has a diameter 0.25m and is moving at 50km/hr. Calculate the drag on the ball if boundary layer separation occurs at 90° [15]. Calculate the drag on the ball if separation occurs at 120° [5]. Calculate the drag on the ball if separation occurs at 180° [5]. For each case, calculate the drag coefficient and compare with that in texts [10]. Outline with words and formulas how you would model the case where the ball is spinning around the vertical axis [10]. Write a short MATLAB code to predict the trajectory of the ball [5].

A propeller for a small airplane has 4 thin plate blades. At the root, the chord is 0.1m, while at the tip it is 0.05m. At the root, the attack angle is 20°, while at the tip it is 10°. The blade span is 1m. The radius of the hub out to the root is 0.25m. The propeller rotational speed is 1800RPM. Calculate the thrust of the propeller while the airplane is stopped [20] and when it is moving forward at 100km/hr [10].

An outlet of a sewage pipe in the ocean can be modeled as a point source. If the flow rate of effluent from the pipe is 25 l/s, determine the diameter of the effluent stream well downstream when it is in a current with speed 1 m/s [20].

QUESTION #1

Superposition of a stream and a doublet gives the potential flow around a sphere. The potential function is

$$\varphi = - S\ r\ Cos[\sigma\ ] - S/2\ R^3/r^2\ Cos[\sigma]$$

On the surface of the sphere it is

$$\varphi = - S\ R\ Cos[c/R] - S/2\ R\ Cos[c/R]$$

Differentiation gives the speed on the sphere

$$\partial\varphi/\partial c = 3/2\ S\ Sin\sigma$$

Application of Bernoulli gives pressure

$$P = \rho/2\ [\ S^2 - (\partial\varphi/\partial c)^2\ ]$$

Substitution into this gives

$$P = \rho/2\ [\ S^2 - 9/4\ S^2\ Sin^2\sigma\ ]$$

This is good up to the where the wake starts. In the wake the pressure is approximately constant and is

$$P = \rho/2\ [\ S^2 - 9/4\ S^2\ Sin^2\sigma_S\ ]$$

A bit of area on the sphere is

$$dA = R \, Sin\sigma \, d\Theta \quad R \, d\sigma$$

Multiplication by pressure gives a bit of force

$$dF = P \, dA = P \, R^2 \, Sin\sigma \, d\sigma \, d\Theta$$

The drag component of the bit of force is

$$dD = + \, dF \, Cos\sigma$$

Integration gives the total drag

$$\int\int + \, P \, R^2 \, Sin\sigma \, Cos\sigma \, d\sigma \, d\Theta$$

For a sphere spinning about a vertical axis, the speed of the flow on its surface would have potential flow components and spin components. The potential flow speed is

$$G = 3/2 \, S \, Sin\sigma$$

Its components are

$$U_P = + \, G \, Sin\sigma \qquad V_P = - \, G \, Cos\sigma \, Sin\Theta \qquad W_P = + \, G \, Cos\sigma \, Cos\Theta$$

