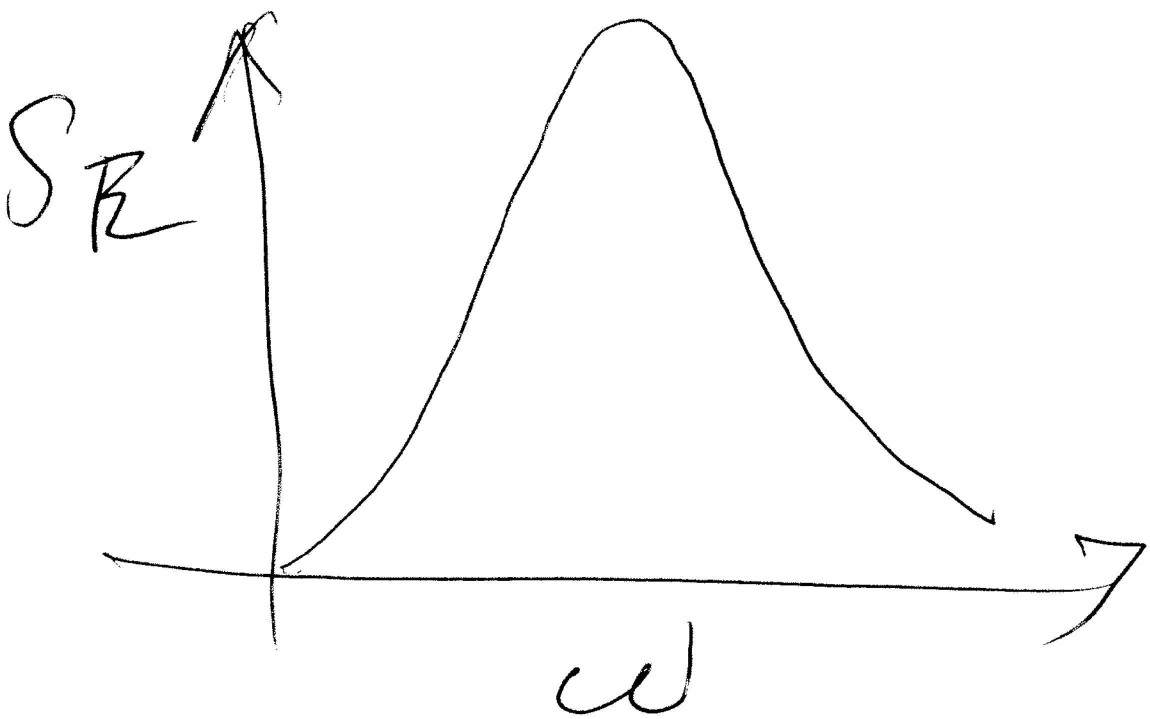
Wave Spectra

An infinite number of waves are present in a random or storm sea state. Each wave has an infinitesimal amplitude and a different frequency.

A wave energy spectrum shows how the energy is spread over a range of frequencies.



By definition the energy per unit water surface area is equal to the area under the curve.

The energy in a small range $\delta\omega$ centered around frequency ω is

$$S_E(\omega)\delta\omega = \frac{1}{2}\rho g \eta_0^2 = \frac{1}{8}\rho g H^2$$

One can also define
a wave amplitude spectrum
$S_\eta(\omega)$ and a wave
height spectrum $S_H(\omega)$

$$S_\eta(\omega)\,\delta\omega = \eta_o^2$$
$$S_H(\omega)\,\delta\omega = H^2$$

These can be obtained
directly from wave

profile data using either a Band Pass Filter or a Fast Fourier Transform.

The shape of a Spectrum depends on the strength duration and fetch of the wind which generated it.

Most fits to spectral data are a function of at least two parameters: the significant wave height H_s and the significant wave period T_s. If N wave height measurements are ordered in a list from largest to smallest the

Significant wave height is the average of the first N/3 measurements in list.

In a more physical sense Hs is roughly an average as sensed by an observer.

Ts is the period associated with Hs. For a storm

Sea state typical values of H_s and T_s are,

$H_s = 5\,m \quad T_s = 10\,s$

Some two parameter fits are Rayleigh and the International Towing Tank Conference or ITTC fit.

eg ITTC Pkt

$$S_\eta(\omega) = \frac{A}{\omega^5} e^{-\frac{B}{\omega^4}}$$

$$A = \frac{346 H_s^2}{T_s^4} \quad B = \frac{691}{T_s^4}$$

Some three parameter

fits are the Joint North Sea Wave Project or JONSWAP fit and the Pierson-Moskowitz or PM Fit. The extra parameter accounts for the influence of fetch.

Statistical information can be obtained from spectral data. The theory for this assumes that the data follows a Rayleigh Distribution and it makes use of various moments of the distribution. For a wave amplitude spectrum these moments are given by

$$M_n = \frac{1}{2} \int_0^\infty S_m(\omega)\, \omega^n \, d\omega$$

It turns out that

$$H_s = 4\sqrt{M_0} \quad T_s = 2\pi \frac{M_0}{M_1}$$

where M_0 & M_1 are

$$M_0 = \frac{1}{2}\int_0^\infty S_\eta(\omega)\, d\omega$$

$$M_1 = \frac{1}{2}\int_0^\infty S_\eta(\omega)\, \omega\, d\omega$$

Various probabilities can also be obtained. For example the probability of M_0

exceeding some level M_o is

$$P(M_o > M_o) = e^{-\frac{M_o^2}{2M_0}}$$

This implies that out of N waves

$$N e^{-\frac{M_o^2}{2M_0}}$$

would have an M_o greater than M_o.

Structure Spectra

When a floating body is being forced by a pure sinusoidal wave the amplitude Ro of its motion in a particular direction is related to the wave amplitude

η_0 by a response amplitude operator or RAO.

$$R_0/\eta_0 = RAO$$

Manipulation gives

$$\frac{R_0^2(\omega)/\delta\omega}{\eta_0^2(\omega)/\delta\omega} = RAO^2$$

Recall that

$$S_\eta(\omega)\,\delta\omega = \eta_o^2(\omega)$$

By analogy

$$S_R(\omega)\,\delta\omega = R_o^2(\omega)$$

Substitution into RAO^2 equation gives

$$\frac{S_R(\omega)}{S_\eta(\omega)} = RAO^2$$

$$S_R(\omega) = RAO^2 \, S_\eta(\omega)$$

Generally $S_R(\omega)$ is not a Rayleigh Distribution. It often has two peaks; one due to peak in S_m and another due to resonance in RAO.

Because of this

Statistics based on Rayleigh Distribution must be corrected.

Correction Factor $CF = \sqrt{1 - \varepsilon^2}$

Broadness Parameter $\varepsilon = \dfrac{M_0 M_4 - M_2^2}{M_0 M_4}$

Significant Amplitude of Motion

$R_S = R_S^{*}\ CF$ 5%

Significant Wave Height

$H_S = H_S^{*}\ CF$ 1%

* Rayleigh Distribution Estimates