FLUID STRUCTURE INTERACTIONS

QUIZ #1 2013

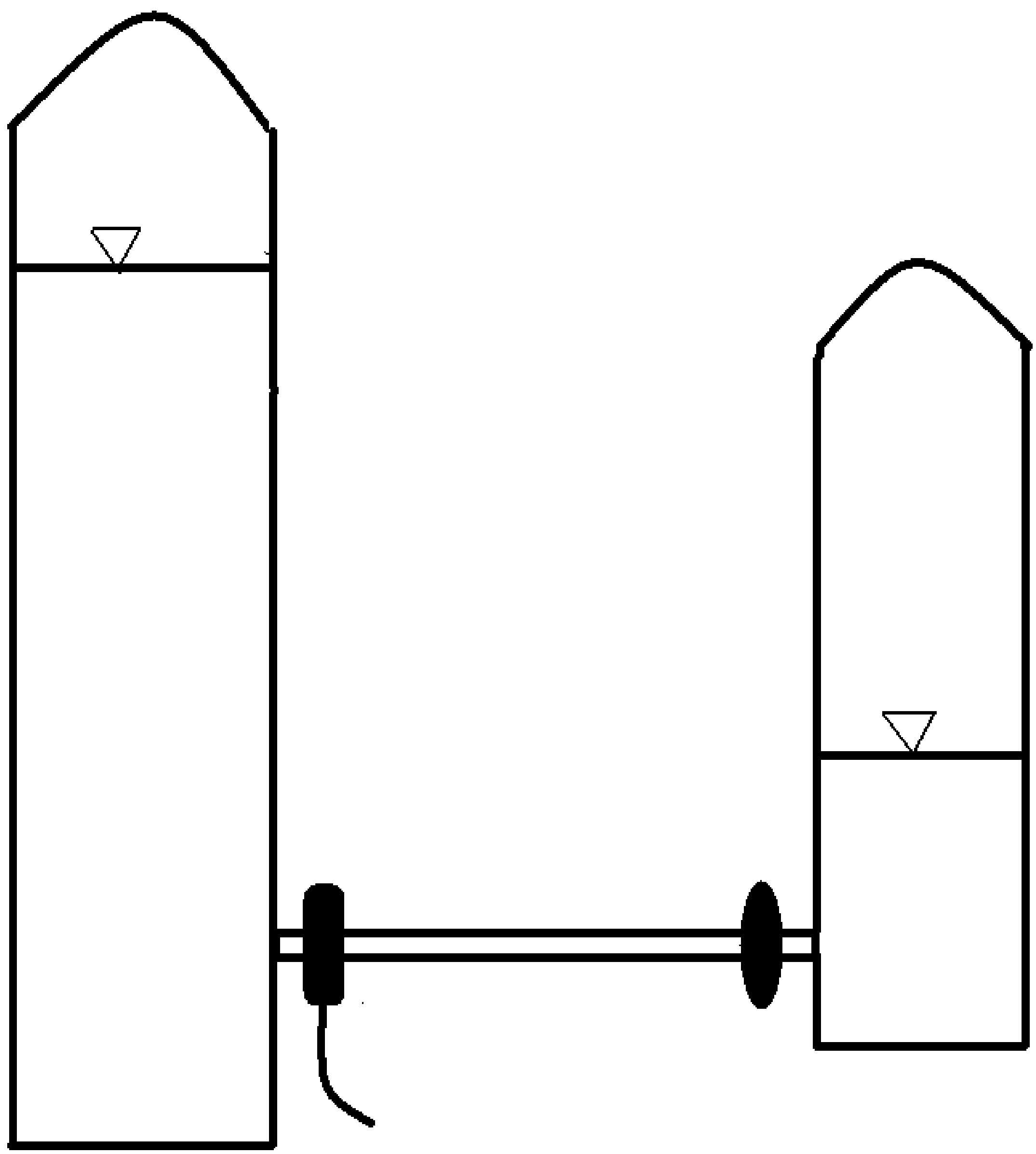
The sketch on the next page shows two water tanks connected by a pipe. The pipe is 100m long and its diameter is 0.1m. There is a pump at the upstream end of the pipe and a valve at the downstream end. At the start, the valve is open, and the pressure in the pipe is 100 BAR and the flow speed is zero. The wave speed is 1000 m/s. The density of water is 1000 kg/m^3. The pump characteristic is:

$$U = A + B \, \mathrm{Cos}[2\pi t/T] \qquad T=0.4\mathrm{s}$$

A) Determine the pressure and the velocity at the ends of the pipe for 8 pipe transit times following a sudden pump start up with the valve open. For this case, let A be 2m/s and let B be 1m/s. [40]