The spin speed is

$$H = \mathbf{R} \, \omega$$

$$\mathbf{R} = \sqrt{ [(R \cos\sigma)^2 + (R \sin\sigma \sin\Theta)^2] }$$

Its components are

$$U_S = + H \sin\varepsilon \qquad V_S = - H \cos\varepsilon \qquad W_S = 0$$

$$\varepsilon = \tan^{-1} ([R \sin\sigma \sin\Theta] / [R \cos\sigma])$$

The total speed components are

$$U = U_P + U_S \qquad V = V_P + V_S \qquad W = W_P + W_S$$

The total speed is

$$\mathbf{S} = \sqrt{ [U^2 + V^2 + W^2] }$$

The pressure is

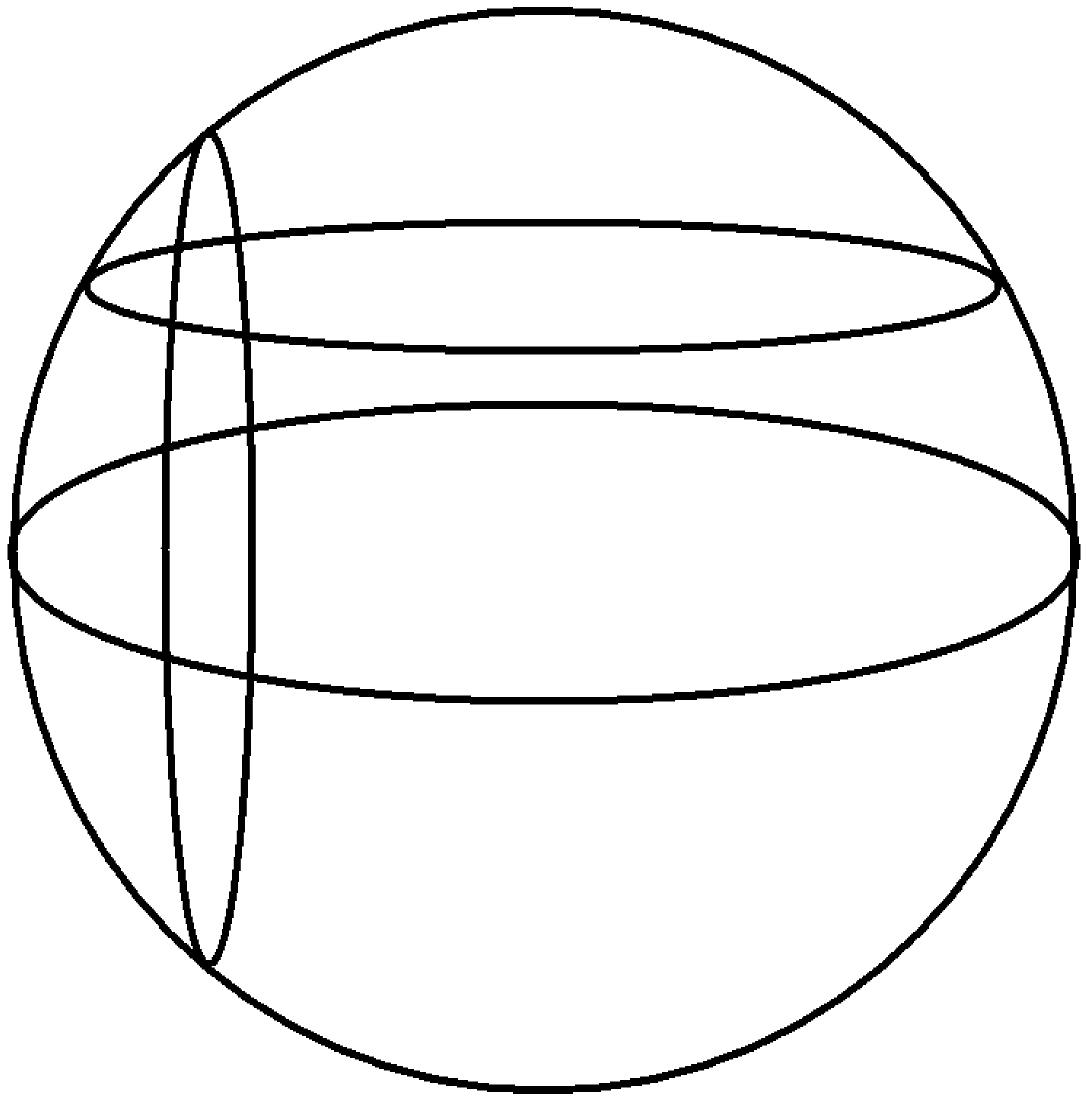
$$P = \rho/2 [S^2 - \mathbf{S}^2]$$

The incremental side force is

$$P R^2 \sin\sigma \sin\sigma \sin\Theta \, d\sigma \, d\Theta$$

Numerical integration gives the side force

$$\Sigma\Sigma [+ P R^2 \sin\sigma \sin\sigma \sin\Theta] \Delta\sigma \Delta\Theta$$



