ENGINEERING 6961

FLUID MECHANICS II

HOMEWORK #0

Write brief summaries of any three of the following efluids videos: (1) Waves in Fluids (2) Flow Instabilities (3) Low Reynolds Number Flows (4) Rotating Flows. Each summary should be 6 pages long, not counting sketches or screen shots.

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LAB #0

Develop a FLOW-3D simulation of a propeller wind turbine. Use SOLIDWORKS to create the turbine. Use thick elliptic cross section blades.

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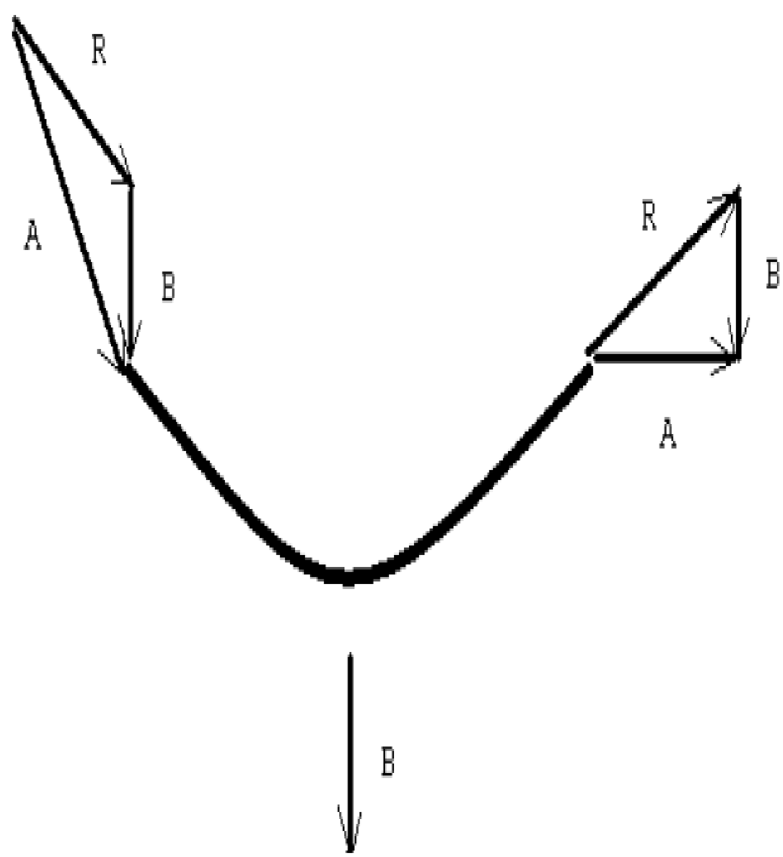
HOMEWORK #1

A dentist drill has 16 blades and 16 jets. A typical blade is shown in the sketch below: R stands for relative speed, B stands for blade speed and A stands for absolute speed. The jets are angled at 15° relative to the direction of motion of the blades. The inlet angle of the blades relative to the blade direction is 45° while the outlet angle of 135°. The mean radius out to the jets is 4mm. The jet diameter is 1mm. The nominal flow rate per jet is 0.1 l/s and the density of air is 2kg/m^3. Determine the power output of the drill as a function of its rotational speed.



The power output of a turbine is

$$\mathbf{P} = T\ \omega = \Delta\ [\ \rho Q\ V_T\ V_B\ ]$$

For a dentist drill this reduces to

$$\mathbf{P} = T\ \omega = \rho Q\ V_B\ \ \Delta[V_T]$$

The absolute velocity at the inlet is

$$V = Q/A$$

The components of this are

$$V_T = V\ \mathrm{Cos}\alpha \qquad V_N = V\ \mathrm{Sin}\alpha$$

Assuming same inlet and outlet areas, the normal component is the same at the inlet and the outlet. The tangential component at the outlet is

$$V_T = V_B + V_N\ \mathrm{Cot}\beta$$

So the power is

$$\rho Q\ V_B\ \ [\ Q/A\ \mathrm{Cos}\alpha\ \ -\ [V_B + V_N\ \mathrm{Cot}\beta]\ ]$$

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HOMEWORK #2

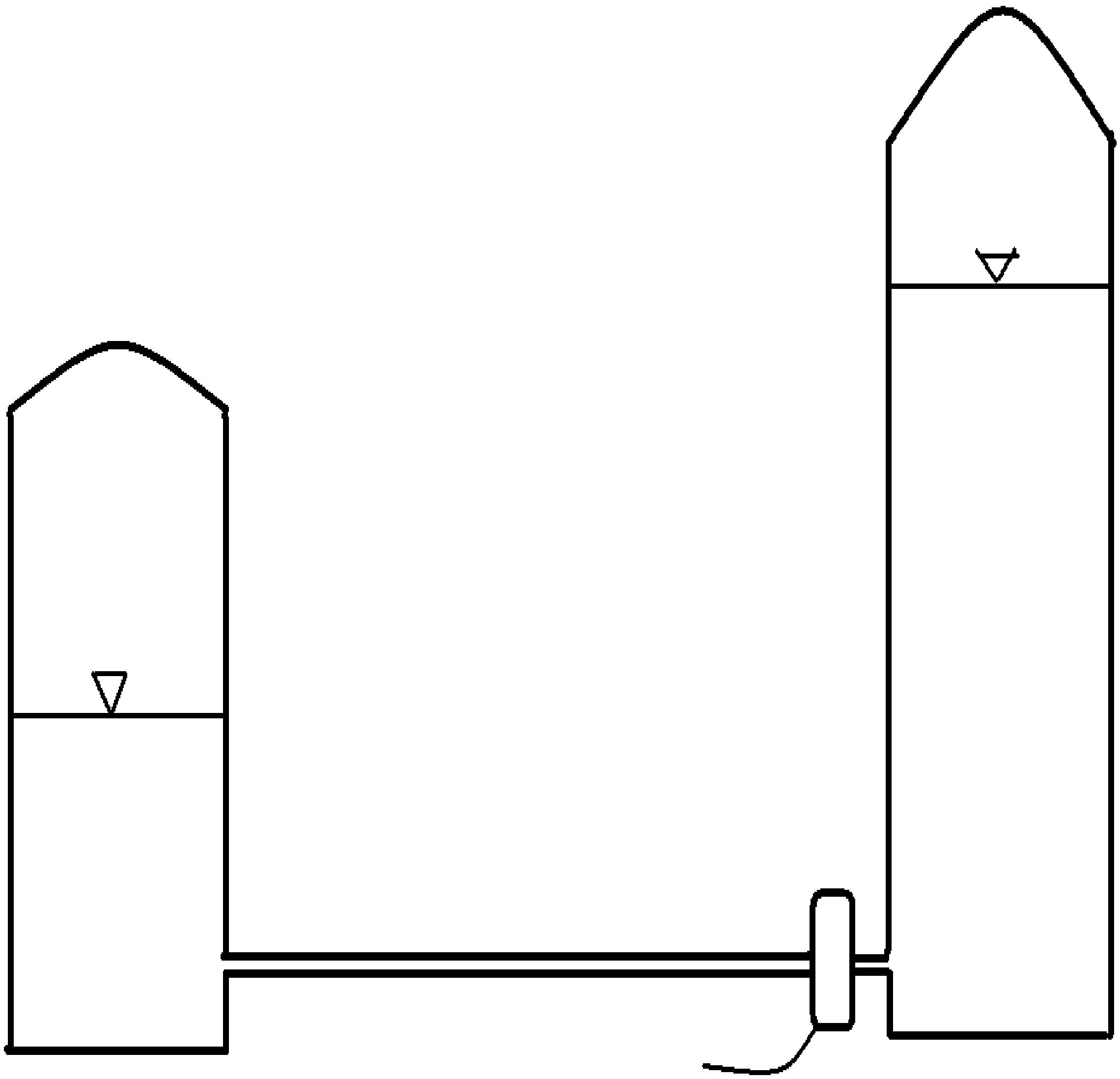
A pipe connects two pressurized water tanks and has a length L=1000m and diameter D=0.5m. There is a positive displacement pump at the downstream end of the pipe. Initially the pump is stopped and the conditions in the pipe upstream of it are P_o=30BAR U_o=0m/s. At time t=0, the pump starts and generates a flow U=A+B*Cos(2πt/**T**] where A=1 B=0.25 **T**=4. Water density ρ is 1000kg/m^3 and the pipe wave speed **a** is 1000m/s. The transit time T of the pipe is 1s. The pipe period **T** is 4T or 4.

Using wave propagation concepts, explain what happens in the upstream pipe following a sudden pump start up.

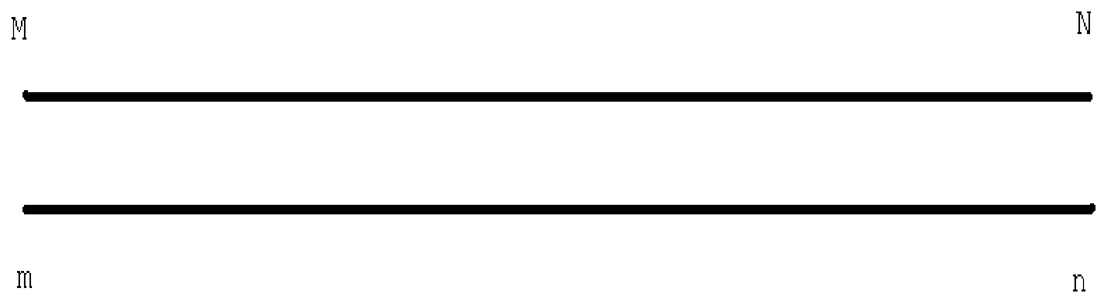
Using algebraic water hammer analysis, determine the pressure and velocity at the ends of the pipe for 6 steps in time.

