Assignment 4 (Optional)

Algorithms: Correctness and Complexity

Due April 13 18:00.

This assignment is optional. If you submit it, your assignment grade will be based on the best 4 of the 5 assignments.

To be submitted via Brightspace. Please consider typing your assignment.

Q0 [10] Design a greedy algorithm for the following problem.

We are given a finite set of points P on the real number line $P \subset \mathbb{R}$, and a finite set of intervals $S \subset \mathbb{R} \times \mathbb{R}$. We say each interval (a, b) **covers** a point p if $a \leq p \leq b$. We wish to find a smallest subset of S that covers every point in P.

(a) [5] Show that this is an example of the schematic optimal subset selection problem which is in the notes.

(b) [5] Create a greedy algorithm for it by reifying the schematic greedy algorithm for optimal subset selection which is in the notes. Be clear about how you choose to define "suitable" and "best suitable". Note that greedily picking the interval that covers the most uncovered points does not work. Try working left to right.

(c) [optional] Use a cut and paste (fairy godmother) argument to show your algorithm is correct.

(d) [optional] Implement (c) in the programming language of your choice.

Q1 [20] In the ancient board game of Yegimeli Zeri, players win by moving their token to square 0. On each turn, they can move forward by 1, 2, or 5 spaces. For example, if the state is this

Direction of travel $\longrightarrow$

$\maltese$					
5	4	3	2	1	0

,

The White Knight, represented by the $\maltese$, can finish by making any of the following sequences of moves

$$\{[1,1,1,1,1], [1,1,1,2], [1,1,2,1], [1,2,1,1], [2,1,1,1], \\ [1,2,2], [2,1,2], [2,2,1], [5]\}$$

Landing on square i earns the player an integer number of points $points(i)$ (which could be negative, zero, or positive). The player must reach square 0 with a given number of moves or fewer. We wish to know the highest score that a player can make if they start on square n and have m moves left. For example suppose the points are as follows

10	1	7	-2	4	0
$\maltese$					
5	4	3	2	1	0

,

If $n = 5$ and $m \geq 4$, the best sequence is $[1,1,2,1]$ and scores 12 points — we say the value of the sequence is 12. If $n = 5$ and $m = 3$, the best sequence is $[2,2,1]$, with a value of 11 points. If $n = 5$ and $m \in \{1,2\}$ then the best sequence is $[5]$, with a value of 0 points. If $n = 5$ and $m = 0$, there is no best sequence, and the value of this position can be considered to be $-\infty$.

(a)[5] Design an inefficient recursive procedure to find the value of the best sequence from square n with m moves remaining.

(b)[5] Design a top-down dynamic programming algorithm for the same problem.

(c)[5] Design a bottom-up dynamic programming algorithm for the same problem.

(d)[5] Based on your answer to either (b) or (c), design an algorithm to compute the best sequence from square n with m moves remaining, if there is one. If there is no such path, the algorithm should return the special value Nil.

(e) [optional] Implement (d) in the programming language of your choice.