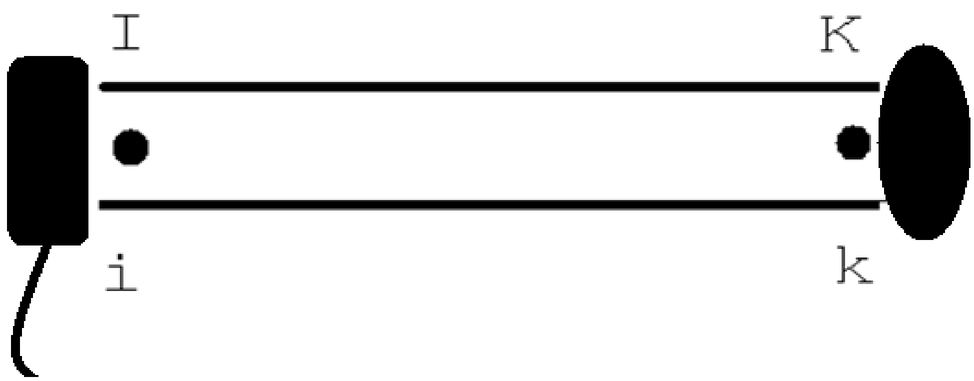
B) Determine the pressure and the velocity at the ends of the pipe for 6 pipe transit times following a sudden pump start up with the valve closed. For this case, let A be 2m/s and let B be 0m/s. [20]

C) For the valve open case, determine the pressure and the velocity at the ends of the pipe for 2 pipe transit time when friction factor f is 0.02. [10]



PART A: PUMP START UP WITH VALVE OPEN

The sketch below shows the setup. In the sketch upper case letters indicate NEW while lower case letters indicate OLD.



An f wave from upstream to downstream gives:

$$P_K - P_i = - \rho a \, [U_K - U_i]$$

For a wide open valve case, P_K is 100BAR.

An F wave from downstream to upstream gives:

$$P_I - P_k = + \rho a \, [U_I - U_k]$$

The pump equation gives

$$U_I = A + B \, Cos[2\pi t/T]$$

STEP #1

$P_k=100$ $U_k=0$ $U_i=0$ $P_i=100$

$P_K=100$ $U_K=U_i-[P_K-P_i]/\rho a = 0$

$U_I=A+BCos[2\pi t/T]=2+1=3$

$P_I=P_k+\rho a[U_I-U_k]=100+10*3=130$

STEP #2

$P_k=100$ $U_k=0$ $U_i=3$ $P_i=130$

$P_K=100$ $U_K=U_i-[P_K-P_i]/\rho a=3+3=6$

$U_I=A+BCos[2\pi t/T]=2+0=2$

$P_I=P_k+\rho a[U_I-U_k]=100+10*2=120$

STEP #3

$P_k=100$ $U_k=6$ $U_i=2$ $P_i=120$

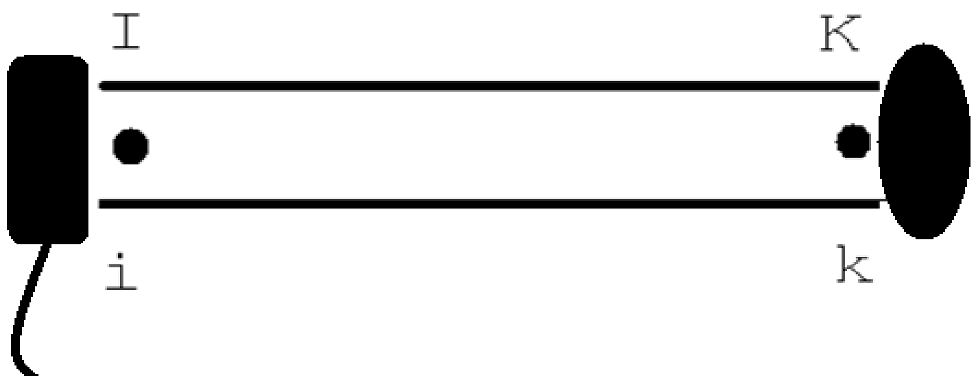
$P_K=100$ $U_K=U_i-[P_K-P_i]/\rho a=2+2=4$

$U_I=A+BCos[2\pi t/T]=2-1=1$

$P_I=P_k+\rho a[U_I-U_k]=100-10*5=50$

PART B: PUMP START UP WITH VALVE CLOSED

The sketch below shows the setup. In the sketch upper case letters indicate NEW while lower case letters indicate OLD.



An f wave from upstream to downstream gives:

$$P_K - P_i = - \rho a \, [U_K - U_i]$$

For a closed valve case, U_K is zero.

An F wave from downstream to upstream gives:

$$P_I - P_k = + \rho a \, [U_I - U_k]$$

The pump equation gives

$$U_I = A + B \, Cos[2\pi t/T]$$

STEP #1

$U_k=0$ $P_k=100$ $U_i=0$ $P_i=100$

$U_K=0$ $P_K=P_i-\rho a[U_K-U_i]=100$

$U_I=A+BCos[2\pi t/T]=2$

$P_I=P_k+\rho a[U_I-U_k]=100+10*2=120$

STEP #2

$U_k=0$ $P_k=100$ $U_i=2$ $P_i=120$

$U_K=0$ $P_K=P_i-\rho a[U_K-U_i]=140$

$U_I=A+BCos[2\pi t/T]=2$

$P_I=P_k+\rho a[U_I-U_k]=100+10*2=120$

STEP #3

$U_k=0$ $P_k=140$ $U_i=2$ $P_i=120$

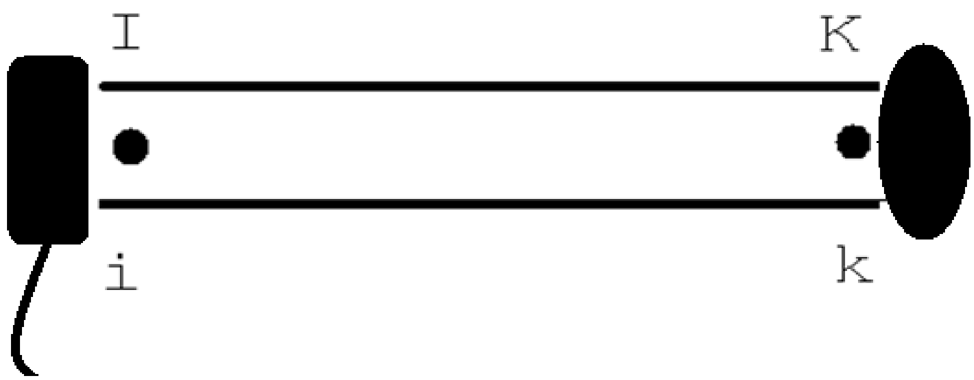
$U_K=0$ $P_K=P_i-\rho a[U_K-U_i]=140$

$U_I=A+BCos[2\pi t/T]=2$

$P_I=P_k+\rho a[U_I-U_k]=140+10*2=160$

PART C: PUMP START UP WITH FRICTION

The sketch below shows the setup. In the sketch upper case letters indicate NEW while lower case letters indicate OLD.



An f wave from upstream to downstream gives:

$$P_K - P_i = - \rho a \, [U_K - U_i] - \rho a \, \Delta t \, C$$

For a wide open valve case, P_K is 100BAR.

An F wave from downstream to upstream gives:

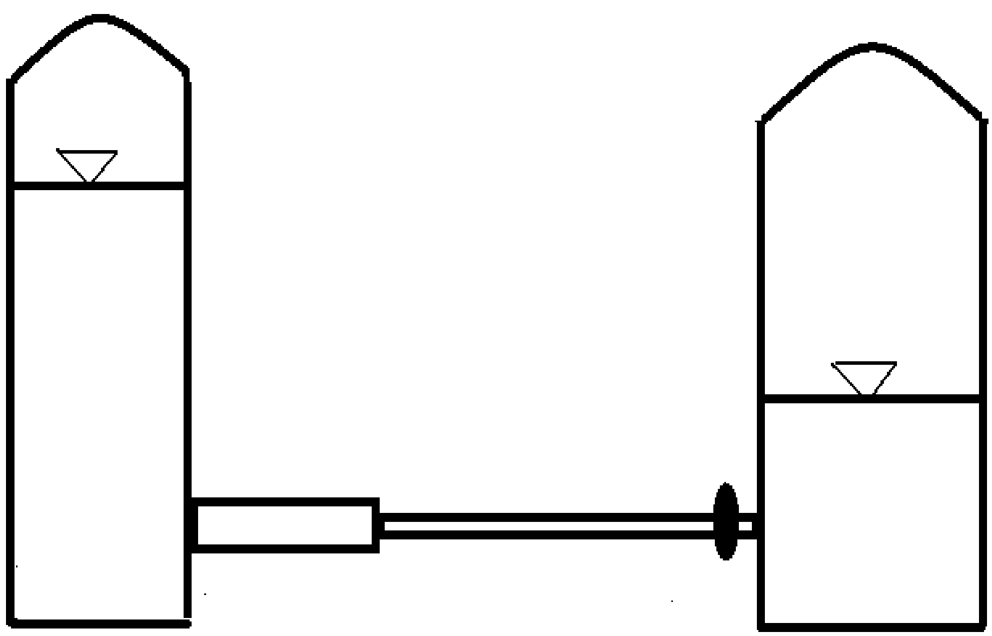
$$P_I - P_k = + \rho a \, [U_I - U_k] + \rho a \, \Delta t \, C$$

The pump equation gives

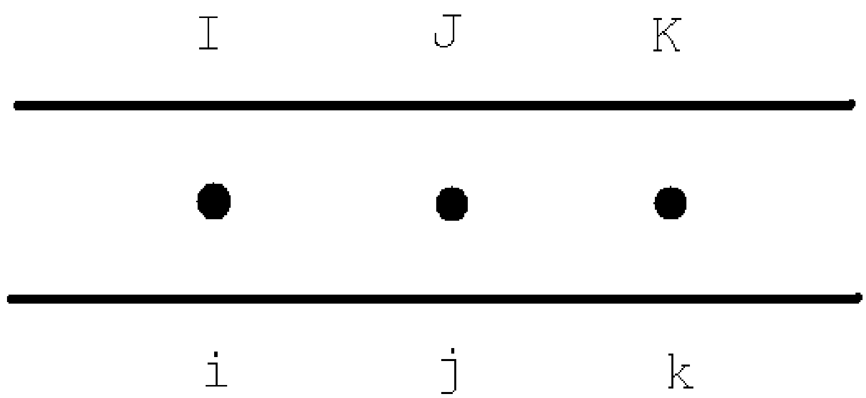
$$U_I = A + B \, Cos[2\pi t/T]$$

The sketch below shows two water tanks connected by two pipes in series. The diameter of the downstream pipe is half that of the upstream pipe. The length of the downstream pipe is twice that of the upstream pipe. The wave speed for the downstream pipe is half that of the upstream pipe. There is a valve at the downstream end of the pipes. At the start, the valve is open and the pressure in the pipes is P_O and the flow speed is U_O.