Using graphical waterhammer analysis, determine the pressure and velocity at the ends of the pipe for 6 steps in time,

A check valve at the upstream end of the pipe would allow flow into the pipe but stop flow out. Determine the pressure and velocity at the ends of the pipe for 6 steps in time,



Pipe and pump flow mismatches at the pump cause a series of pressure waves at the pump and flow waves at the tank which gradually build up. The pump and pipe periods are the same which causes resonance.



$$U_N = A + B \cos[2\pi t/T]$$

$$P_N - P_m = - [\rho a] [U_N - U_m]$$

$$P_N = P_m - [\rho a] [U_N - U_m]$$

$$P_M - P_n = + [\rho a] [U_M - U_n]$$

$$U_M = U_n + [P_M - P_n] / [\rho a]$$

#0 $U_N = 0.0$ $P_N = 30.0$ $P_M = 30$ $U_M = 0.0$

#1 $U_N = 1.25$ $P_N = 17.5$ $P_M = 30$ $U_M = 0.0$

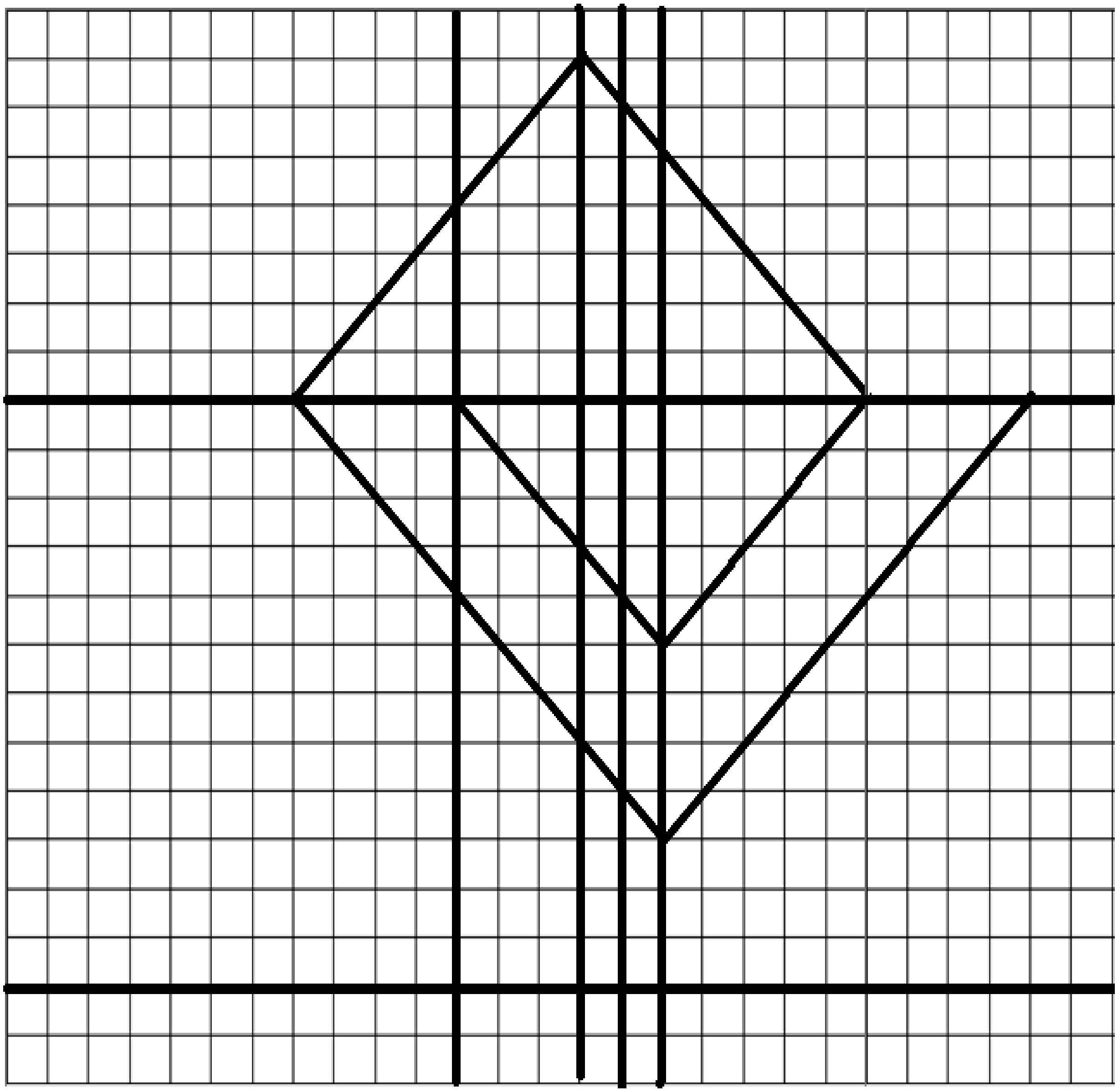
#2 $U_N = 1.0$ $P_N = 20.0$ $P_M = 30$ $U_M = 2.5$

#3 $U_N = 0.75$ $P_N = 47.5$ $P_M = 30$ $U_M = 2.0$

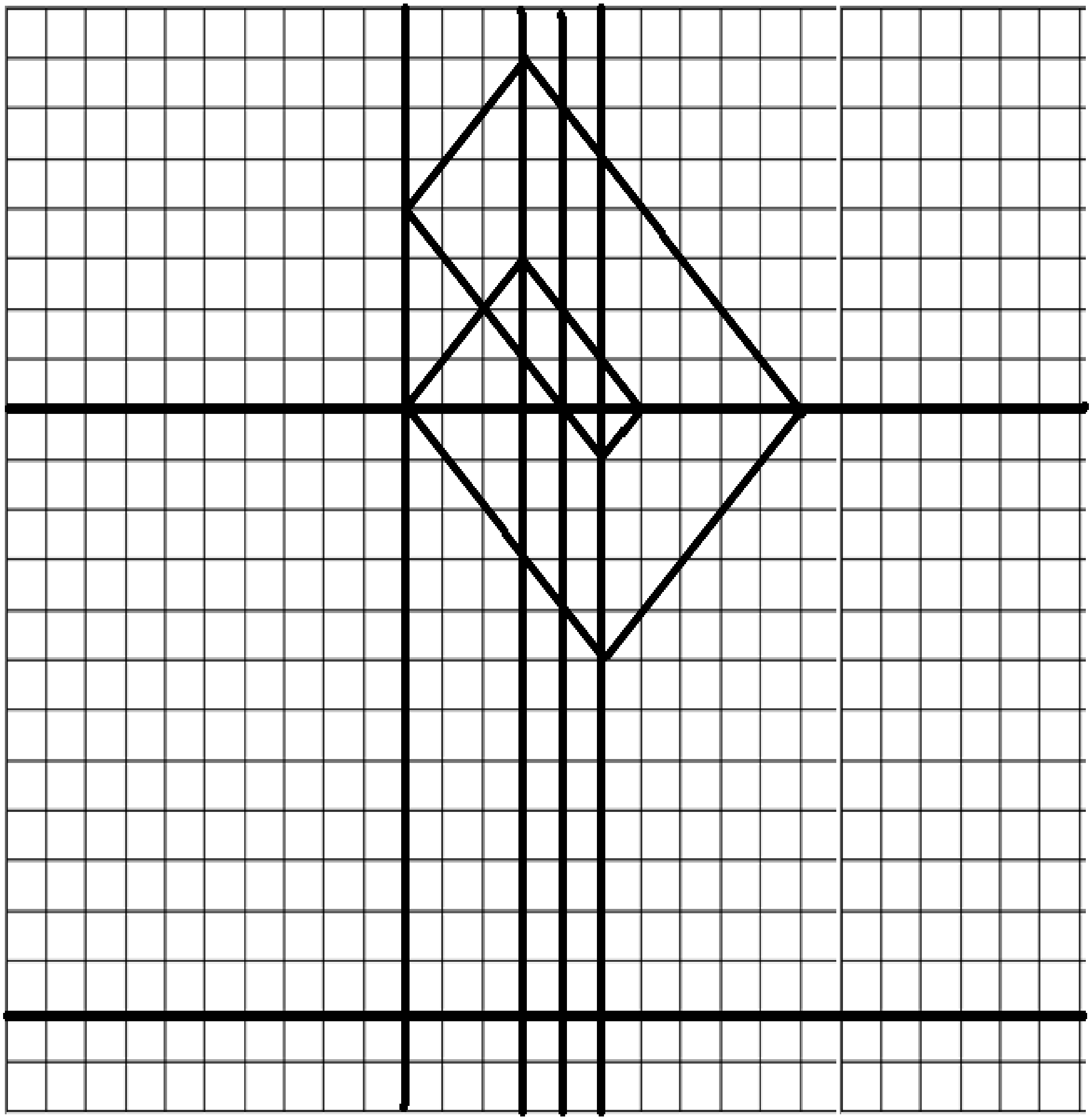
#4 $U_N = 1.0$ $P_N = 40.0$ $P_M = 30$ $U_M = -1.0$

#5 $U_N = 1.25$ $P_N = 7.5$ $P_M = 30$ $U_M = 0.0$

#6 $U_N = 1.0$ $P_N = 20.0$ $P_M = 30$ $U_M = 3.5$



NO VALVE CASE



VALVE CASE

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FLUID MECHANICS II

HOMEWORK #3

A certain steel rod has a 5cm diameter and a 1m length. It falls like a spear under gravity in standard atmosphere. Determine its terminal speed. Determine its terminal Mach number. Assume it has a wake drag coefficient of 1 for the wake at the back. Assume that stagnation pressure acts over its front area.

