Wave Scattering

Differential Formulation

Governing Equations

Conservation of Mass

$$\nabla^2 \phi = 0$$

within fluid,

Potential Components

$$\phi = \phi_I + \phi_S$$

Incident Wave ϕ_I

Scattered Wave ϕ_S

Seabed Constraint

$$\frac{\partial \phi}{\partial n} = 0$$

Water Surface Constraints

Kinematic Constraint

$$\frac{\delta \eta}{\partial t} = \frac{\delta \phi}{\delta z}$$

Dynamic Constraint

$$\frac{\partial \phi}{\partial t} + g\eta = 0$$

$$\frac{\partial^2 \phi}{\partial t^2} + g\frac{\partial \phi}{\partial z} = 0$$

Structure Constraint

$$\frac{\partial \phi}{\partial n} = 0$$

Radiation Condition?

One would expect that at an infinite distance from the

Structure scattered waves would be found moving radially away from it. One would not expect scattered waves to be moving in towards structure.

Incoming waves are mathematically possible. Radiation condition makes only outgoing waves possible.

$$\frac{\partial \phi_s}{\partial t} + c_p \frac{\partial \phi_s}{\partial r} = 0$$

Integral Formulation

Consider a fluid region R with volume V surrounded by a surface S. For ideal fluid the flow within R is taken to be irrotational

Such that a scalar ϕ exists which satisfies

$$\nabla^2 \phi = 0$$

within R. Let there be another scalar ϕ^* which also satisfies

$$\nabla^2 \phi^* = 0$$

within R. Now consider

$$\int_S \left[\phi \frac{\partial \phi^*}{\partial n} - \phi^* \frac{\partial \phi}{\partial n} \right] ds$$

Manipulation gives

$$\int_S \left[\phi \nabla \phi^* - \phi^* \nabla \phi \right] \cdot \vec{n}\, ds$$

Gauss' Theorem gives

$$\int_V \nabla \cdot \left[\phi \nabla \phi^* - \phi^* \nabla \phi \right] dV$$

Expansion gives

$$\int_{V} \left[\phi \nabla^2 \phi^* + \nabla\phi \cdot \nabla\phi^* - \phi^* \nabla^2 \phi - \nabla\phi^* \cdot \nabla\phi \right] dV$$

$$= \int_{V} \left(\phi \nabla^2 \phi^* - \phi^* \nabla^2 \phi \right) dV = 0$$

This implies

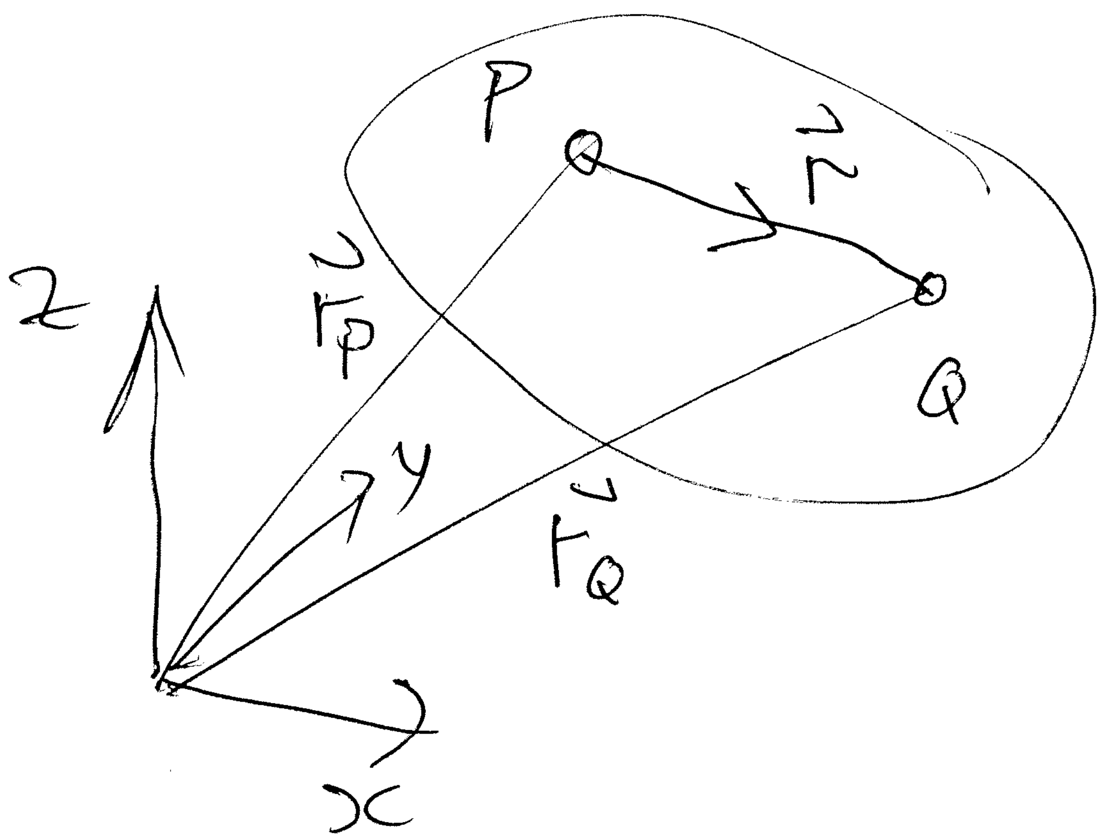
$$\int_{S} \left[\phi \frac{\partial \phi^*}{\partial n} - \phi^* \frac{\partial \phi}{\partial n} \right] dS = 0$$