QUESTION #2

The thrust of a slice of a blade is

$$\rho\ S\ \Gamma\ dr$$

For a flat plate blade

$$\Gamma = 4\ \pi\ S\ R\ \mathrm{Sin}\Theta = C\ \pi\ S\ \mathrm{Sin}\Theta$$

Using the small angle approximation

$$\Gamma = C\ \pi\ S\ \Theta$$

The chord and angle of attack are

$$C = a + b\ r \qquad \Theta = n + m\ r$$

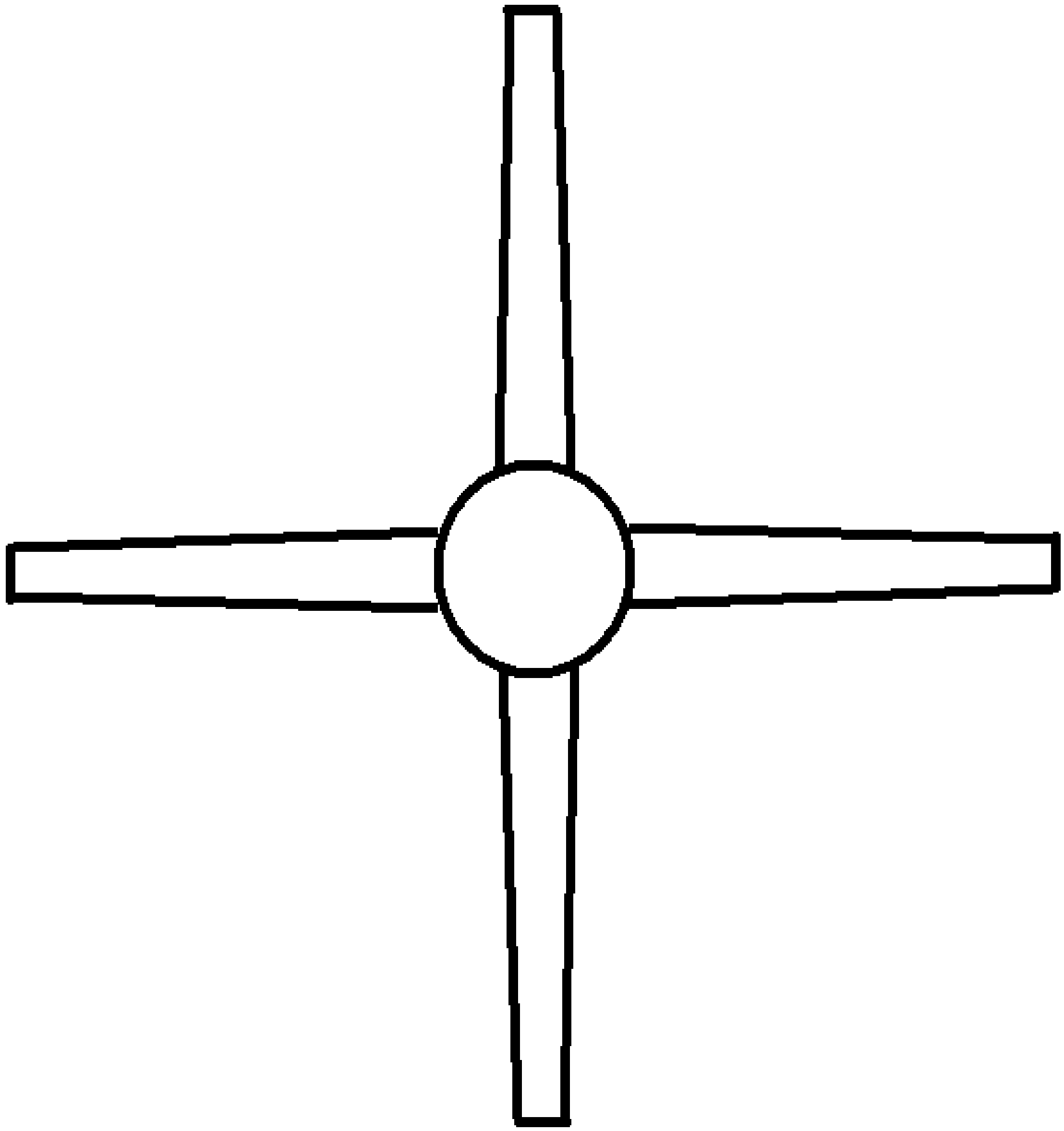
The speed of a slice is

$$S = r\ \omega$$

The total thrust is

$$4 \int \rho\ r\omega\ [a+br]\ \pi\ r\omega\ [n+mr]\ dr$$

When the airplane is moving forward, the speed of the flow coming at the blade consists of the rotational component S and the forward speed component U. The speed is