A certain divers air bottle has a pressure of 20 MPa and a temperature of 20 ^{o}C when fully charged. It weighs 18 kg. Imagine it has a valve with a throat diameter of 1.27 cm. How much thrust would the bottle generate if the valve was suddenly opened. Note that there is no cone downstream of the throat. Would it be able to lift its own weight if oriented vertically?

A certain air pipe has a diameter of 15cm. Its friction factor is 0.01. The Mach number at one location in the pipe is 5. The pressure at this location is 50 BAR. At what location downstream would the flow become sonic? Do adiabatic and isothermal cases? Repeat for a starting Mach number of 0.2. Use space stepping on the M^2 and P equations to find how pressure varies from the starting location to the final location.

QUESTION #1

One could use stagnation point flow theory to get an accurate estimate of the load on the bottom face. One could use conical expansion wave theory to get the pressure in the wake. Both are beyond the scope of this homework. Here we assume the stagnation pressure acts over the bottom face and we assume the wake contains standard atmosphere air. We can calculate the weight of the rod. We assume a terminal speed. Knowing the temperature of air, we can calculate the sound speed. This allows us to calculate a Mach Number. If it is less than 1, we use isentropic equations to get pressure at the stagnation point. If it is greater than 1, we use the blunt object equations to get the pressure. We iterate on terminal speed until the pressure load balances the weight.

Isentropic Equations

$$T_S/T_U = [\ (1 + [(k-1)/2]\ M_U M_U) / (1 + [(k-1)/2]\ M_S M_S)\ ]$$

$$P_S/P_U = [T_S/T_U]^x \qquad x = k/(k-1)$$

Blunt Object Equations

$$P_D/P_U = 1 + [2k/(k+1)]\ (M_U M_U - 1)$$

$$M_D M_D = [(k-1)\ M_U M_U + 2]\ /\ [2k\ M_U M_U - (k-1)]$$

$$T_S/T_D = [\ (1 + [(k-1)/2]\ M_D M_D) / (1 + [(k-1)/2]\ M_S M_S)\ ]$$

$$P_S/P_D = [T_S/T_D]^x \qquad x = k/(k-1)$$

QUESTION #2

Flow through an ideal rocket nozzle is isentropic. The equations connecting an upstream point G to a downstream point H are:

$$T_H/T_G = [\ (1 + [(k-1)/2]\ M_G M_G) \ /\ (1 + [(k-1)/2]\ M_H M_H)\ ]$$

$$P_H/P_G = [T_H/T_G]^x \qquad x = k/(k-1)$$

The thrust of a nozzle with no cone is:

$$F = \dot{M}\, U_T + (P_T - P_A)\, A_T$$

It contains a momentum part and a pressure part.

At the throat where M=1 the mass flow rate is

$$\dot{M} = \rho_T\, A_T\, U_T = [P_T/RT_T]\, A_T\, \sqrt{[kRT_T]}$$

The isentropic equations with G equal to the combustion chamber and H equal to the throat give P_T and T_T.

QUESTION #3

Thermodynamics gives for pipe flow with friction

$$\Delta M^2/M^2 = + kM^2[1+[(k-1)/2]M^2] /[1-M^2] f\Delta x/D$$

$$\Delta P/P = - kM^2[1+(k-1) M^2]/[2(1-M^2)] f\Delta x/D$$

Each equation is of the form

$$\Delta G = H \Delta x$$

Simple space stepping gives

$$[G_{NEW} - G_{OLD}] = H_{OLD} [x_{NEW} - x_{OLD}]$$

$$G_{NEW} = G_{OLD} + \Delta x H_{OLD}$$

For adiabatic flow, the distance to M=1 follows from

$$fL^*/D = (1-M^2)/(kM^2)+[(k+1)/(2k)] \ln[(k+1)M^2/(2+(k-1)M^2)]$$

while for isothermal flow it follows from

$$fL^*/D = (1-kM^2)/(kM^2) + \ln[kM^2]$$

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HOMEWORK #4

Map 16 evenly spaced points on a circle with radius R=0.25 and horizontal offset n=0.05 and vertical offset m=0 to a foil plane using the Joukowsky mapping function. The foil is moving through water at a speed S equal to 10km/hr. The angle of attack of the foil θ is 15°. Calculate the pressure midway between consecutive points on the foil. Use the pressures to estimate the lift on the foil. Do the calculations using Matlab m code. Compare the estimated lift with the theoretical lift.

A long green house has radius R equal to 5m and length L equal to 50m. A strong wind with speed S equal to 100km/hr is blowing perpendicular to the long axis of the green house. Assume that the flow separation angle σ is 110°. Determine the lift and drag on the green house. Check the answers using numerical integration. Do the calculations using Matlab m code.

The coordinates of each point in the standard frame are:

$$\mathbf{X} = -R \, \mathrm{Cos}[\gamma] \qquad \mathbf{Y} = + R \, \mathrm{Sin}[\gamma]$$

The coordinates in the offset frame are:

$$x = \mathbf{X} - n \qquad y = \mathbf{Y} + m$$

The foil coordinates are:

$$\alpha = x + xa^2/(x^2+y^2) \qquad \beta = y - ya^2/(x^2+y^2)$$

where

$$a = \sqrt{[R^2-m^2]} - n$$

Application of Bernoulli gives

$$\rho/2 \; [\; S^2 - (\partial\varphi/\partial c)^2 \;]$$

An approximation of this

$$\rho/2 \; [\; S^2 - (\Delta\varphi/\Delta c)^2 \;]$$

where

$$\Delta c = \sqrt{[\Delta\alpha^2 + \Delta\beta^2]}$$

The potential for the flow around the circle is

$$\varphi = S\ X + S\ XR^2/[X^2+Y^2] \ + \ \Gamma/[2\pi]\ \sigma$$

where

$$X = \mathbf{X}\ \mathrm{Cos}\Theta + \mathbf{Y}\ \mathrm{Sin}\Theta$$

On the circle the potential is

$$\varphi = 2\ S\ X + \ \Gamma/[2\pi]\ \sigma$$

When a point on the circle is mapped to the foil plane, it carries its φ with it. This implies

$$\Delta\varphi = 2\ S\ \Delta X + \ \Gamma/[2\pi]\ \Delta\sigma$$

The pressure lift is

$$\Sigma\ \ P\Delta c\ \mathrm{Sin}(\boldsymbol{\theta}-\Theta)$$

The circulation needed to make the flow look realistic at the trailing edge of the foil is

$$\Gamma = 4\pi SR\ \mathrm{Sin}\kappa$$

where

$$\kappa = \Theta + \varepsilon \qquad \varepsilon = \tan^{-1}\ [m/(n+a)]$$

The theoretical lift is: $\rho S\ \Gamma$.

The potential function for the greenhouse flow is

$$\varphi = S\ X + S\ XR^2/[X^2+Y^2]$$

On the cylinder this reduces to

$$\varphi = 2\ S\ X$$

On the cylinder, geometry gives

$$X = -\ R\ \cos\sigma \qquad c=R\sigma$$

This allows us to rewrite the potential function as

$$\varphi = -2\ S\ R\ \cos[c/R]$$

The speed of the fluid over the cylinder is

$$\partial\varphi/\partial c = 2\ S\ \sin\sigma$$

Application of Bernoulli gives

$$P/\rho \ + \ (\partial\varphi/\partial c)^2/2 \ = \ S^2/2$$

Manipulation gives

$$P = \rho/2\ [\ S^2 - (\partial\varphi/\partial c)^2\ ]$$

$$P = \rho/2\ S^2\ [\ 1 - 4\ \sin^2\sigma\ ]$$

This is only good up to the separation angle σ. In the wake downstream, the pressure is approximately constant and is:

$$P = \rho/2\ S^2\ [\ 1 - 4\ \mathrm{Sin}^2\sigma\ ]$$

Pressure acts over the incremental area:

$$dA = L\ Rd\sigma$$

This gives the incremental force:

$$dF = P\ dA = P\ L\ R\ d\sigma$$

The incremental lift is:

$$-\ dF\ \mathrm{Sin}\sigma\ =\ -\ P\ L\ R\ \mathrm{Sin}\sigma\ d\sigma$$

and the incremental drag is:

$$+\ dF\ \mathrm{Cos}\sigma\ =\ +\ P\ L\ R\ \mathrm{Cos}\sigma\ d\sigma$$

Integration gives the total lift

$$-\int P\ L\ R\ \mathrm{Sin}\sigma\ d\sigma$$

and the total drag

$$+\int P\ L\ R\ \mathrm{Cos}\sigma\ d\sigma$$

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HOMEWORK #5

A certain hydrodynamic lubrication thrust bearing has 3 pads. Each pad has a 60° wedge angle. The outer radius of each pad is 2m and the inner radius is 1m. The front gap is 2mm and the back gap is 1mm. The shaft speed is 500 RPM. The oil viscosity is 0.1 Ns/m^2. Use a 9 point CFD scheme to determine the load supported by the bearing. Check load using code pad. Repeat for the case where the sides of the bearing are blocked.