Write down equations for pressure and velocity in the network following a sudden valve closure. [30]



The downstream pipe is twice as long as the upstream pipe, and it has a wave speed that is half that of the upstream pipe. This means that its transit time is 4 times that of the upstream pipe. This implies it must be divided into 4 reaches. The sketch below shows a typical reach point:



Water hammer analysis gives for a reach point:

$$P_J = (P_k+P_i)/2 - [\rho a][U_k-U_i]/2$$
$$U_J = (U_k+U_i)/2 - [P_k-P_i]/[2\rho a]$$

The junction between the pipes is quite simple. Let the positive direction for each pipe be down the pipe. In this case, water hammer analysis gives for the junction pressure:

$$P_J = [X - Y] / Z$$

where

$$X = [\ A_W/a_W\ P_v + A_N/a_N\ P_m]$$

$$Y = \rho\ [\ A_W U_v - A_N U_m\ ]$$

$$Z = [\ A_W/a_W + A_N/a_N\ ]$$

The junction velocities are:

$$U_N = U_m - [P_J - P_m]/[\rho a_W]$$

$$U_W = U_v + [P_J - P_v]/[\rho a_X]$$

An f wave for the downstream pipe gives:

$$P_K - P_i = -\ \rho a\ [U_K - U_i]$$

For a closed valve, U_K is zero.

An F wave for the upstream pipe gives:

$$P_I - P_k = +\ \rho a\ [U_I - U_k]$$

For a tank, P_I is P_O.

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QUIZ #1 2013

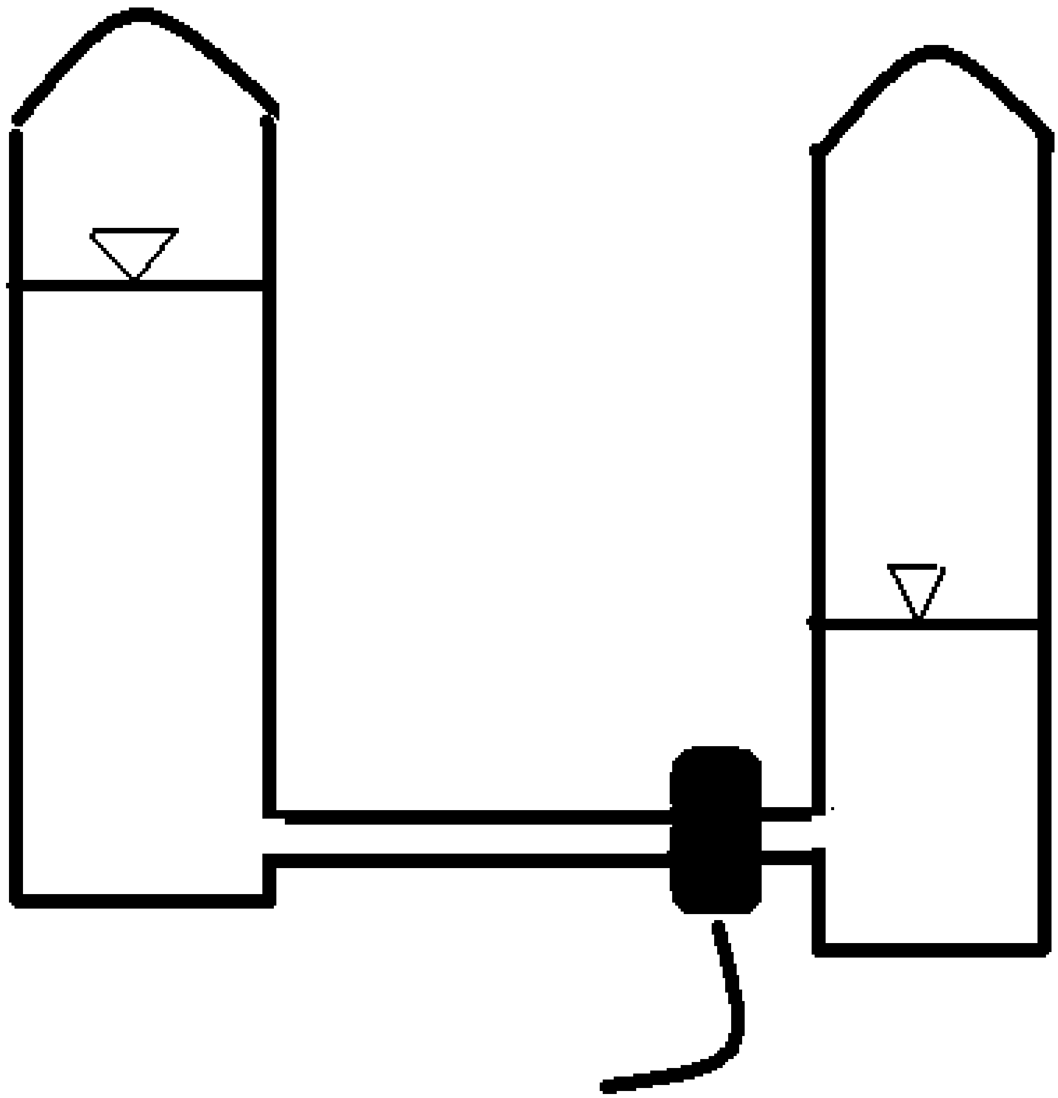
The sketch on the next page shows two water tanks connected by a pipe. The pipe is 100m long and its diameter is 0.1m. There is a pump at the downstream end of the pipe. At the start, the pressure in the pipe is 100 BAR and the flow speed is zero. The wave speed is 1000 m/s. The density of water is 1000 kg/m^3. The pump characteristic is:

$$U = A + B \cos[2\pi t/T] \qquad T=0.4s$$

A) Determine the pressure and the velocity at the ends of the pipe for 8 pipe transit times following a sudden pump start. For this case, let A be 2m/s and let B be 1m/s. [40]

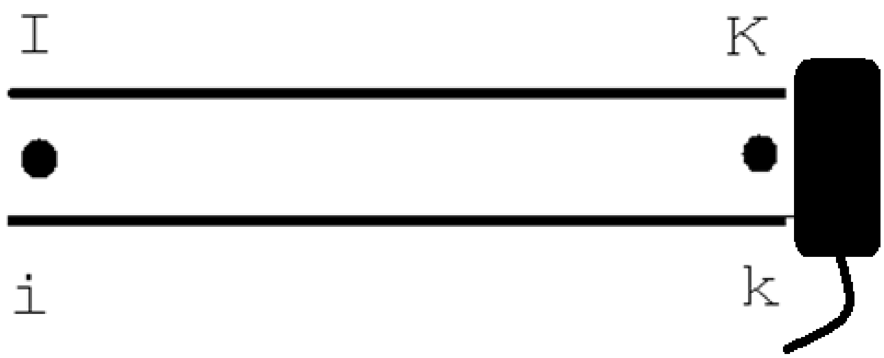
B) Determine the pressure and the velocity at the ends of the pipe for 6 pipe transit times following a sudden pump start up if the upstream end of the pipe is a dead end. For this case, let A be 2m/s and let B be 0m/s. [20]

C) Determine the pressure and the velocity at 3 points in the pipe for 2 pipe transit time. [10]



PART A: PUMP START UP WITH OPEN END

The sketch below shows the setup. In the sketch upper case letters indicate NEW while lower case letters indicate OLD.



An f wave from upstream to downstream gives:

$$P_K - P_i = - \rho a \ [U_K - U_i]$$

The pump equation gives

$$U_K = A + B \ Cos[2\pi t/T]$$

An F wave from downstream to upstream gives:

$$P_I - P_k = + \rho a \ [U_I - U_k]$$

For a tank upstream, P_I is 100BAR.

STEP #1

$P_i=100$ $U_i=0$ $U_k=0$ $P_k=100$

$P_I=100$ $U_I=U_k+[P_I-P_k]/\rho a=0$

$U_K=A+BCos[2\pi t/T]=3$

$P_K=P_i-\rho a[U_K-U_i]=70$

STEP #2

$P_i=100$ $U_i=0$ $U_k=3$ $P_k=70$

$P_I=100$ $U_I=U_k+[P_I-P_k]/\rho a=3+3=6$

$U_K=A+BCos[2\pi t/T]=2$

$P_K=P_i-\rho a[U_K-U_i]=100-10*2=80$

STEP #3

$P_i=100$ $U_i=6$ $U_k=2$ $P_k=80$

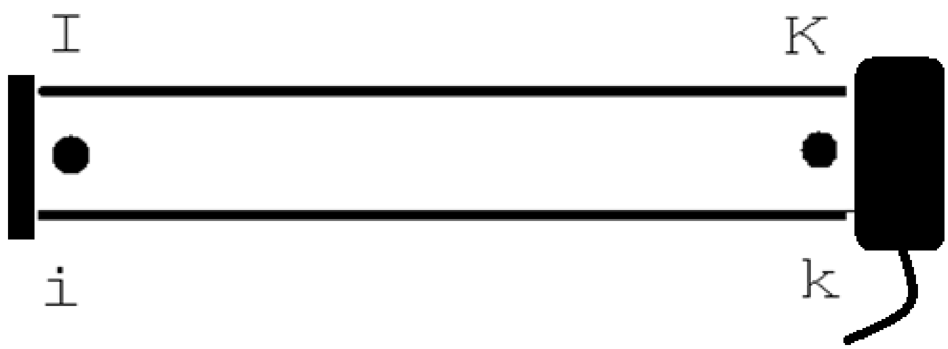
$P_I=100$ $U_I=U_k+[P_I-P_k]/\rho a=2+2=4$

$U_K=A+BCos[2\pi t/T]=1$

$P_K=P_i-\rho a[U_K-U_i]=100+10*5=150$

PART B: PUMP START UP WITH DEAD END

The sketch below shows the setup. In the sketch upper case letters indicate NEW while lower case letters indicate OLD.



