Assignment 0

Advanced Computing Concepts for Engineering

Due January 23, 2017

Note that the work that you turn in for this assignment must represent your individual effort. You are welcome to help your fellow students to understand the material of the course and the meaning of the assignment questions, however, the answer that you submit must be created by you alone.

Please consider preparing your assignment with a typesetting program such as TeX, LaTeX, LyX, Scientific Word, or MS Word.

Q0 [10] Propositional logic.

Using the laws that appear above them in the notes, prove the following distributivity laws

(a)[5] $(p \wedge q \Rightarrow r) = ((p \Rightarrow r) \vee (q \Rightarrow r))$

(b)[5] $(p \Rightarrow q \wedge r) = ((p \Rightarrow q) \wedge (p \Rightarrow r))$

Q1 [10] Substitutions

(a)[4] Underline all bound occurrences of variables in the following formulae. Circle all free occurrences of variables.

$$\{i \in \mathbb{N} \mid i < f(i) \cdot g(i)\}$$

$$(\forall x \in \mathbb{R} \cdot g(x) < f(y))$$

(b)[6] Make the following substitutions

$$\{i \in \mathbb{N} \mid i < f(i) \cdot g(i)\}\,[x, i, f : y, j, g]$$

$$(\forall x \in \mathbb{R} \cdot g(x) < f(y))\,[y : y + 1]$$

$$(\forall x \in \mathbb{R} \cdot g(x) < f(y))\,[y : x + 1]$$

Q2 [12] Quantifiers and sets

Ranter is a social network in which users sent out short messages called rants. Let U be the set of all users on the social network and let follows : $U \times U \xrightarrow{\text{tot}} \mathbb{B}$ be a boolean function expressing that the first user follows the second.

(a)[3] Use quantifiers to say that following is irreflexive, i.e., that no one follows themself.

(b)[3] If, for some users a and b, follows(a, b), we say that a is a degree 1 follower of b. If, for some users a, b,and c, a follows c and c follows b, we say that a is a degree 2 follower of b. And so on. Use quantifiers to say that a is a degree k follower of b. You may assume $k > 0$. [Hint: You may need to quantify over a function. Hint: check that the free variables of your expression are a, b, and k.]

(c)[3] Explain the meaning of the following expression in clear English

$$\forall x \in R \cdot \exists y \in S \cdot \text{follows}(x, y)$$

[Hint: check that the free variables of your English sentence are the same as the free variables of the expression.]

(d)[3] Explain the meaning of the following expression in clear English

$$\exists x \in R \cdot \forall y \in S \cdot \text{follows}(x, y)$$

[Hint: check that the free variables of your English sentence are the same as the free variables of the expression.]

Q3 [10] Refinement

(a)[5] Make a table of all (9) behaviours belonging to $\Sigma \dagger \Sigma$ where[1]

$$\Sigma = \{\text{“}x\text{”} \mapsto \{1, 2, 3\}\}$$

For each behaviour, indicate whether it is accepted (✓) or rejected (×) by

[1] $\Sigma \dagger \Sigma$ is $\{\text{“}x\text{”} \mapsto \{1, 2, 3\}, \text{“}x'\text{”} \mapsto \{1, 2, 3\}\}$

each of the following specifications (on $\Sigma \dagger \Sigma$)

$$\begin{aligned} a &= \langle x < 3 \Rightarrow x' < 3 \rangle \\ b &= \langle x < 2 \Rightarrow x' < 3 \rangle \\ c &= \langle x \leq 3 \Rightarrow x' < 3 \rangle \\ e &= \langle x < 3 \Rightarrow x' \leq 3 \rangle \\ f &= \langle x < 3 \Rightarrow x' < 2 \rangle \\ g &= \langle x < 3 \wedge x' < 3 \rangle \\ \textbf{magic} &= \langle \mathfrak{false} \rangle \\ \textbf{abort} &= \langle \mathfrak{true} \rangle \end{aligned}$$

(b) [5] What are the refinement relations between the specifications in part (a). Illustrate these relationships with a Hasse diagram.

Q4 [12] Specification

Write formal for the following informal specifications. In each case the signature is $\{\text{“}x\text{”} \mapsto \mathbb{N}, \text{“}x'\text{”} \mapsto \mathbb{N}\}$. Use angle bracket notation.

(a) x' is bigger than x.

(b) x' twice as big as x if x is smaller than 100; otherwise it doesn't matter what x' is.

(c) x' twice as big as x if x is smaller than 100; otherwise x' is 200.

(d) x is at least 1 and x' is smaller than x.