A certain slab of porous rock is very wide but just 1 kilometer long. Its depth is 10m. One can assume that its sides and top and bottom are blocked. The slab contains oil with viscosity $1Ns/m^2$. The permeability of the slab rock is 10^{-12} m^2. The hydrostatic pressure at the inlet to the slab is 60BAR and at its outlet is 30BAR. Determine the flow rate of oil through a kilometer wide section of the slab in barrels per minute.

The die of a certain wire coating machine is long and has a diameter 5mm. The wire diameter is 2mm. The speed of the wire is 1m/s. What is the diameter of the coating?

Derive the viscosity equation for a disk viscosity meter. Derive the viscosity equation for a drum viscosity meter. Derive the viscosity equation for a capillary tube viscosity meter.

HYDRODYNAMIC LUBRICATION THRUST BEARING

Reynolds Equation for a cylindrical geometry is

$$r\ \partial/\partial c\ (h^3\ \partial P/\partial c) + \partial/\partial r\ (rh^3\ \partial P/\partial r) = 6\mu S\ \partial h/\partial\Theta$$

Application of a North South East West CFD scheme gives

$$r_P\ [\ [(h_E+h_P)/2]^3\ (P_E-P_P)/\Delta c - [(h_W+h_P)/2]^3\ (P_P-P_W)/\Delta c\ ]\ /\ \Delta c$$

$$+$$

$$[\ [h_P]^3[r_N+r_P]/2\ (P_N-P_P)/\Delta r - [h_P]^3[r_S+r_P]/2\ (P_P-P_S)/\Delta r\ ]\ /\ \Delta r$$

$$= 6\mu S\ (h_E - h_W)/[2\Delta\Theta]$$

Manipulation gives the template

$$P_P = (A\ P_E + B\ P_W + C\ P_N + D\ P_S + H)\ /\ (A + B + C + D)$$

where

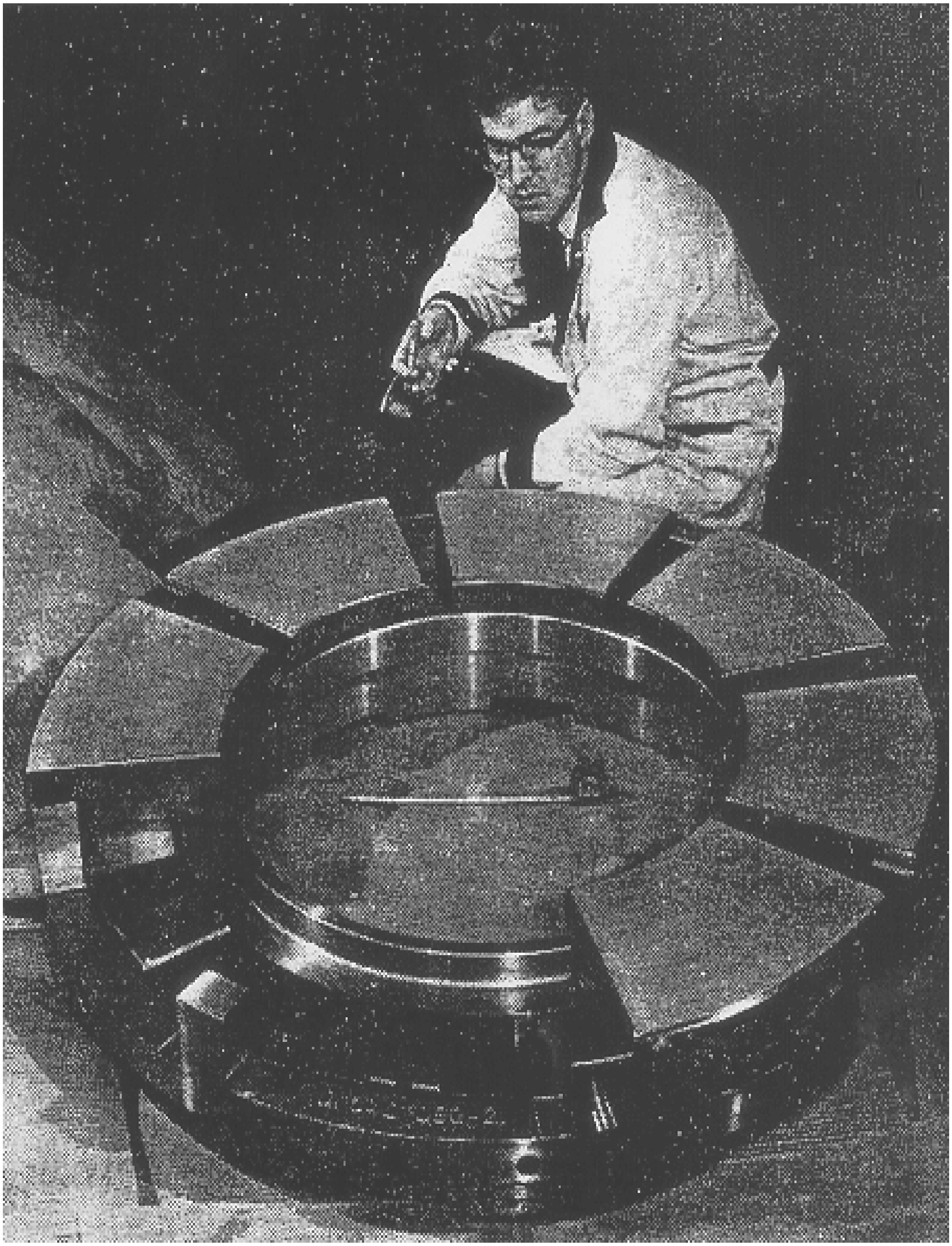
$$A = [(h_E+h_P)/2]^3\ r_P\ /\ [\Delta c]^2$$

$$B = [(h_W+h_P)/2]^3\ r_P\ /\ [\Delta c]^2$$

$$C = [h_P]^3\ [(r_N+r_P)/2]\ /\ [\Delta r]^2$$

$$D = [h_P]^3\ [(r_S+r_P)/2]\ /\ [\Delta r]^2$$

$$H = -\ 6\mu\ r_P\omega\ (h_E-h_W)/[2\Delta\Theta]$$



```
%
%     HYDRODYNAMIC THRUST BEARING
%
      clear all
      NR=5;NA=5;NIT=222;
      MR=NR-1;MA=NA-1;
      PI=3.14159; DENSITY=880.0;
      GRAVITY=9.81; VISCOSITY=0.1;
      RPM=500.0; RPS=RPM/60.0;
      RIN=1.0;ROUT=2.0;
      AIN=+60.0;AOUT=+120.0;
      AIN=AIN/180.0*PI;
      AOUT=AOUT/180.0*PI;
      ONE=0.002;TWO=0.001;
      DELR=(ROUT-RIN)/MR;
      DELA=(AOUT-AIN)/MA;
      GAP=TWO-ONE;
      SPAN=AOUT-AIN;
      SLOPE=GAP/SPAN;
      PRESSURE=0.0;
      RNODE=RIN;
      for JJ=1:NR
      ANODE=AIN;
      for II=1:NA
      R(II,JJ)=RNODE;
      HEAD(II,JJ)=0.0;
      CHANGE=ANODE-AIN;
      P(II,JJ)=PRESSURE;
      X(II,JJ)=-RNODE*cos(ANODE);
      Y(II,JJ)=+RNODE*sin(ANODE);
      H(II,JJ)=ONE+SLOPE*CHANGE;
      ANODE=ANODE+DELA;
      end
      RNODE=RNODE+DELR;
      end
```

```
        THRUST=0.0;
        for IT=1:NIT
        for JJ=2:MR
        for II=2:MA
        DELC=DELA*R(II,JJ);
        AREA=DELR*DELC;
        SPEED=RPS*2.0*PI*R(II,JJ);
        A=((H(II+1,JJ)+H(II,JJ))/2.0)^3/DELC^2;
        B=((H(II,JJ)+H(II-1,JJ))/2.0)^3/DELC^2;
        C=((H(II,JJ+1)+H(II,JJ))/2.0)^3/DELR^2;
        D=((H(II,JJ)+H(II,JJ-1))/2.0)^3/DELR^2;
        C=C*(R(II,JJ+1)+R(II,JJ))/2.0;
        D=D*(R(II,JJ)+R(II,JJ-1))/2.0;
        A=A*R(II,JJ); B=B*R(II,JJ);
%       if(JJ==2) D=0.0;end;
%       if(JJ==MR) C=0.0;end;
        S=-6.0*VISCOSITY*SPEED*SLOPE;
        AA=A*P(II+1,JJ); BB=B*P(II-1,JJ);
        CC=C*P(II,JJ+1); DD=D*P(II,JJ-1);
        P(II,JJ)=(S+AA+BB+CC+DD)/(A+B+C+D);
        DELP=P(II,JJ)-PRESSURE;
        HEAD(II,JJ)=DELP/DENSITY/GRAVITY;
        FORCE=(P(II,JJ)-PRESSURE)*AREA;
        if(IT==NIT) THRUST=THRUST+FORCE;end;
        end
        end
        end
        THRUST=THRUST*3.0
        surf(X,Y,HEAD)
        P
```