An f wave from upstream to downstream gives:

$$P_K - P_i = - \rho a \, [U_K - U_i]$$

The pump equation gives

$$U_K = A + B \, \mathrm{Cos}[2\pi t/T]$$

An F wave from downstream to upstream gives:

$$P_I - P_k = + \rho a \, [U_I - U_k]$$

For a dead end upstream, U_I is zero.

STEP #1

$U_i=0 \quad P_i=100 \qquad U_k=0 \quad P_k=100$

$U_I=0 \quad P_I=P_k+\rho a[U_I-U_k]=100+0=100$

$U_K=A+BCos[2\pi t/T]=2$

$P_K=P_i-\rho a[U_K-U_i]=100-10*2=80$

STEP #2

$U_i=0 \quad P_i=100 \qquad U_k=2 \quad P_k=80$

$U_I=0 \quad P_I=P_k+\rho a[U_I-U_k]=80-10*2=60$

$U_K=A+BCos[2\pi t/T]=2$

$P_K=P_i-\rho a[U_K-U_i]=100-10*2=80$

STEP #3

$U_i=0 \quad P_i=60 \qquad U_k=2 \quad P_k=80$

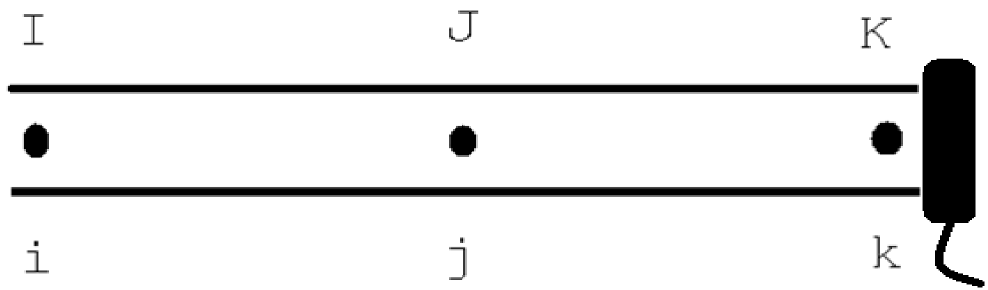
$U_I=0 \quad P_I=P_k+\rho a[U_I-U_k]=80-10*2=60$

$U_K=A+BCos[2\pi t/T]=2$

$P_K=P_i-\rho a[U_K-U_i]=60-10*2=40$

PART C: PUMP START UP WITH 3 POINTS IN PIPE

The sketch below shows the setup. In the sketch upper case letters indicate NEW while lower case letters indicate OLD.



