Assignment 1

Advanced Computing concepts for Engineering

Due Feb 12, 2015.

Note that the work that you turn in for this assignment must represent your individual effort. You are welcome to help your fellow students to understand the material of the course and the meaning of the assignment questions, however, the answer that you submit must be created by you alone.

Q0[10] Given $\Sigma = \{\text{“}w\text{”} \mapsto \mathbb{R}, \text{“}x\text{”} \mapsto \mathbb{R}, \text{“}y\text{”} \mapsto \mathbb{R}, \text{“}z\text{”} \mapsto \mathbb{R}\}$, implement the following specification using a sequence of (nonparallel) assignments.

$$\langle (x', y') = (x \times y, x + y) \rangle$$

Use forward substitution and erasure laws.

Q1[10] Use the alternation law to implement

$$\langle \{x', y'\} = \{x, y\} \wedge x' \leq y' \rangle$$

Q2.[25] Up edge. Given a constant function $C : \{0, .., N\} \xrightarrow{\text{tot}} \mathbb{B}$, an up edge is an index $i \in \{0, ..N\}$ such that $\neg C(i) \wedge C(i+1)$.To ensure an up edge exists, we will assume $\neg C(0)$ and $C(N)$. Consider the specification

$$f = \langle \mathcal{B} \rangle \text{ where } \mathcal{B} \text{ is } \neg C(i') \wedge C(i'+1)$$

Derive a solution using the binary search technique and the method of invariants. Make sure you use plenty of English prose as well as formal derivation to explain your solution. Here is a suggested outline: Propose an invariant $\mathcal{I}$. Find a command that refines $m = \langle \mathcal{I}' \rangle$. Propose a loop guard $\mathcal{A}$ so that $\mathcal{I} \wedge \neg \mathcal{A} \Rightarrow \widetilde{\mathcal{B}}$ is valid. (Show it is.). Find a loop body that implements $h = \langle \mathcal{A} \wedge \mathcal{I} \Rightarrow \mathcal{I}' \rangle$. (Show that it does.)

Q3[25] The log base 2

Use the method of invariants to derive an algorithm for the following specification where x and y are integer variables.

$$\left\langle y > 0 \Rightarrow z' = 2^{x'} \leq y < 2^{x'+1} \right\rangle$$