POINT #1

A = 5.0086e-08

B = 7.6941e-08

C = 1.1791e-07

D = 9.6469e-08

S = 0.0375

POINT #2

A = 3.0343e-08

B = 5.0086e-08

C = 7.4250e-08

D = 6.0750e-08

S = 0.0375

POINT #3

A = 1.6619e-08

B = 3.0343e-08

C = 4.2969e-08

D = 3.5156e-08

S = 0.0375

POINT #4

A = 4.1738e-08

B = 6.4117e-08

C = 1.3934e-07

D = 1.1791e-07

S = 0.0450

POINT #5

A = 2.5286e-08

B = 4.1738e-08

C = 8.7750e-08

D = 7.4250e-08

S = 0.0450

POINT #6

A = 1.3849e-08

B = 2.5286e-08

C = 5.0781e-08

D = 4.2969e-08

S = 0.0450

POINT #7

A = 3.5775e-08

B = 5.4958e-08

C = 1.6078e-07

D = 1.3934e-07

S = 0.0525

POINT #8

A = 2.1674e-08

B = 3.5775e-08

C = 1.0125e-07

D = 8.7750e-08

S = 0.0525

POINT #9

A = 1.1871e-08

B = 2.1674e-08

C = 5.8594e-08

D = 5.0781e-08

S = 0.0525

```
>> pad

THRUST =

1.8489e+06


P =

1.0e+06 *

         0         0         0         0         0
    0    0.3798    0.4904    0.3692         0
    0    0.6858    0.8822    0.6565         0
    0    0.8473    1.1096    0.8611         0
         0         0         0         0         0
```

```
>> pad

THRUST =

6.2074e+06


P =

1.0e+06 *

     0         0         0         0         0
0    1.3684    1.4718    1.5569         0
0    2.4782    2.6625    2.8129         0
0    2.6233    2.8676    3.0724         0
     0         0         0         0         0
```

POROUS MEDIA FLOWS

The Darcy Law gives

$$\mathbf{v} = - k/\mu \, \nabla P = - K \, \nabla P$$

where **v** is the velocity vector, P is pressure, k is permeability and μ is viscosity. Conservation of Mass gives

$$\nabla . \; \mathbf{v} = 0$$

Substitution into Mass gives

$$\nabla . [K \; \nabla P] = 0$$

$$\partial/\partial x \; [K \; \partial P/\partial x] + \partial/\partial y \; [K \; \partial P/\partial y] + \partial/\partial z \; [K \; \partial P/\partial z] = 0$$

For a rectangular slab with blocked sides **v** is constant. This implies that the pressure gradient is also constant. So the pressure at the midpoint is an average of the inlet and outlet pressures. The flow rate is

$$Q = \mathbf{v} \; A = - K \; [P_O - P_I]/L \quad [H \; W]$$

WIRE COATING

Assume that the die is long and there is no axial variation inside it. In this case Conservation of Momentum is:

$$0 = 1/r \, d/dr \, [\, r \, \mu \, dU/dr \,]$$

Integration gives

$$U = W \ln[r/R_D] / \ln[R_W/R_D]$$

The volumetric flow rate within the die is

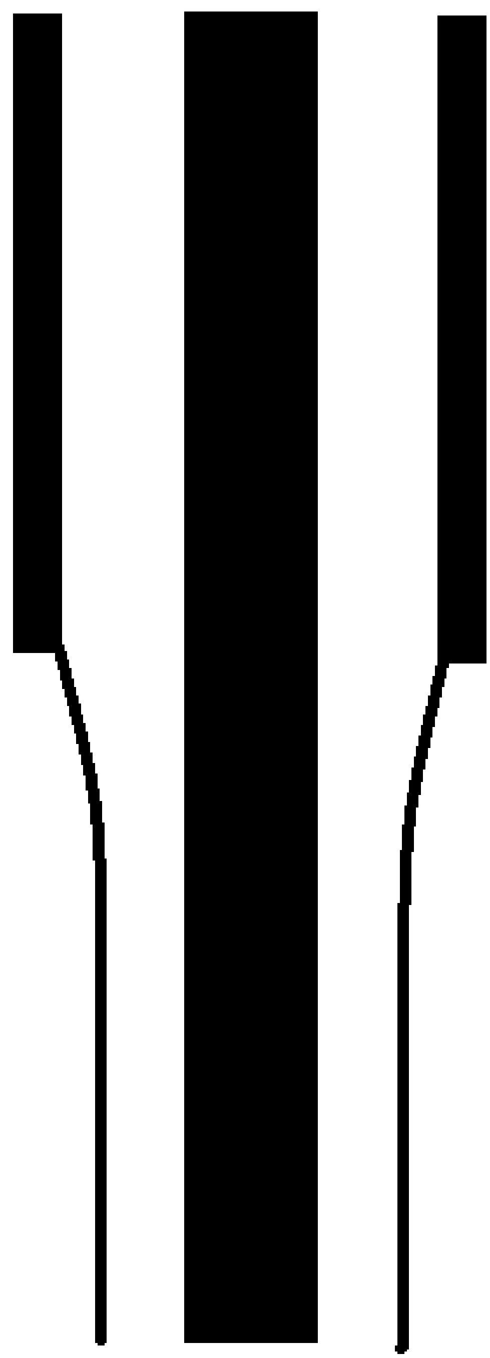
$$Q = \int 2\pi r \, U \, dr$$

The volumetric flow rate far downstream is

$$Q = W \, (\, \pi \, R_C \, R_C \; - \; \pi \, R_W \, R_W \,)$$

Equating these flow rates gives

$$R_C \, R_C = [\, R_D \, R_D \; - \; R_W \, R_W \,] / [\, 2 \ln[R_D/R_W] \,]$$



DISK VISCOMETER

A disk viscometer consists of a disk which rotates inside a can. A liquid fills the gap between them. Let the gap be h. The torque required to rotate the disk is:

$$T = \int r \ \mu \ r\omega/h \ 2\pi \ r \ dr$$

With known geometry and measured torque, one gets

$$\mu = [\ 2\ T\ h\ ] \ / \ [\pi\ R^4\ \omega]$$

DRUM VISCOMETER

A drum viscometer consists of a drum which rotates inside a sleeve. A liquid fills the gap between them. Let the gap be h. The torque required to rotate the drum is:

$$T = R \ \mu \ R\omega/h \ 2\pi \ RL$$

With known geometry and measured torque, one gets

$$\mu = [\ T\ h\ ] \ / \ [\ 2\pi\ R^3\ L\ \omega\ ]$$

CAPILLARY TUBE VISCOMETER

Conservation of Mass considerations give

$$\partial U/\partial s = 0$$

Conservation of Momentum considerations give

$$\partial P/\partial s = 1/r\ \partial/\partial r\ (r\ \mu\ \partial U/\partial r)$$

Integration gives

$$U = -\ [R^2-r^2]/[4\mu]\ \partial P/\partial s$$

The volumetric flow rate

$$Q = \int U\ 2\pi r\ dr = -\ [\pi R^4]/[8\mu]\ \partial P/\partial s$$

$$Q = -\ [\pi R^4]/[8\mu]\ [-\rho g H]/L = [\pi R^4]/[8\mu]\ [\rho g H]/L$$

Manipulation gives

$$\mu = [\rho g H][\pi R^4]\ /\ [8QL]$$

TURBOMACHINE TUTORIAL

A certain water sprinkler has 4 pipes each with length 0.5m. The diameter of each pipe is 2cm. Each pipe has a 90° bend at its outlet. This bend makes an angle of 45° up from the horizontal. The overall flow rate Q is 8 L/s. Derive an equation for the power of the turbine and the peak power RPM. Determine the RPM for peak power. Determine the no load or free wheel RPM of the turbine. Plot the power versus RPM of the turbine?