Duplications

When the surface integral is zero mass is conserved within R.

Let $\phi^* = \frac{1}{r}$ where

$$\vec{r} = \vec{r}_Q - \vec{r}_P$$

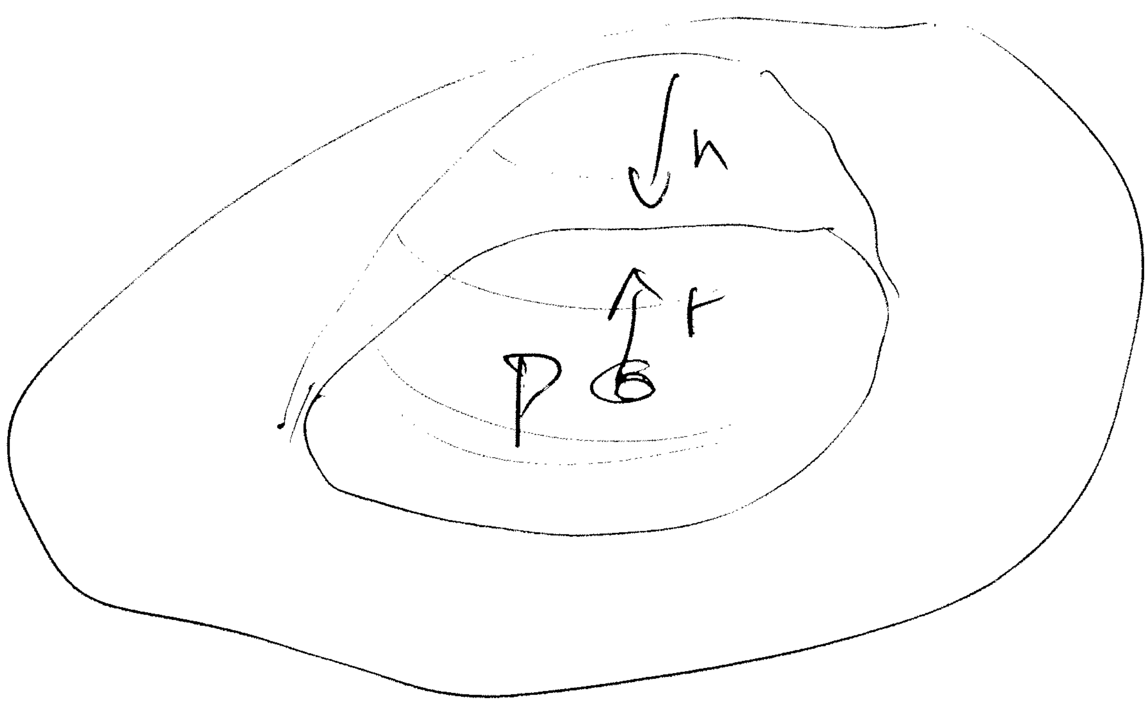


where P and Q are points on the surface. To evaluate the integral P is set and Q is allowed to move over the surface. The surface integral becomes

$$\int_S \left[\phi(Q) \frac{\partial 1/r}{\partial n} - \frac{1}{r} \frac{\partial \phi(Q)}{\partial n} \right] ds = 0$$

Care must be taken when Q approaches P because P is a singular point: ϕ^* goes to infinity as r goes to zero.

A small hemisphere is used to indent the surface to make P external.