$$\mathbf{S} = \sqrt{\ [\ S^2 + U^2\ ]}$$

The angle of the flow is

$$\alpha = \tan^{-1} [U/S]$$

The apparent angle of attack is

$$\beta = \Theta - \alpha$$

$$= e + f\ r$$

The incremental load is now

$$\rho\ \mathbf{S}\ [a+br]\ \pi\ \mathbf{S}\ [e+fr]\ dr$$

This will have the thrust component

$$\rho\ \mathbf{S}\ [a+br]\ \pi\ \mathbf{S}\ [e+fr]\ \mathrm{Cos}\beta\ dr$$

$$\rho\ \mathbf{S}\ [a+br]\ \pi\ \mathbf{S}\ [e+fr]\ dr$$

and the drag component

$$\rho\ \mathbf{S}\ [a+br]\ \pi\ \mathbf{S}\ [e+fr]\ \mathrm{Sin}\beta\ dr$$

$$\rho\ \mathbf{S}\ [a+br]\ \pi\ \mathbf{S}\ [e+fr]\ \beta\ dr$$

Integration gives the total thrust and drag.

FLUID MECHANICS II

HOMEWORK #4

A hydrodynamic lubrication journal bearing for truck has a shaft diameter of 50mm and a sleeve diameter of 51mm. The span of the sleeve is 50mm. During operation, the minimum gap between the shaft and the sleeve is 0.1mm. The rotational speed of the shaft is 2400RPM. The viscosity of the bearing oil is $0.1Ns/m^2$. Derive a 5 point CFD template for the bearing. Use the template and Gauss Seidel iteration to determine the pressures at 9 points in the bearing. Use the pressures to calculate the load supported by the bearing. Determine the location of the minimum gap in the bearing. Write a short MATLAB code to check the calculations.

A hydrodynamic lubrication thrust bearing for a ship has 4 pads. The outside diameter of each pad is 1m and the inside diameter is 0.5m. The gap at the leading edge of each pad is 2mm and the gap at the trailing edge is 1mm. The angle between the leading and trailing edges is 60°. The rotational speed of the shaft is 600RPM. The viscosity of the bearing oil is $0.2Ns/m^2$. Derive a 5 point CFD template for the bearing. Use the template and Gauss Seidel iteration to determine the pressures at 9 points in the bearing. Use the pressures to calculate the load supported by the bearing. Write a short MATLAB code to check the calculations.

A rectangular slab of porous rock is 1000m long, 25m wide and 5m deep. Pressure at its inlet is 50BAR while pressure at its outlet is 10BAR. The sides of the slab are blocked. The viscosity of the fluid in the rock is $1Ns/m^2$. The permeability of the rock in units m^2 varies linearly from 10^{-12} at the inlet to 10^{-14} at the outlet. Derive a 7 point CFD template for the pressure throughout the slab. Use the template to determine the pressure at the middle of the slab. Compare the CFD result with that based on 1D theory for the slab.

QUESTION #1

Reynolds equation for a Cartesian geometry is

$$\partial/\partial x\ (h^3/12\mu\ \partial P/\partial x) + \partial/\partial y\ (h^3/12\mu\ \partial P/\partial y)$$

$$= \partial[h(U_T+U_B)/2]/\partial x + \partial[h(V_T+V_B)/2]/\partial y + (W_T-W_B)$$

Manipulation gives

$$\partial/\partial x\ (h^3\ \partial P/\partial x) + \partial/\partial y\ (h^3\ \partial P/\partial y) = 6\mu S\ \partial h/\partial x$$