Conservation of Rotational Momentum for fluid moving in an inertial reference frame gives for each pipe:

$$\Delta\ [\rho Q^{*}\ V_T\, R] = T$$

where ρ is the density of water, $Q^{*} = Q/4$ is the flow rate, V_T is the tangential flow velocity and R is the distance out from the axis of rotation. At the outlet, the tangential velocity is

$$Q^{*}/A\ \mathrm{Cos}\beta\ \mathrm{Cos}\theta + R\ \omega$$

where A is the pipe area, ω is the rotational speed, β is 180° and θ is 45°. At the inlet, the tangential velocity is zero. So torque becomes

$$- \rho Q^* [Q^*/A \cos\beta \cos\theta + R \omega] R = T$$

The power output of one pipe is

$$\mathbf{P} = T \omega = - \rho Q^* [Q^*/A \cos\beta \cos\theta + R \omega] R \omega$$

$$\mathbf{P} = T \omega = \rho Q^* [V_J - V_B] V_B$$

$$V_J = Q^*/A \cos\theta \quad V_B = R \omega$$

Differentiation of the power equation with respect to V_B shows that power peaks when V_B is half V_J. So the peak power of one pipe is

$$\mathbf{P}^* = \rho Q^* [V_J]^2/4$$

The power and peak power of the turbine are:

$$\mathbf{P} = \rho Q [V_J - V_B] V_B \qquad \mathbf{P}^* = \rho Q [V_J]^2/4$$

The free wheel speed of the turbine

$$\mathbf{P} = 0 \quad V_J = V_B$$

$$\omega = Q^*/A \cos\theta / R$$

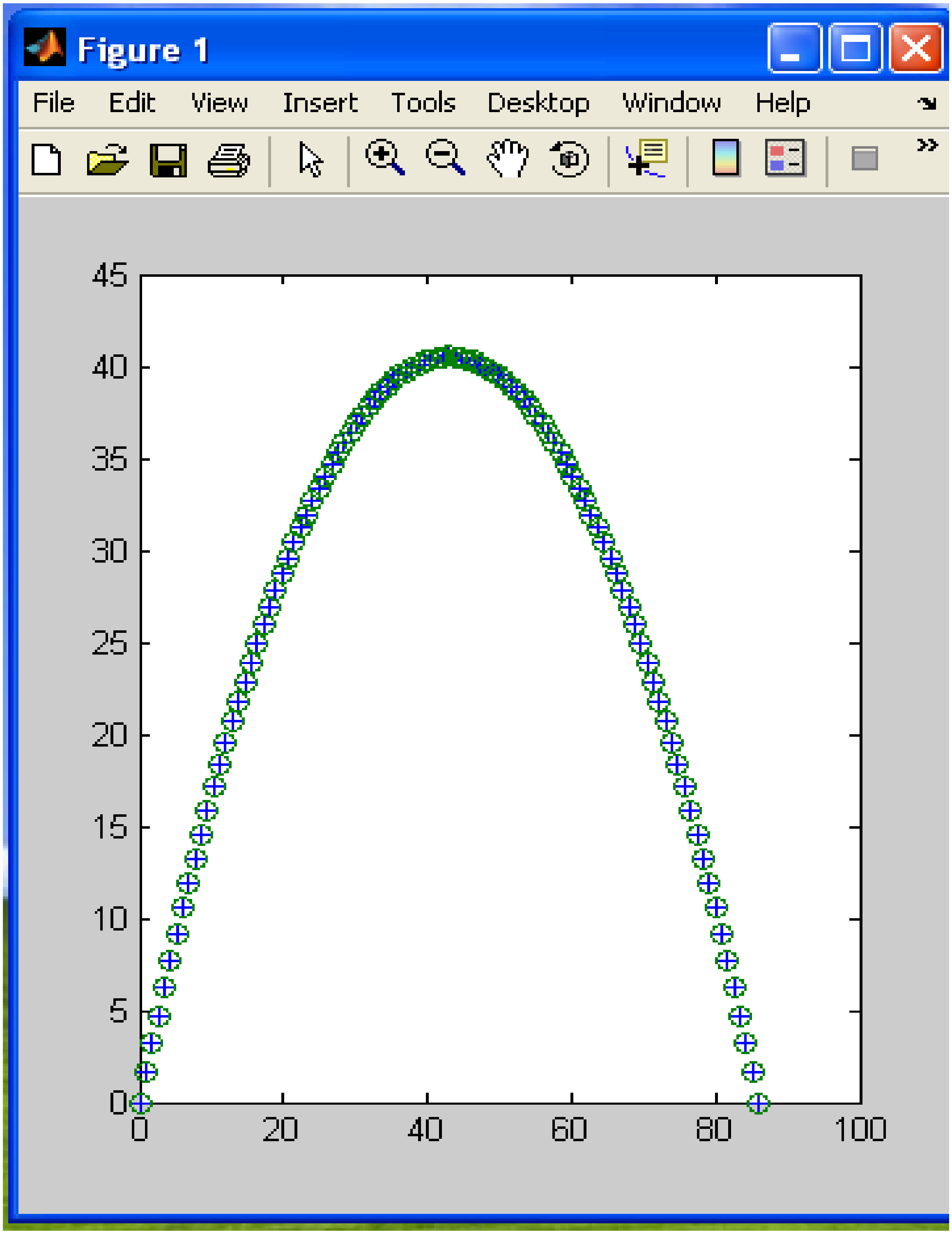


Figure 1
File
Edit
View
Insert
Tools
Desktop
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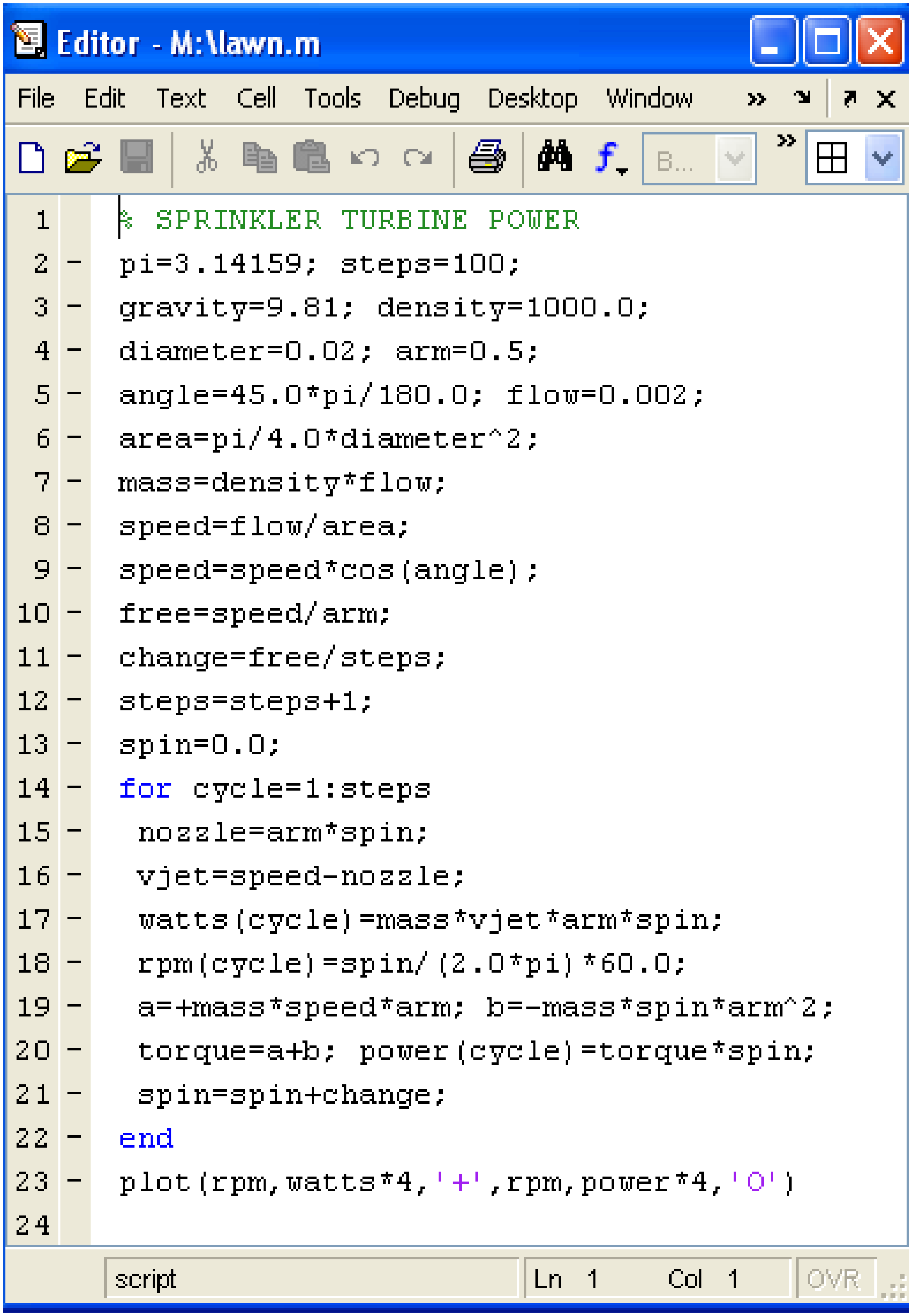
Help
45
40
35
30
25
20
15
10
5
0
0
20
40
60
80
100