For the downstream end

$$P_K - P_j = - \rho a \, [U_K - U_j]$$

$$U_K = A + B \, \mathrm{Cos}[2\pi t/T]$$

For the upstream end

$$P_I - P_j = + \rho a \, [U_I - U_j]$$

$$P_I = 100\mathrm{BAR} \text{ or } U_I = 0$$

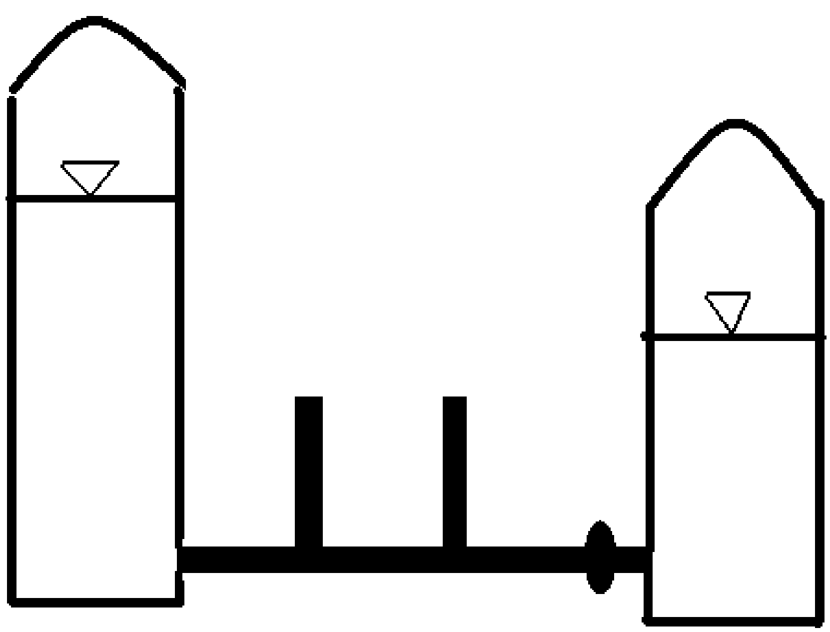
At the reach point

$$P_J = (P_k+P_i)/2 - [\rho a][U_k-U_i]/2$$

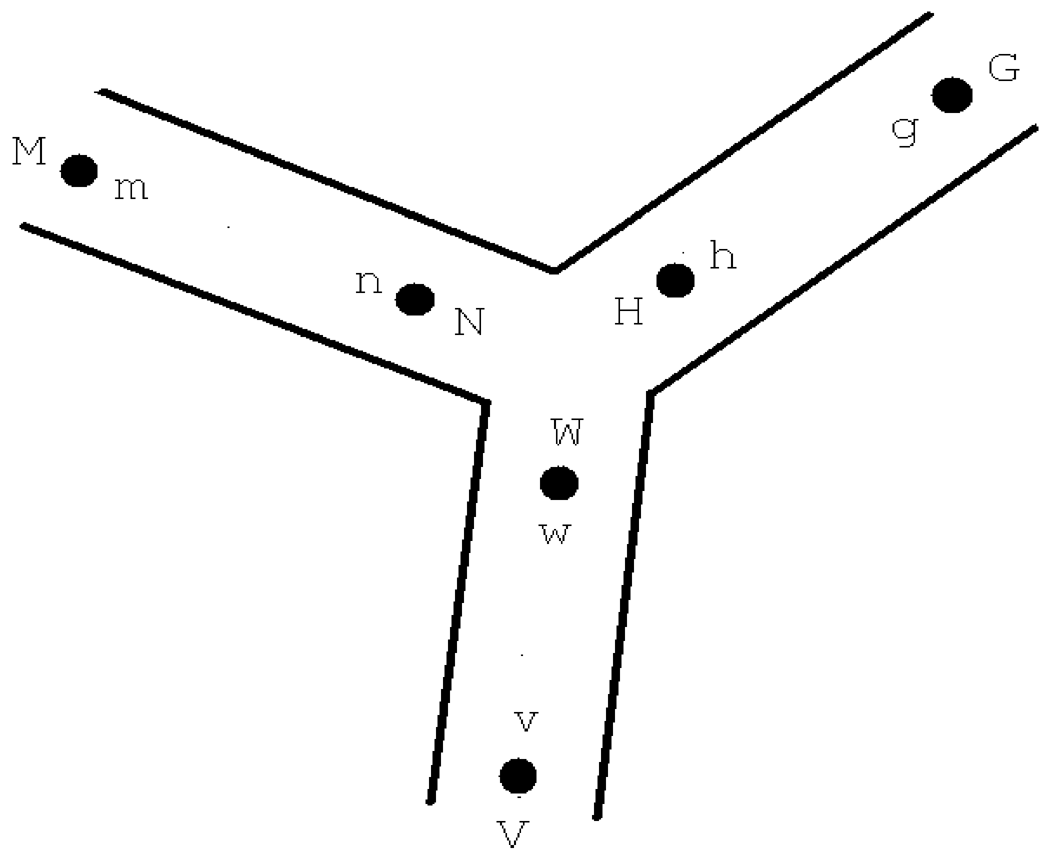
$$U_J = (U_k+U_i)/2 - [P_k-P_i]/[2\rho a]$$

The sketch below shows two water tanks connected by a pipe with 2 junctions. Dead end pipes are attached at the junctions. There is a valve at the downstream end of the pipe. At the start, the valve is partially open and the pressure in the pipes is P_O and the flow speed is U_O. Each section of pipe has the same length, diameter and wave speed.

Write down equations for the pressure and the velocity at the end of the pipes in the network following a sudden valve closure for the case where pipe friction is zero. [20] Outline how you would include friction into the analysis. [10]



The sketch below shows a typical junction.



Water hammer analysis gives for each junction:

$$P_J = [X - Y] / Z$$

where

$$X = [\ A_H/a_H\ P_g + A_W/a_W\ P_v + A_N/a_N\ P_m]$$

$$Y = \rho\ [\ A_H U_g + A_W U_v - A_N U_m\ ]$$

$$Z = [\ A_H/a_H + A_W/a_W + A_N/a_N\ ]$$

The junction velocities are:

$$U_N = U_m - [P_J - P_m]/[\rho a_W]$$

$$U_H = U_g + [P_J - P_g]/[\rho a_E]$$

$$U_W = U_v + [P_J - P_v]/[\rho a_X]$$

An f wave for the downstream pipe gives:

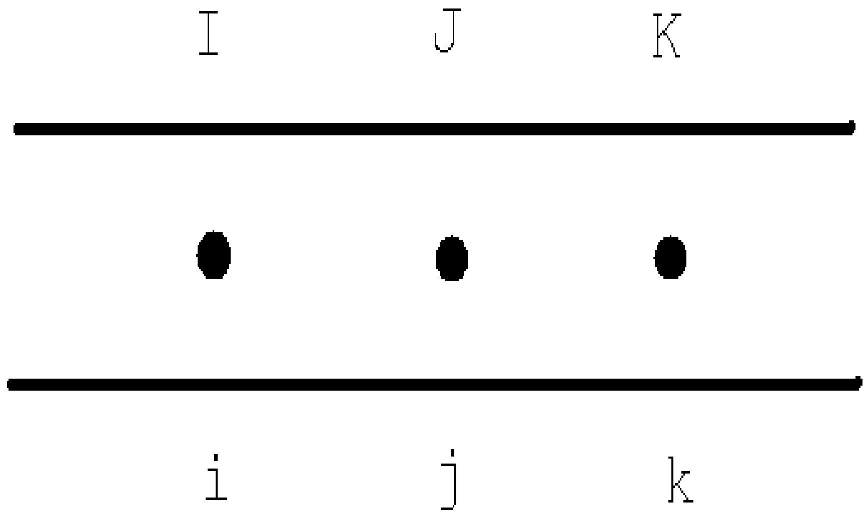
$$P_K - P_i = - \rho a\ [U_K - U_i]$$

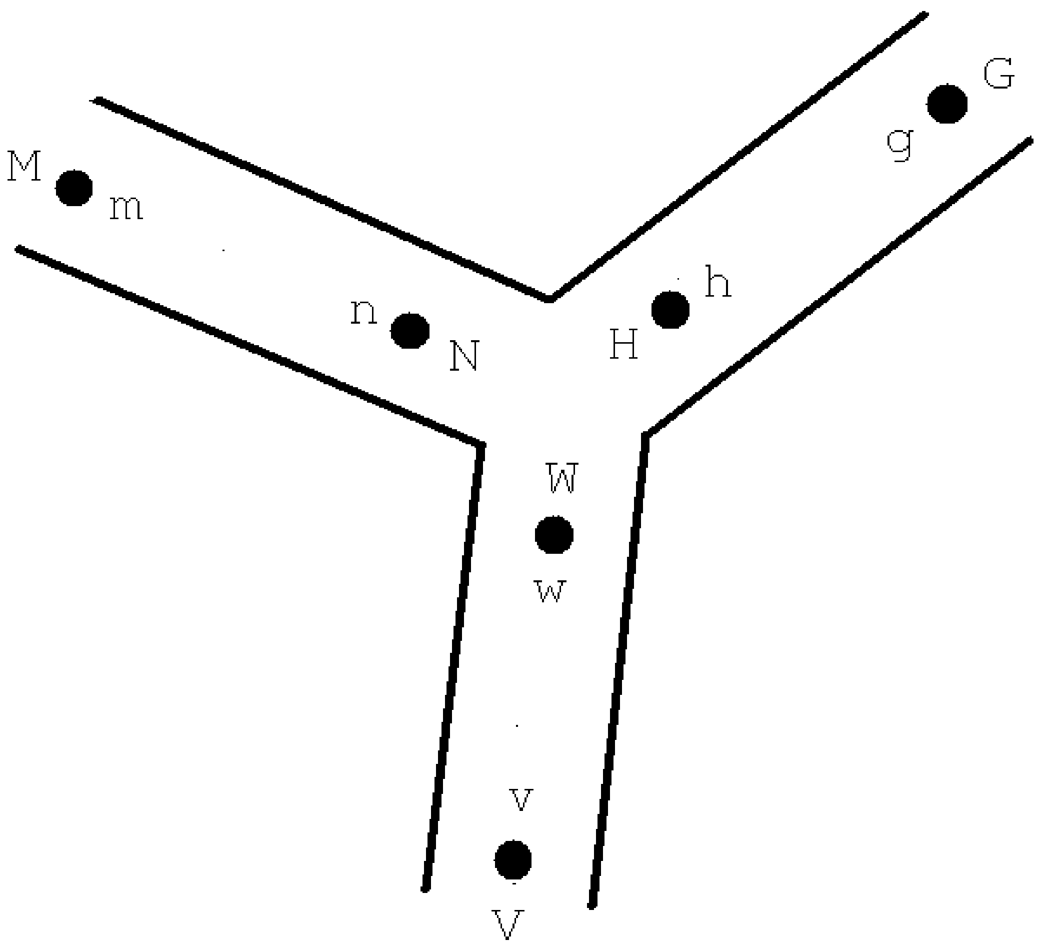
For a closed valve, U_K is zero.

An F wave for the upstream pipe gives:

$$P_I - P_k = + \rho a\ [U_I - U_k]$$

For a tank, P_I is P_O.


I
J
K
i
j
k


G
g
M
m
h
n
H
N
W
w
v
V

$$\Delta P = - \rho a \, \Delta U \qquad P_K - P_i = - \rho a \, [U_K - U_i]$$

$$\Delta P = + \rho a \, \Delta U \qquad P_I - P_k = + \rho a \, [U_I - U_k]$$

$$\Delta P = - \rho a \, \Delta U - \Delta t \, \rho a \, C$$

$$\Delta P = + \rho a \, \Delta U + \Delta t \, \rho a \, C$$

$$C = f/D \, U|U|/2$$

$$P_J = (P_k+P_i)/2 - [\rho a][U_k-U_i]/2$$

$$U_J = (U_k+U_i)/2 - [P_k-P_i]/[2\rho a]$$

$$P_J = [X - Y] / Z$$

$$X = [A_H/a_H \, P_g + A_W/a_W \, P_v + A_N/a_N \, P_m]$$

$$Y = \rho \, [A_H U_g + A_W U_v + A_N U_m]$$

$$Z = [A_H/a_H + A_W/a_W + A_N/a_N]$$

$$U_N = U_m + [P_J-P_m]/[\rho a_W]$$

$$U_H = U_g + [P_J-P_g]/[\rho a_E]$$

$$U_W = U_v + [P_J-P_v]/[\rho a_X]$$

$$a = \sqrt{[\mathbf{K}/\rho]}$$

$$\mathbf{K} = K/[1+[DK]/[Ee]]$$

$$a_M = \sqrt{[K_M/\rho_M]}$$

$$\rho_M = \Sigma[\rho_C V_C]/V_M$$

$$K_M = V_M/\Sigma[V_C/K_C]$$