Application of central differencing gives

$$[\ [(h_E+h_P)/2]^3\ (P_E-P_P)/\Delta x - [(h_W+h_P)/2]^3\ (P_P-P_W)/\Delta x\ ]\ /\Delta x$$

$$+$$

$$[\ [(h_N+h_P)/2]^3\ (P_N-P_P)/\Delta y - [(h_S+h_P)/2]^3\ (P_P-P_S)/\Delta y\ ]\ /\Delta y$$

$$= 6\mu S\ (h_E - h_W)/[2\Delta x]$$

Manipulation gives the template

$$P_P = (A\ P_E + B\ P_W + C\ P_N + D\ P_S + H)\ /\ (A + B + C + D)$$

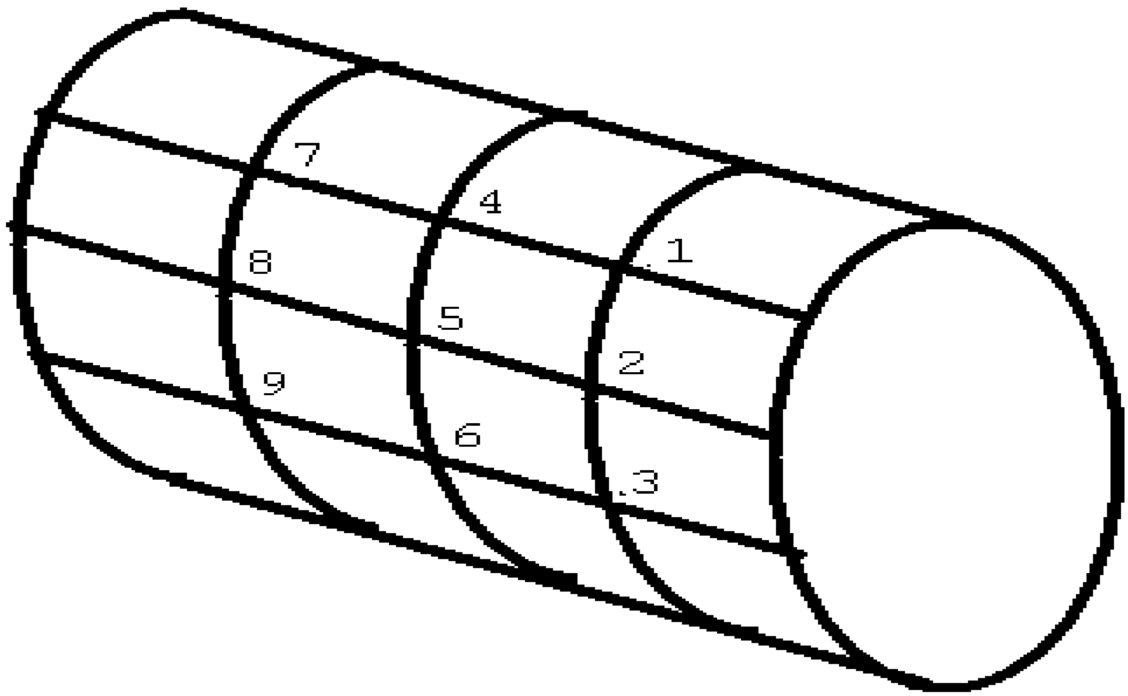
$$A = [(h_E+h_P)/2]^3\ /\ [\Delta x^2]$$