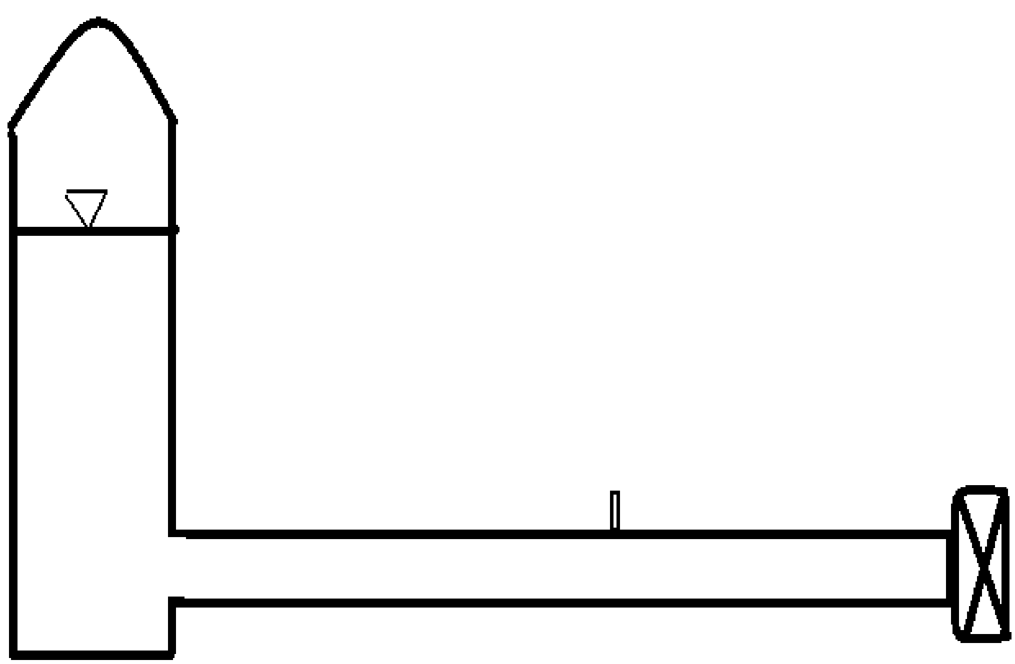


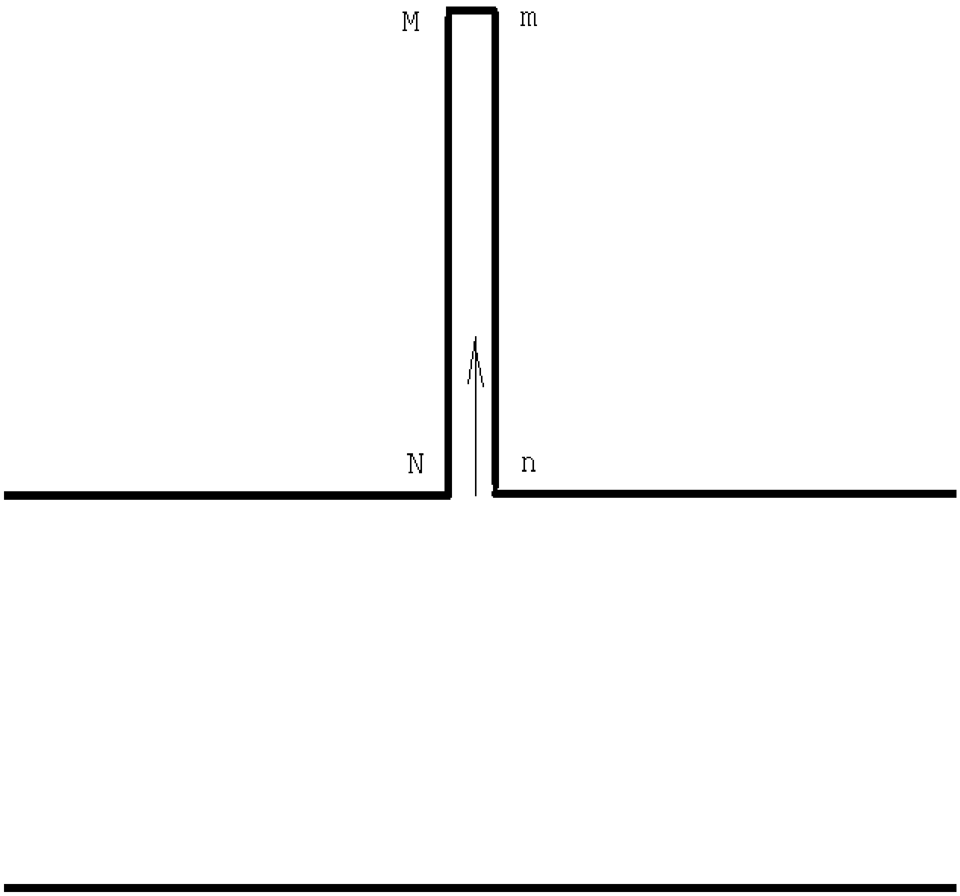
Editor - M:\lawn.m

```
% SPRINKLER TURBINE POWER
pi=3.14159; steps=100;
gravity=9.81; density=1000.0;
diameter=0.02; arm=0.5;
angle=45.0*pi/180.0; flow=0.002;
area=pi/4.0*diameter^2;
mass=density*flow;
speed=flow/area;
speed=speed*cos(angle);
free=speed/arm;
change=free/steps;
steps=steps+1;
spin=0.0;
for cycle=1:steps
 nozzle=arm*spin;
 vjet=speed-nozzle;
 watts(cycle)=mass*vjet*arm*spin;
 rpm(cycle)=spin/(2.0*pi)*60.0;
 a=+mass*speed*arm; b=-mass*spin*arm^2;
 torque=a+b; power(cycle)=torque*spin;
 spin=spin+change;
end
plot(rpm,watts*4,'+',rpm,power*4,'O')
```

WATERHAMMER TUTORIAL

A small dead end pipe is attached to a large water pipe as shown in the sketch. The initial pressure in the large pipe and the dead end pipe is 20 BAR. The initial flow speed in the large pipe is 1 m/s and in the dead end pipe is 0 m/s. The ρa for each pipe is 10 BAR/[m/s] The large pipe undergoes a sudden valve closure. It generates a surge wave followed by a back flow wave followed by a suction wave followed by an inflow wave. As these waves pass the entrance of the small pipe, they generate waves inside the small pipe. Because of friction these waves decay very quickly. The small pipe is so small it does not influence the waves in the large pipe. Using graphical water hammer analysis determine the pressure and velocity at the ends of the small pipe as each wave passes by.





F WAVE

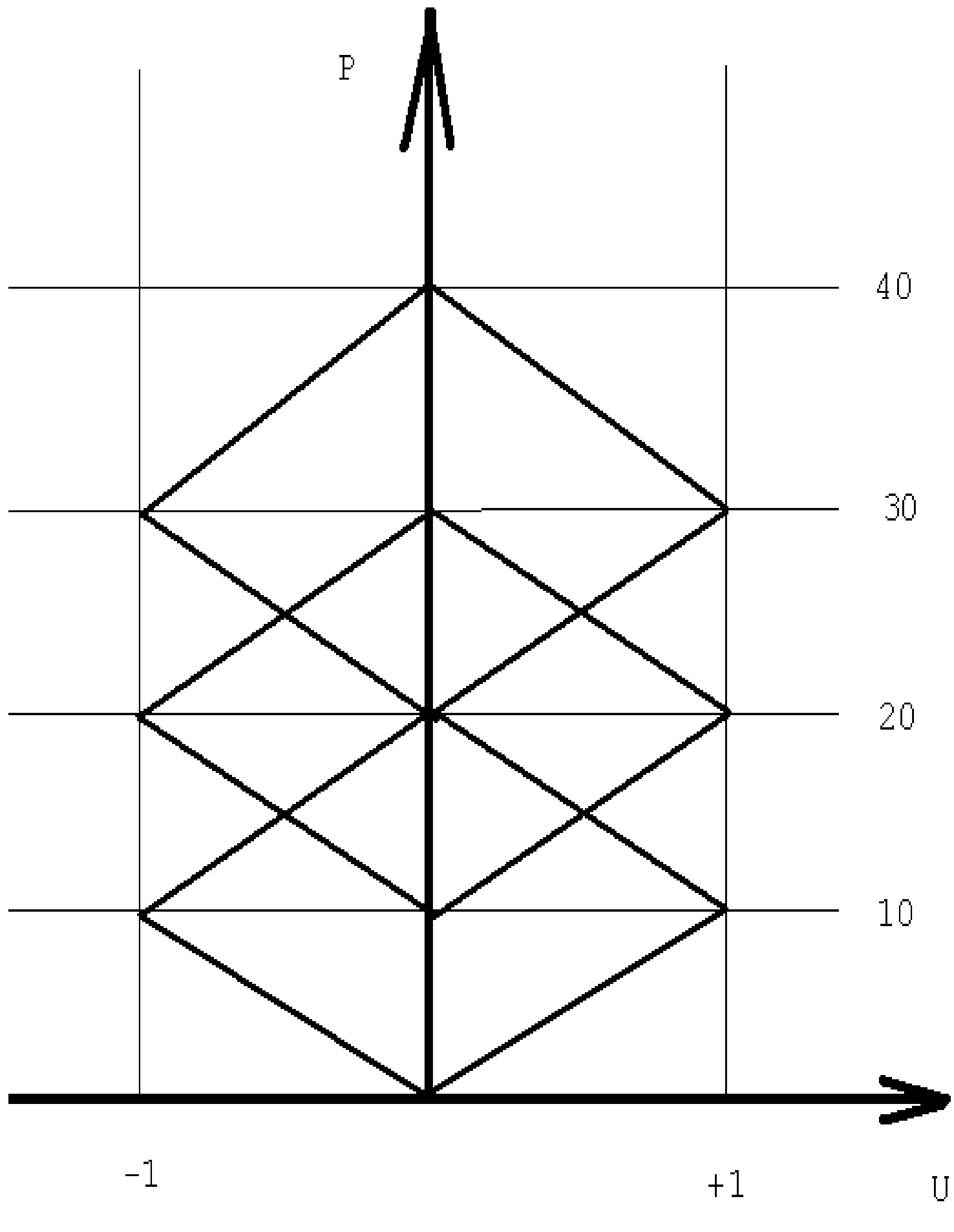
$$[P_N - P_m] = + \rho a\ [U_N - U_m]$$

$$U_N = U_m + [P_N - P_m]/[\rho a]$$

f WAVE

$$[P_M - P_n] = - \rho a\ [U_M - U_n]$$

$$P_M = P_n - \rho a\ [U_M - U_n]$$


P
40
30
20
10
-1
+1
U

SURGE WAVE

When the surge wave in the large pipe passes the entrance of the small pipe, it causes an inflow wave into the small pipe. The surge wave in the large pipe also moves into the small pipe.