The hemisphere radius is allowed to go to zero after the integration over it is completed. For the hemisphere

$$\frac{\partial}{\partial n} = -\frac{\partial}{\partial r}$$

Its surface area is

$2\pi r^2$,

So for the hemisphere

$$\int_S \phi(Q) \frac{\partial 1/r}{\partial n} ds \hat{=} \phi(P) \frac{1}{r^2} 2\pi r^2$$

$$= 2\pi \phi(P)$$

$$\int_S \frac{1}{r} \frac{\partial \phi(Q)}{\partial n} ds \hat{=} \frac{1}{r} \frac{\partial \phi(P)}{\partial n} 2\pi r^2$$

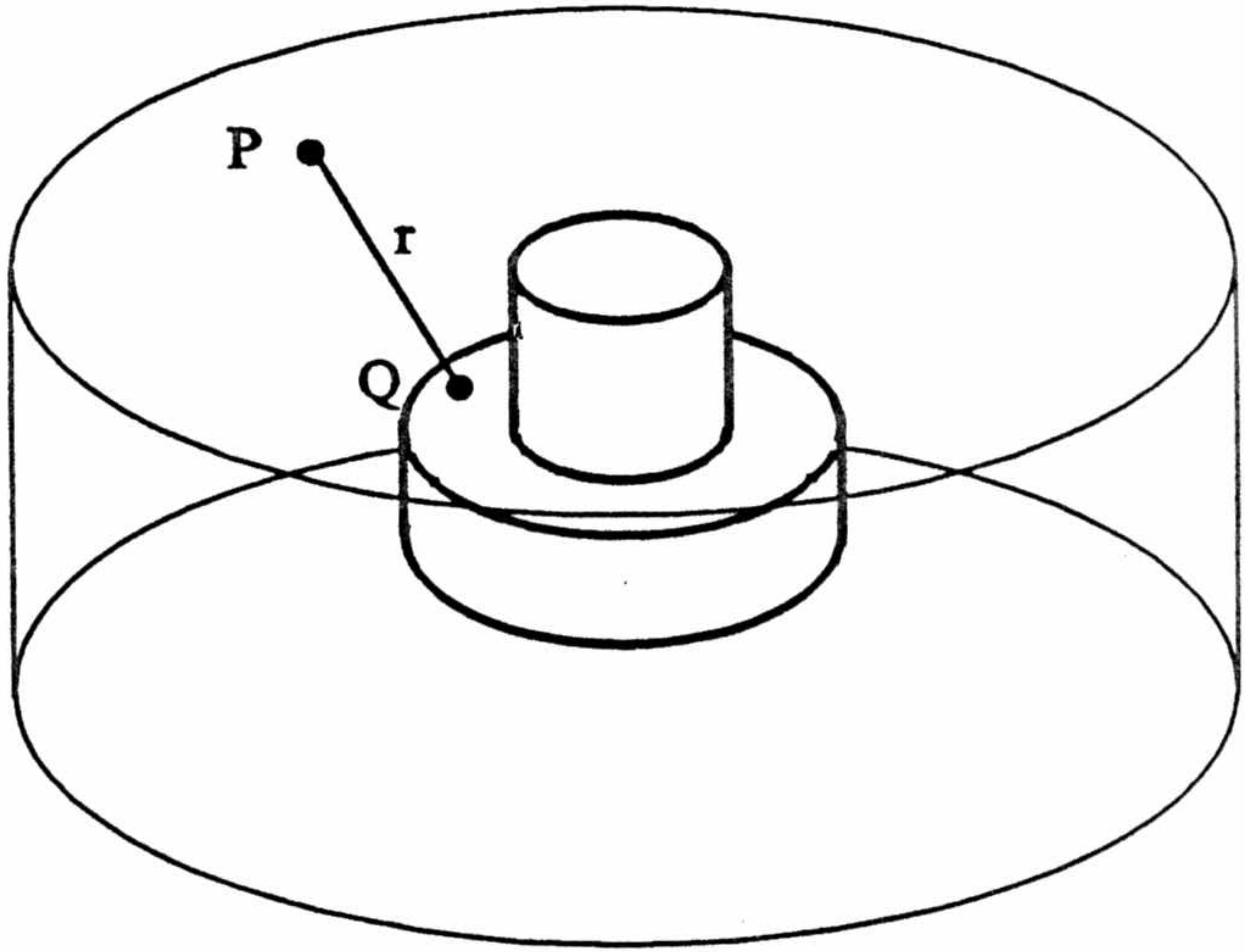
$$= 0$$

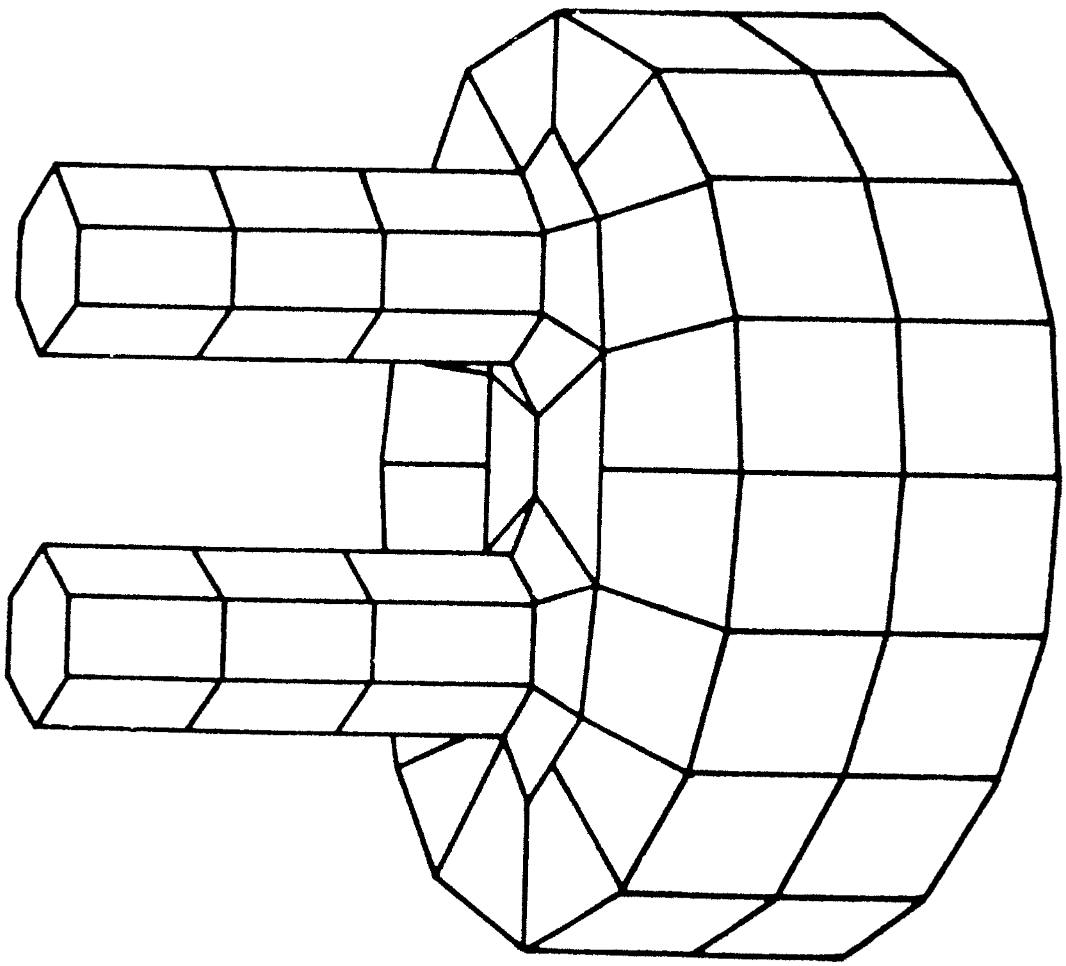
So overall integral becomes

$$2\pi\phi(P)+\int_{S^*}\left[\phi(Q)\frac{\partial 1/r}{\partial n}-\frac{1}{r}\frac{\partial\phi(Q)}{\partial n}\right]ds=0$$

Manipulation gives

$$\phi(P)=\frac{1}{2\pi}\int_{S^*}\left[\frac{1}{r}\frac{\partial\phi(Q)}{\partial n}-\phi(Q)\frac{\partial 1/r}{\partial n}\right]ds$$


P
r
Q



Panel Method Steps 2
Wave Load Case

(1) Discretize Surface
surrounding fluid
with N nonoverlapping
facets or panels.
Handout

② Use constraints to replace $\frac{\partial \phi(Q)}{\partial n}$ terms in integral.

③ Assume that over each panel integrand is constant.

$$\phi(p) = \frac{1}{2\pi} \sum \left[\frac{1}{r} \frac{\partial \phi(q)}{\partial n} - \phi(q) \frac{\partial \, 1/r}{\partial n} \right] \Delta s$$

4. For each panel let

$$\phi = \phi_I + \phi_S$$

$$\phi_I = A \sin \omega t + B \cos \omega t$$

$$\phi_S = a \sin \omega t + b \cos \omega t$$

Known A & B Unknowns a & b

(5) Substitute into each sum to get an equation system of the form

$$I \sin \omega t + J \cos \omega t = 0$$

$$I = 0 \qquad J = 0$$

6 Solve equation system to get the a and b for each panel.

7 Use Unsteady Bernoulli to get pressure and integration to get load.