$$B = [(h_W+h_P)/2]^3\ /\ [\Delta x^2]$$

$$C = [(h_N+h_P)/2]^3\ /\ [\Delta y^2]$$

$$D = [(h_S+h_P)/2]^3\ /\ [\Delta y^2]$$

$$H = -\ 6\mu\ R\omega\ (h_E-h_W)/[2\Delta x]$$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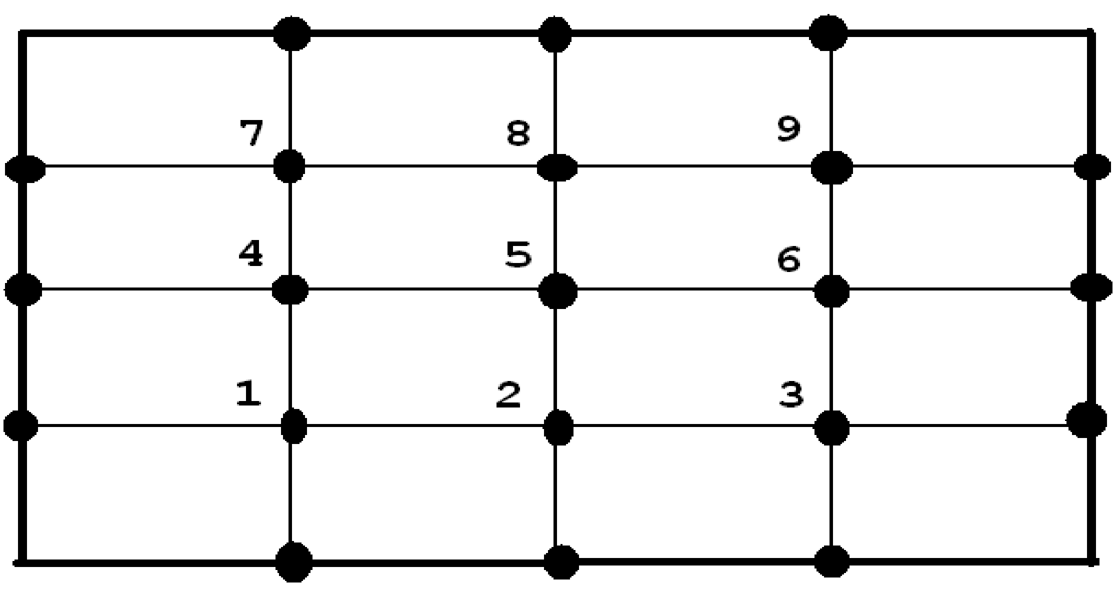
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QUESTION #2

Reynolds equation for a cylindrical geometry is

$$r\ \partial/\partial c\ (h^3/12\mu\ \partial P/\partial c) + \partial/\partial r\ (rh^3/12\mu\ \partial P/\partial r) +$$

$$= \partial[rh(U_T+U_B)/2]/\partial r + \partial[h(V_T+V_B)/2]/\partial\Theta + r(W_T-W_B)$$

Manipulation gives

$$r\ \partial/\partial c\ (h^3\ \partial P/\partial c) + \partial/\partial r\ (rh^3\ \partial P/\partial r) = 6\mu S\ \partial h/\partial\Theta$$

Application of central differencing gives

$$r_P\ [[(h_E+h_P)/2]^3(P_E-P_P)/\Delta c - [(h_W+h_P)/2]^3(P_P-P_W)/\Delta c]/\Delta c$$

$$+$$

$$[[(rh^3)_N+(rh^3)_P]/2](P_N-P_P)/\Delta r - [(rh^3)_S+(rh^3)_P]/2](P_P-P_S)/\Delta r]/\Delta r$$