When the inflow wave hit the dead end of the small pipe, it generates an extra surge wave.

When this surge wave reaches the entrance of the small pipe, it generates a back flow wave.

When the back flow wave reaches the dead end of the small pipe, it creates a suction wave.

When the suction wave reaches the entrance of the small pipe, it creates an inflow wave.

If there was no friction, the cycle would repeat over and over. However, because of friction, the waves quickly decay. The velocity becomes zero and the pressure becomes the surge pressure level.

BACKFLOW WAVE

When the backflow wave in the large pipe passes the entrance of the small pipe, the pressure drops to its initial level and this causes a backflow flow wave from the small pipe.

When the backflow wave hit the dead end of the small pipe, it generates a suction wave.

When the suction wave reaches the entrance of the small pipe, it generates an inflow wave.

When the inflow wave reaches the dead end of the small pipe, it creates a surge wave.

When the surge wave reaches the entrance of the small pipe, it creates a back flow wave.

If there was no friction, the cycle would repeat over and over. However, because of friction, the waves quickly decay. The velocity becomes zero and the pressure becomes the initial pressure level.

SUCTION WAVE

When the suction wave in the large pipe passes the entrance of the small pipe, it causes a backflow flow wave from the small pipe. The suction wave in the large pipe also moves into the small pipe.

When the backflow wave hit the dead end of the small pipe, it generates an extra suction wave.

When this suction wave reaches the entrance of the small pipe, it generates an inflow wave.

When the inflow wave reaches the dead end of the small pipe, it creates a surge wave.

When the surge wave reaches the entrance of the small pipe, it creates a backflow wave.

If there was no friction, the cycle would repeat over and over. However, because of friction, the waves quickly decay. The velocity becomes zero and the pressure becomes the suction pressure level.

INFLOW WAVE

When the inflow wave in the large pipe passes the entrance of the small pipe, the pressure rises to its initial level and this causes an inflow wave into the small pipe.

When the inflow wave hit the dead end of the small pipe, it generates a surge wave.

When the surge wave reaches the entrance of the small pipe, it generates a backflow wave.

When the backflow wave reaches the dead end of the small pipe, it creates a suction wave.

When the suction wave reaches the entrance of the small pipe, it creates an inflow wave.

If there was no friction, the cycle would repeat over and over. However, because of friction, the waves quickly decay. The velocity becomes zero and the pressure becomes the initial pressure level.

GAS DYNAMICS TUTORIAL

The pressure ratio across a shock wave generated by an explosion was measured to be 4.5. The explosion took place in standard atmosphere. Determine: the Mach number of the shock wave; the absolute flow speed behind the shock wave; the stagnation point pressure on an object placed in the flow.

The normal shock plot gives the Mach number to be around 2.0. It gives the ratio of the temperature behind the shock wave over the temperature in front of it to be around 1.6. It gives the ratio of the velocity in front over the velocity behind to be around 2.6. It gives the relative Mach number behind to be around 0.55. Calculations give:

$$T_U = 293K \qquad a_U = \sqrt{[kRT_U]} = 343 \text{ m/s}$$

$$T_D = 1.6\ T_U = 468K \qquad a_D = \sqrt{[kRT_D]} = 443 \text{ m/s}$$

$$U_U = M_U\ a_U = 686 \text{ m/s} \qquad U_D = M_D\ a_D = 238 \text{ m/s}$$

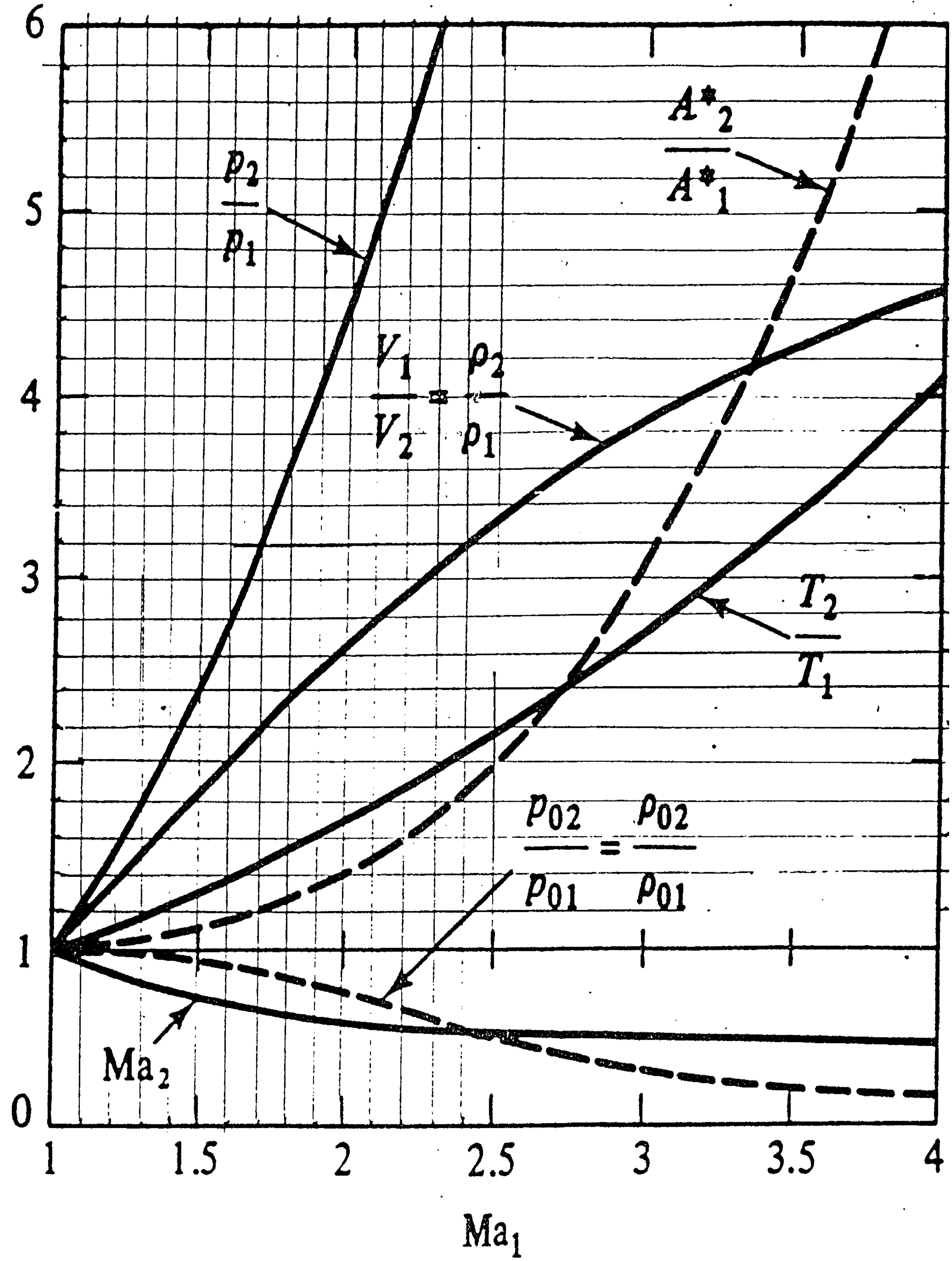
$$U_F = U_U - U_D = 448 \text{ m/s} \qquad T_F = T_D \qquad a_F = a_D$$

$$M_F \text{ approx } 1 \qquad P_F \text{ approx } 4.5 \text{ BAR}$$

$$T_S/T_F = (1 + 0.2 * 1^2) / (1 + 0.2 * 0^2) = 1.2$$

$$P_S/P_F = [T_S/T_F]^x \qquad x = k/(k-1)$$

$$P_S = P_F\ [1.2]^{3.5} = 8.5 \text{ BAR}$$



6
5
4
3
2
1
0
$\frac{p_2}{p_1}$
$\frac{A^*_2}{A^*_1}$
$\frac{V_1}{V_2} = \frac{\rho_2}{\rho_1}$
$\frac{T_2}{T_1}$
$\frac{p_{02}}{p_{01}} = \frac{\rho_{02}}{\rho_{01}}$
Ma_2
1
1.5
2
2.5
3
3.5
4
Ma_1