$$= 6\mu S\ (h_E - h_W)/[2\Delta\Theta]$$

Manipulation gives the template

$$P_P = (A\ P_E + B\ P_W + C\ P_N + D\ P_S + H)\ /\ (A + B + C + D)$$

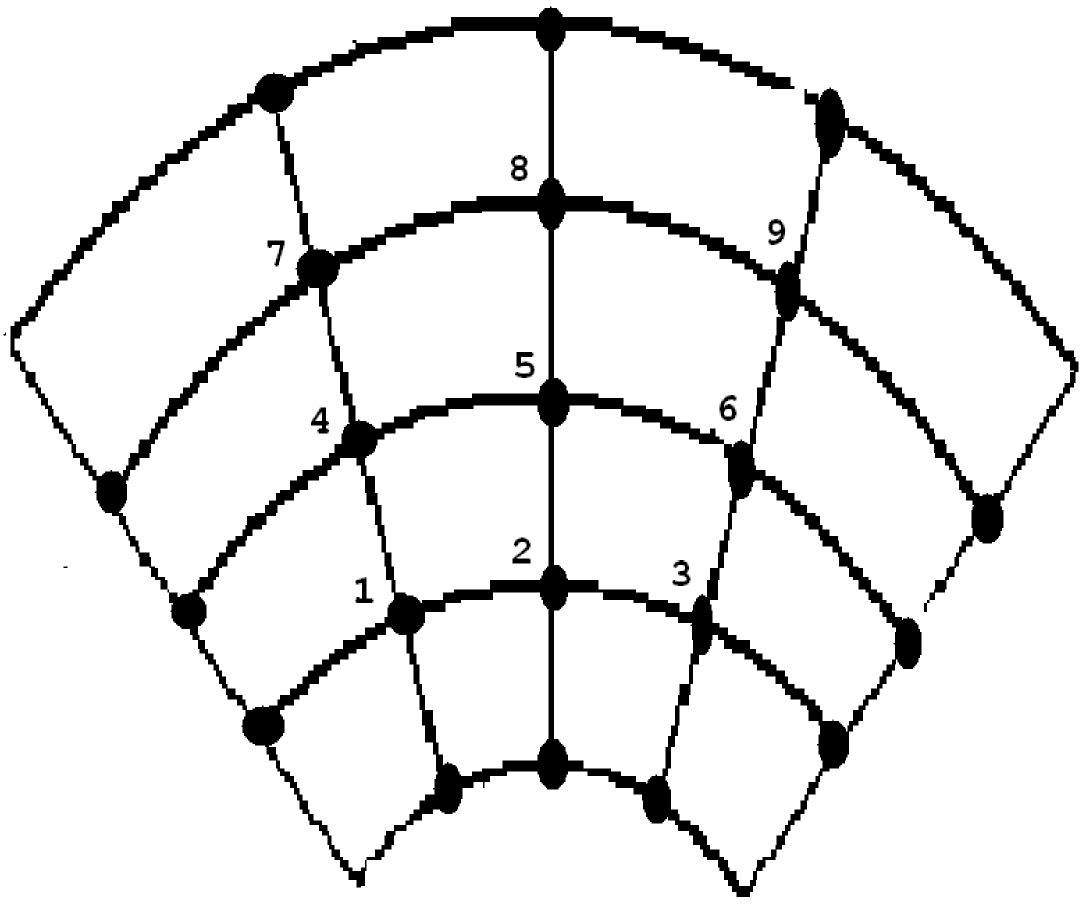
$$A = [(h_E+h_P)/2]^3\ r_P\ /\ [\Delta c^2]$$

$$B = [(h_W+h_P)/2]^3\ r_P\ /\ [\Delta c^2]$$

$$C = [(rh^3)_N+(rh^3)_P]/2]\ /\ [\Delta r^2]$$

$$D = [(rh^3)_S+(rh^3)_P]/2]\ /\ [\Delta r^2]$$

$$H = -\ 6\mu\ r_P\omega\ (h_E-h_W)/[2\Delta\Theta]$$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QUESTION #3

The pressure equation for porous media is

$$\nabla . [K \nabla P] = 0$$

$$\partial/\partial x [K \partial P/\partial x] + \partial/\partial y [K \partial P/\partial y] + \partial/\partial z [K \partial P/\partial z] = 0$$

Application of central differencing gives

$$[(K_E+K_P)/2 (P_E-P_P)/ \Delta x - (K_W+K_P)/2 (P_P-P_W)/ \Delta x] / \Delta x$$
$$+$$
$$[(K_N+K_P)/2 (P_N-P_P)/ \Delta y - (K_S+K_P)/2 (P_P-P_S)/ \Delta y] / \Delta y$$
$$+$$
$$[(K_J+K_P)/2 (P_J-P_P)/ \Delta z - (K_I+K_P)/2 (P_P-P_I)/ \Delta z] / \Delta z$$

$$= 0$$

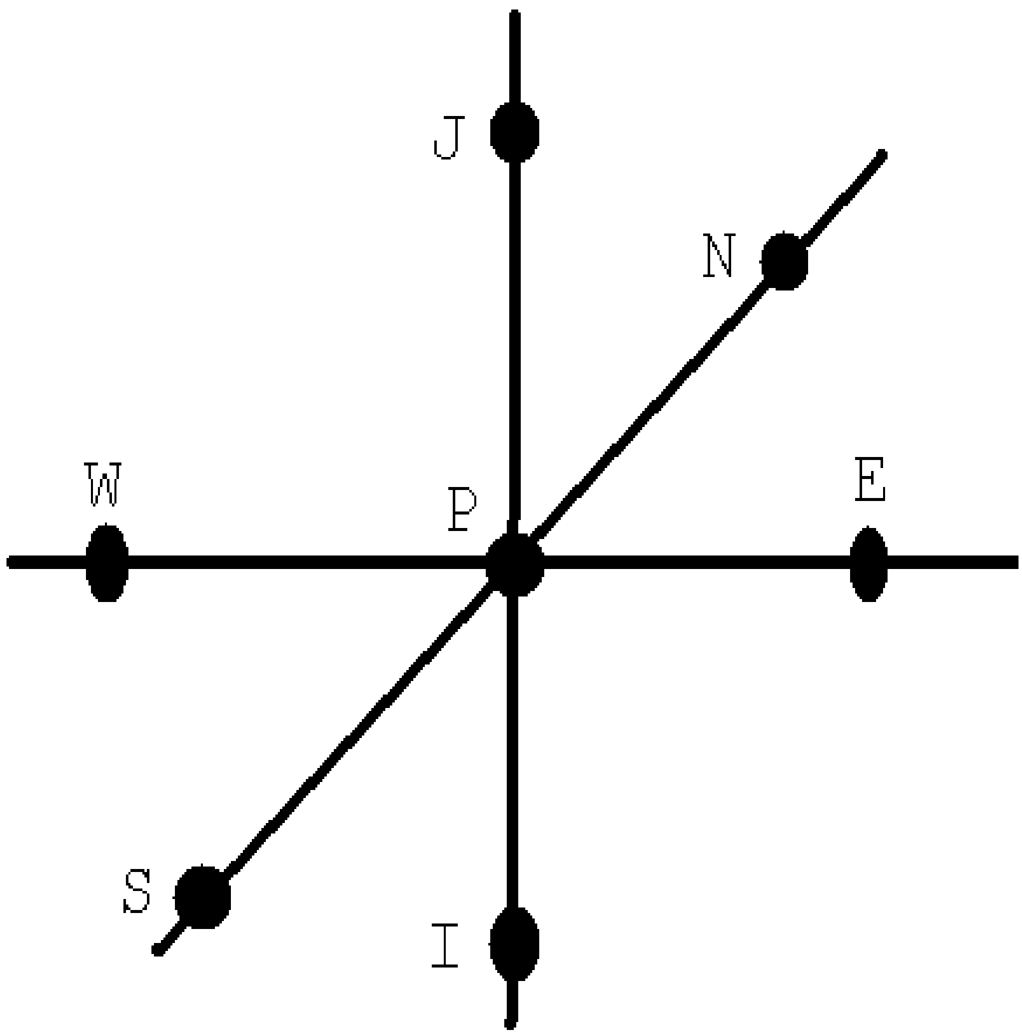
Manipulation gives the template

$$P_P = \frac{(A\ P_E + B\ P_W + C\ P_N + D\ P_S + G\ P_J + H\ P_I)}{(A + B + C + D + G + H)}$$

$A = [(K_E+K_P)/2]/[\Delta x^2]$ $B = [(K_W+K_P)/2]/[\Delta x^2]$

$C = [(K_N+K_P)/2]/[\Delta y^2]$ $D = [(K_S+K_P)/2]/[\Delta y^2]$

$G = [(K_J+K_P)/2]/[\Delta z^2]$ $H = [(K_I+K_P)/2]/[\Delta z^2]$


J
N
W
P
E
S
I

For a blocked sides case, the pressure equation is

$$d/dx\ [K\ dP/dx] = 0$$

Integration gives

$$K\ dP/dx = G \qquad dP/dx = G/K$$

For a linear variation in permeability

$$K = ax + b$$

The pressure gradient equation becomes

$$dP/dx = G\ /\ [ax + b]$$

Integration gives

$$P = G/a\ \ln[ax+b] + H$$

The boundary conditions are

$$P=P_{IN} \text{ at } x=0 \qquad P=P_{OUT} \text{ at } x=d$$

This allows one to find the constants of integration.

The blocked sides CFD template for one point is

$$P_P = (A\ P_E + B\ P_W)\ /\ (A + B)$$